International Transmission of the Business Cycle and Environmental Policy - Supplementary Material

June 2019

Equilibrium Conditions in the No-Policy Model

In the absence of environmental policy $p_{E,t} = 0$ and $\mu_t = 0$. It follows that the equilibrium conditions of the model are:

$$C_t^{-\varphi_C} = \lambda_t, \tag{1}$$

$$\begin{aligned} q_t \;=\; & \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[r_{K,t+1} + \gamma_I \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} - \frac{\gamma_I}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 \right] \right\} + \\ & + \beta(1-\delta)\mathbb{E}_t \left(\frac{q_{t+1}\lambda_{t+1}}{\lambda_t} e^{u_{K,t+1}} \right), \end{aligned} \tag{2}$$

$$-\xi_L L_t^{\varphi L} + \lambda_t w_t = 0, \tag{3}$$

$$\gamma_I \left(\frac{I_t}{K_t} - \delta \right) = q_t - 1, \tag{4}$$

$$\frac{1}{R_t} = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1}} \right), \tag{5}$$

$$K_{t+1} = I_t + (1-\delta) e^{u_{K,t}} K_t, \tag{6}$$

$$r_{K,t} = \alpha \Psi_t \frac{Y_t^D}{K_t}, \tag{7}$$

$$w_t = (1-\alpha)\Psi_t \frac{Y_t^D}{L_t}, \tag{8}$$

$$Y_t^D = \Lambda_t A_t (e^{u_{K,t}} K_t)^\alpha L_t^{1-\alpha}, \tag{9}$$

$$(1-\sigma) + \sigma \Psi_t \frac{1}{p_t^D} - \gamma_P \left(\Pi_t^D - 1 \right) \Pi_t^D + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \gamma_P \left(\Pi_{t+1}^D - 1 \right) \left(\Pi_{t+1}^D \right)^2 \frac{Y_{t+1}^D}{Y_t^D} \frac{1}{\Pi_{t+1}} \right] = 0, \tag{10}$$

$$Y_t = [\kappa^{\frac{1}{\rho}} (Y_t^H)^{\frac{\rho-1}{\rho}} + (1-\kappa)^{\frac{1}{\rho}} (M_t)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}, \tag{11}$$

$$Y_t^H = \kappa Y_t \left(\frac{1}{p_t^D} \right)^\rho, \tag{12}$$

$$Y_t^D = Y_t^H + X_t, \tag{13}$$

$$M_t = Y_t (1-\kappa) \left(\frac{1}{p_t^{D*} S_t^R} \right)^\rho, \tag{14}$$

$$\Pi_t = \frac{p_{t-1}^D}{p_t^D} \Pi_t^D, \tag{15}$$

$$X_t = Y_t^*(1-\kappa)\left(\frac{S_t^R}{p_t^D}\right)^{\rho}, \tag{16}$$

$$p_t^D Y_t^D = C_t + I_t + p_t^D X_t - S_t^R p_t^{D*} M_t + \frac{\gamma_I}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + p_t^D \frac{\gamma_P}{2}(\Pi_t^D - 1)^2 Y_t^D, \tag{17}$$

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\iota_\Pi} e^{u_{R,t}}, \tag{18}$$

$$E_t = \epsilon (Y_t^D)^{1-\gamma}, \tag{19}$$

$$u_{K,t} = \rho_K u_{K,t-1} + \varepsilon_{K,t}, \tag{20}$$

$$u_{A,t} = \rho_A u_{A,t-1} + \varepsilon_{A,t}, \tag{21}$$

$$u_{R,t} = \rho_R u_{R,t-1} + \varepsilon_{R,t}. \tag{22}$$

Common equations determine the time path of the depreciation rate of the domestic currency s_t, the net external asset position $f_t^* = \frac{S_t F_t^*}{P_t}$, the real exchange rate S_t^R, the stock of pollution Z_t in the atmosphere and the related damage Λ_t:

$$\frac{1}{R_t^*} = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}(1+s_{t+1})}{\Pi_{t+1}\lambda_t}\right], \tag{23}$$

$$f_t^* = R_t^* \left[\frac{(1+s_t)}{\Pi_t} f_{t-1}^* + p_t^D X_t - S_t^R p_t^{D*} M_t\right], \tag{24}$$

$$S_t^R = S_{t-1}^R (1+s_t) \frac{\Pi_t^*}{\Pi_t}, \tag{25}$$

$$Z_t = \eta Z_{t-1} + E_t + E_t^* + E_t^{NI}, \tag{26}$$

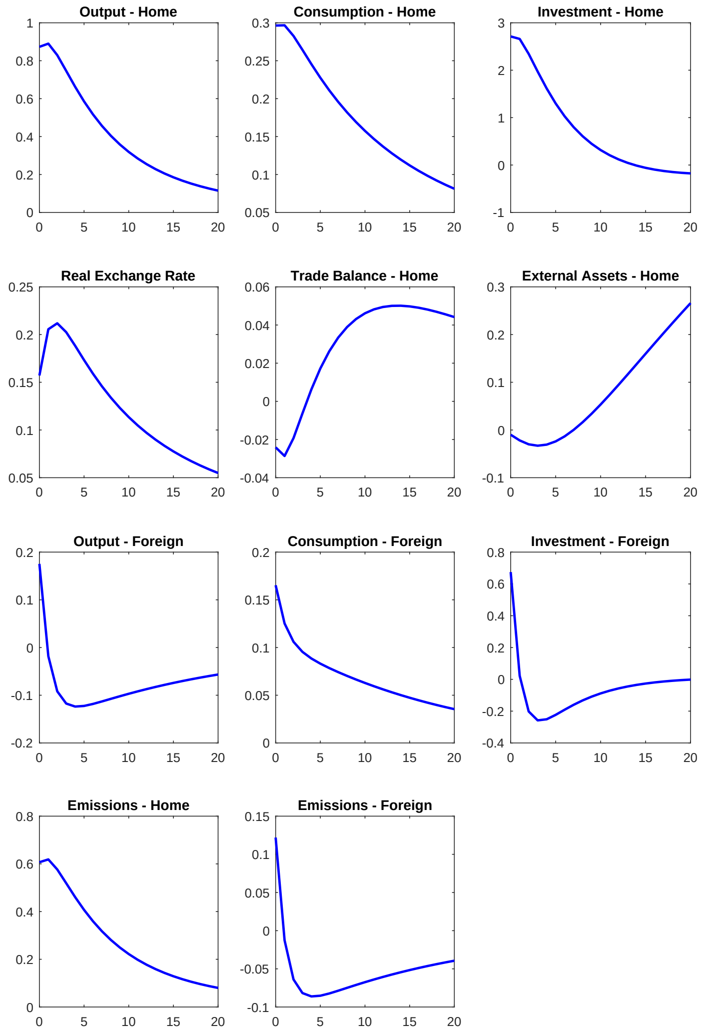
$$\Lambda_t = \exp[-\chi(Z_t - \overline{Z})]. \tag{27}$$

The overall economy is then described by 19 variables related to Home, $\{C_t$, E_t, I_t, K_t, L_t, M_t, p_t^D, q_t, R_t, $r_{K,t}$, w_t, X_t, Y_t, Y_t^D, Y_t^H, λ_t, Π_t, Π_t^D, $\Psi_t\}$, 19 variables related to Foreign $\{C_t^*$, E_t^*, I_t^*, K_t^*, L_t^*, M_t^*, $p_t^{D^*}$, q_t^*, R_t^*, $r_{K,t}^*$, w_t^*, X_t^*, Y_t^*, Y_t^{D*}, Y_t^{H*}, λ_t^*, Π_t^*, Π_t^{D*}, $\Psi_t^*\}$, and 5 common variables, $\{f_t^*$, s_t, S_t^R, $Z_t, \Lambda_t\}$.

Dynamic Analysis in the No-Policy Model

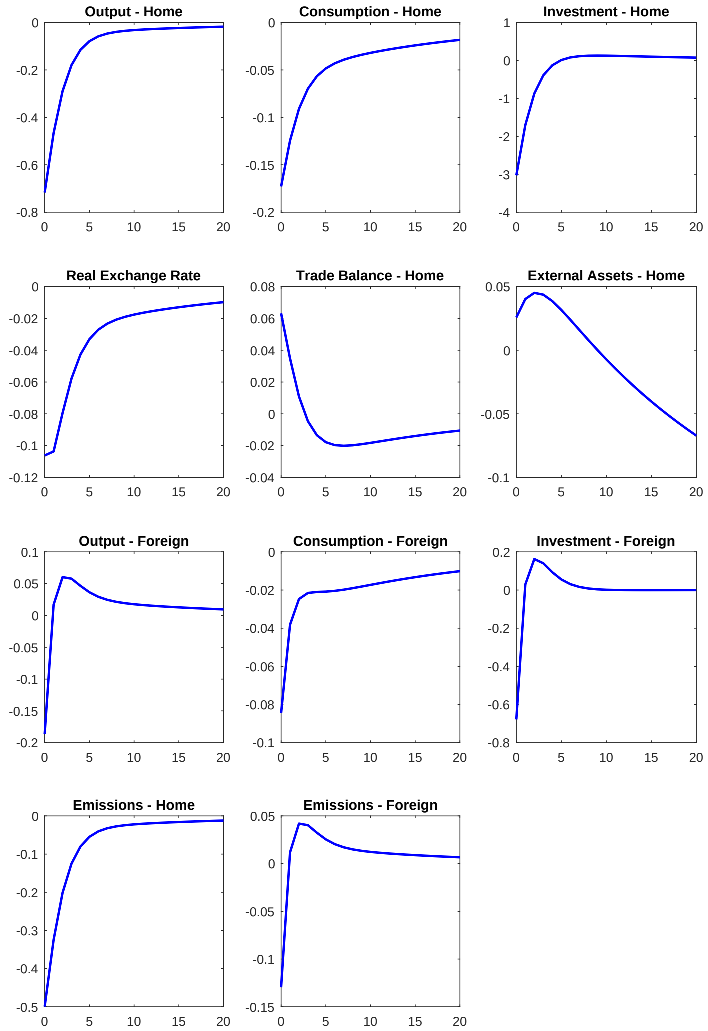
Figures 1-3 report the impulse responses of the main variables to shocks in the no-policy scenario.

Figure 1: Dynamic Response to a 1% TFP Shock - No Policy Scenario



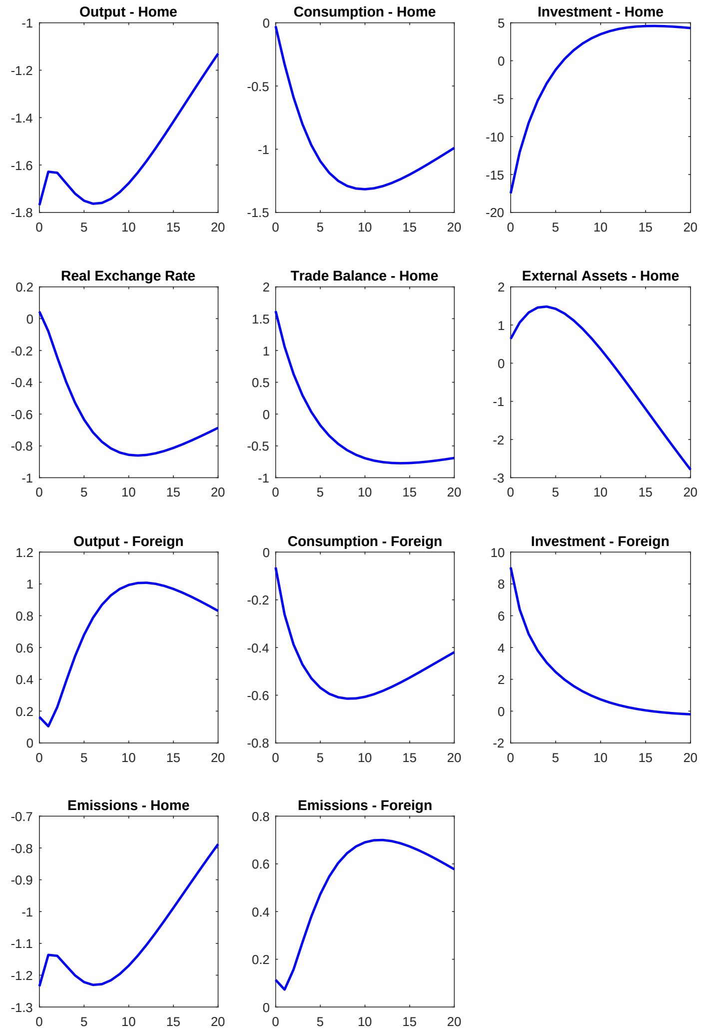
Note: the figure plots the impulse responses to a positive shock on TFP for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state with the exception of the trade balance and of the external asset position that are reported in percentage points from the zero steady state.

Figure 2: Dynamic Response to a 0.5% Monetary Policy Shock- No Policy Scenario



Note: the figure plots the impulse responses to a positive shock on the risk-free interest rate for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state with the exception of the trade balance and of the external asset position that are reported in percentage points from the zero steady state.

Figure 3: Dynamic Response to a -1% Capital Quality Shock - No Policy Scenario

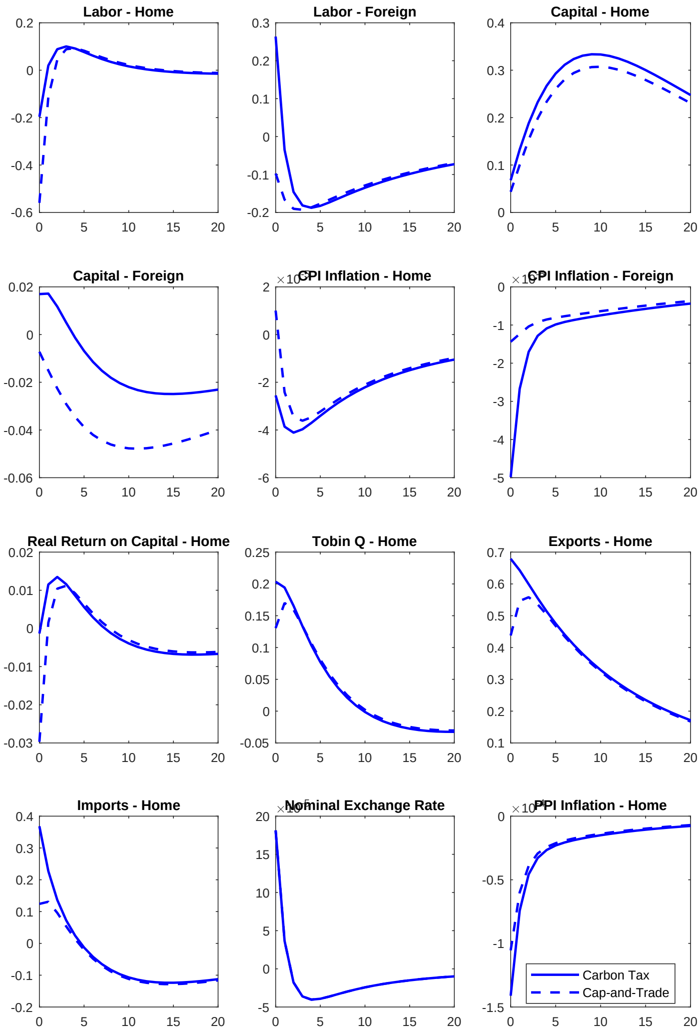


Note: the figure plots the impulse responses to a negative shock on the quality of capital for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state with the exception of the trade balance and of the external asset position that are reported in percentage points from the zero steady state.

Dynamic Response of other Macroeconomic Variables

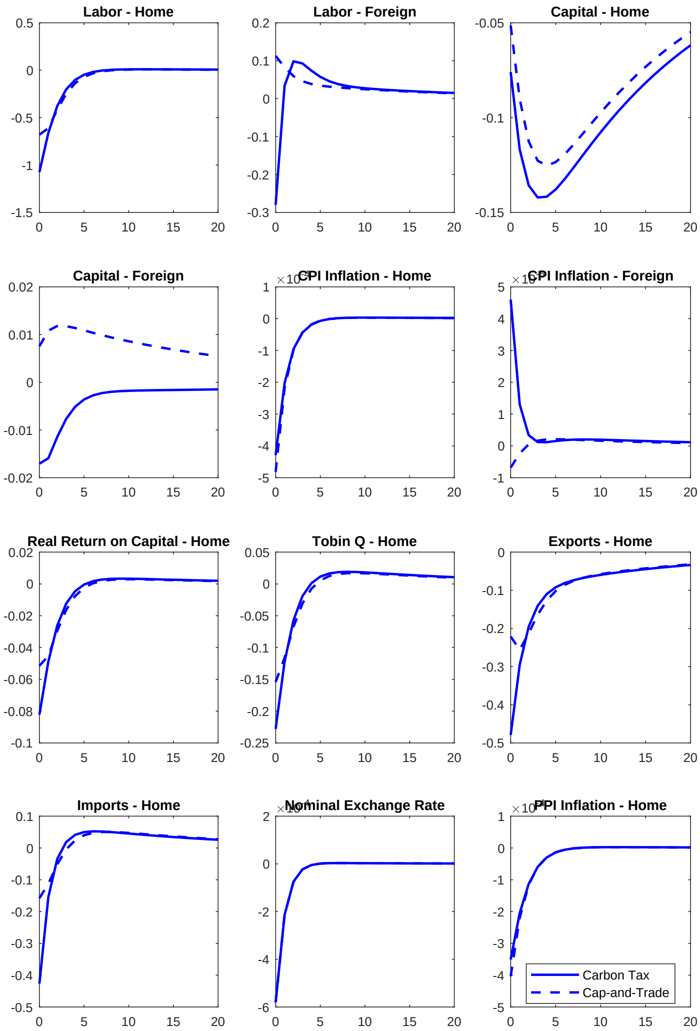
Figures 4-6 report the impulse responses of other main variables to shocks in the two policy scenario cases.

Figure 4: Dynamic Response to a 1% TFP Shock



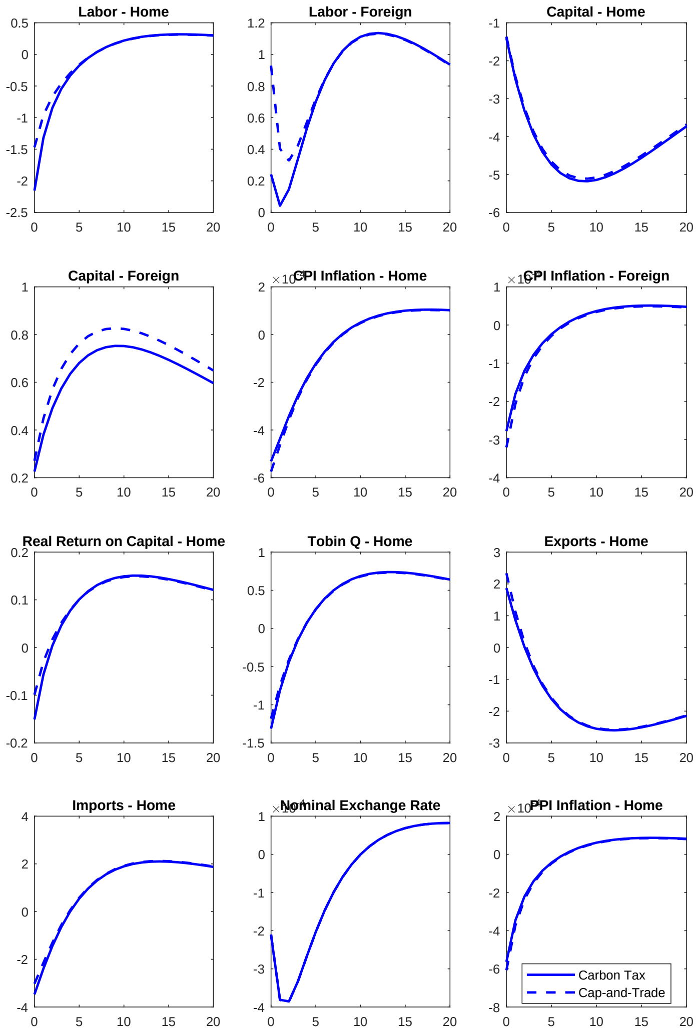
Note: the figure plots the impulse responses to a positive shock on TFP for a 20-quarter time horizon (horizontal axis); results are reported as percentage deviations from the initial steady state with the exception of inflation and the real return on capital which are reported in percentage points.

Figure 5: Dynamic Response to a 0.5% Monetary Policy Shock



Note: the figure plots the impulse responses to a positive shock on the risk-free interest rate for a 20-quarter time horizon (horizontal axis); results are reported as percentage deviations from the initial steady state with the exception of inflation and the real return on capital which are reported in percentage points.

Figure 6: Dynamic Response to a -1% Capital Quality Shock

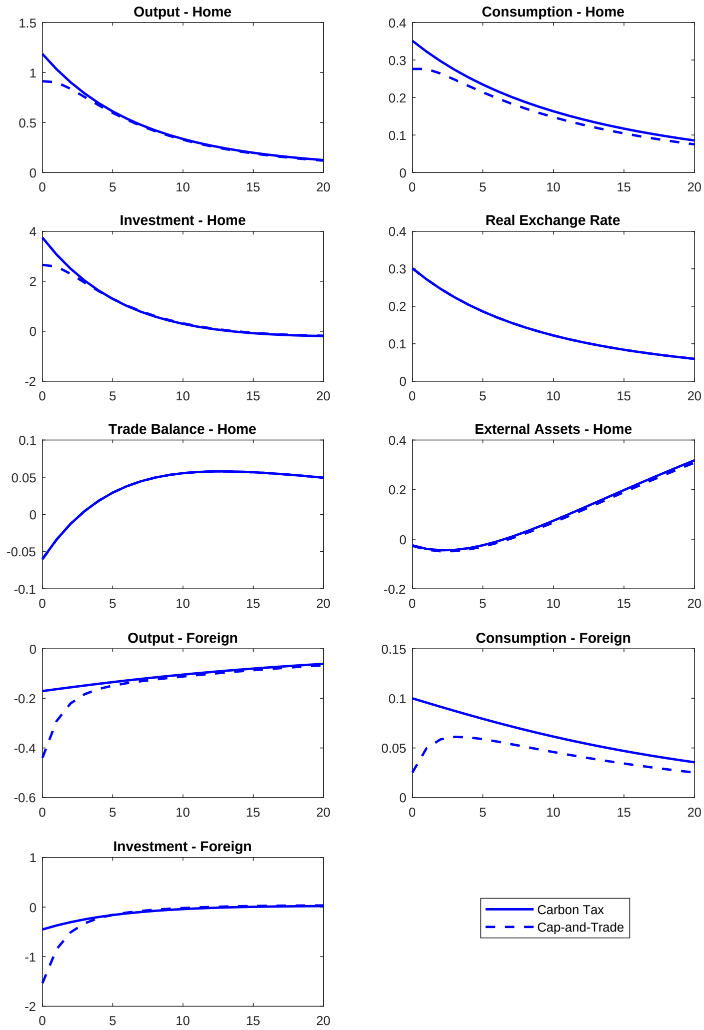


Note: the figure plots the impulse responses to a negative shock on the quality of capital for a 20-quarter time horizon (horizontal axis); results are reported as percentage deviations from the initial steady state with the exception of inflation and the real return on capital which are reported in percentage points.

Flexible Prices

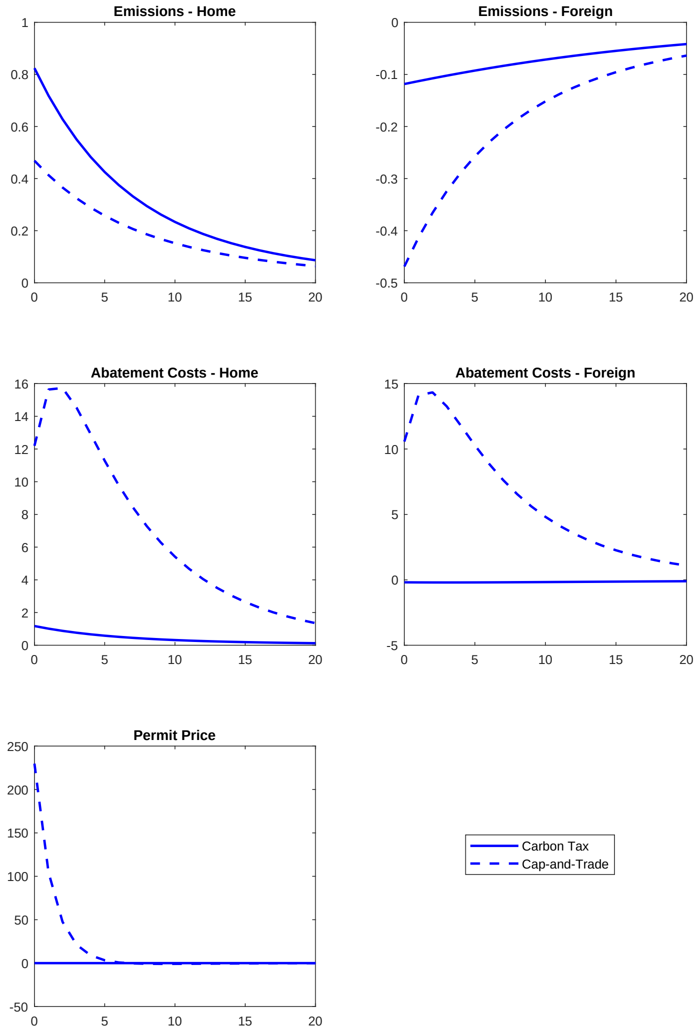
Figures 7 and 8 report the impulse responses of the main variables to a TFP shock in the two policy scenarios under flexible prices. We solve the model under the assumption that $\gamma_P = 0$. Table 1 reports the summary statistics under TFP and capital quality shocks. Monetary policy shocks are neutral under flexible prices.

Figure 7: Dynamic Response to a 1% TFP Shock under Flexible Prices - Macroeconomic Variables



Note: the figure plots the impulse responses to a positive shock on TFP for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state with the exception of the trade balance and of the external asset position that are reported in percentage points from the zero steady state.

Figure 8: Dynamic Response to a 1% TFP Shock under Flexible Prices - Environmental Variables



Note: the figure plots the impulse responses to a positive shock on TFP for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state.

Table 1: International Transmission of Shocks - Flexible Prices (%)

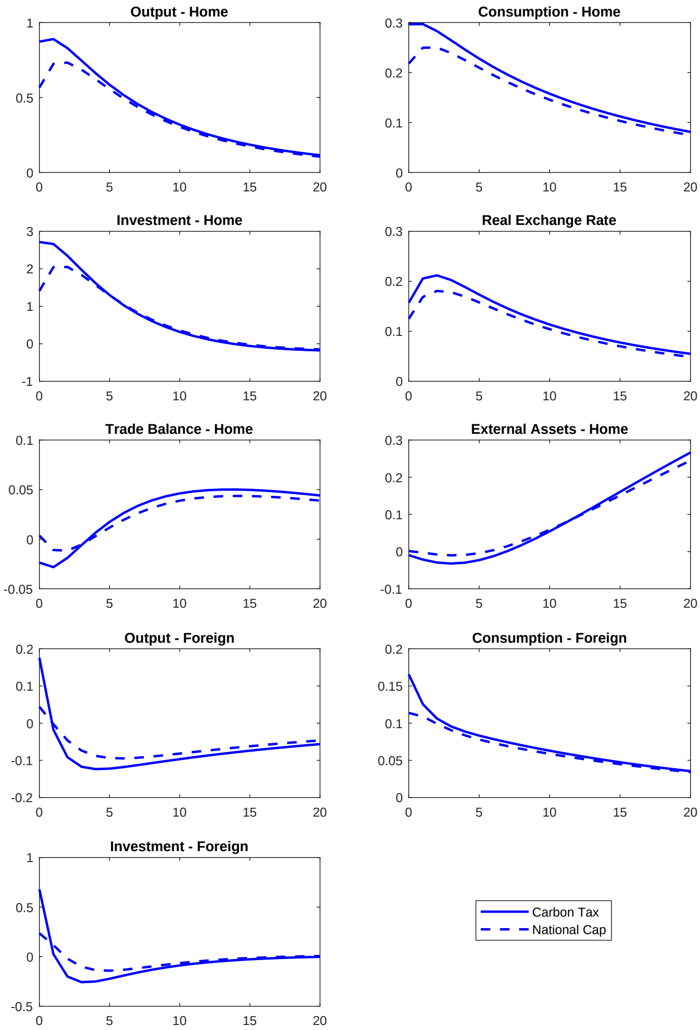
	σ_{Y^D}	$\sigma_{Y^{D*}}/\sigma_{Y^D}$	$\rho(Y^D, Y^{D^*})$
Carbon Tax			
TFP shock	5.3475	14.4875	-97.0302
Capital quality shock	6.3258	61.5678	-75.0830
Cap-and-Trade			
TFP shock	4.6675	32.8416	-91.6000
Capital quality shock	6.2492	57.5552	-89.4927
No Policy			
TFP shock	5.3599	14.4934	-97.0150
Capital quality shock	6.3359	61.6688	-75.0425

Note: the table reports moments generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%.

National Cap

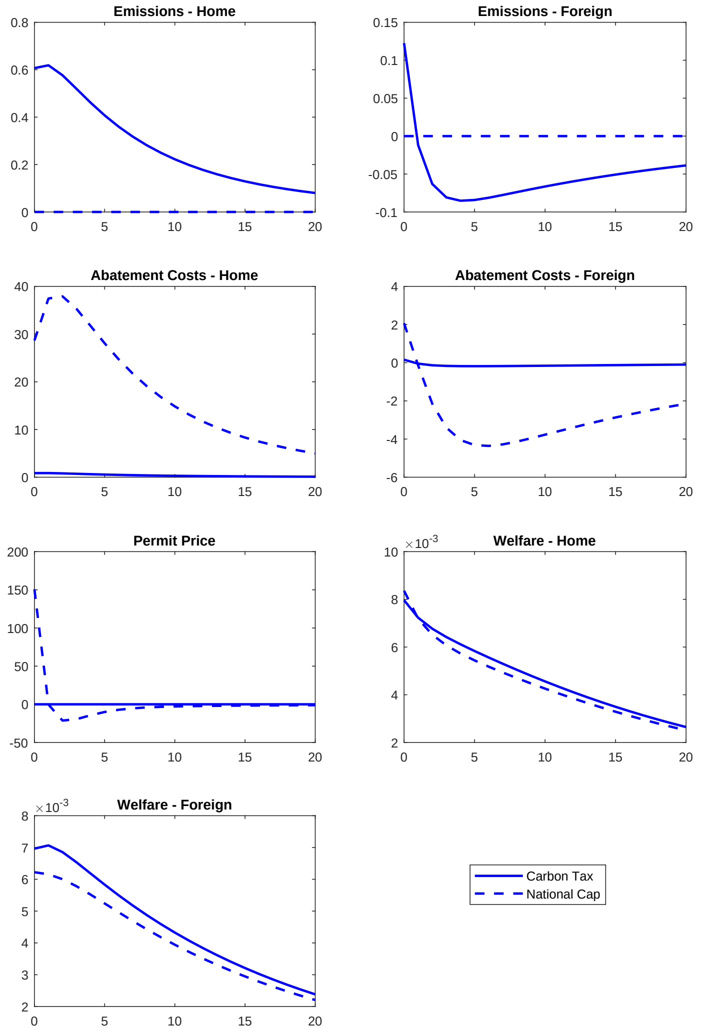
Figures 9-12 report the impulse responses of the main variables to a TFP shock and to a monetary policy shock under a carbon tax and a national cap regime, where the cross-border exchange of emission permits is not allowed. Tables 2 shows the summary statistics under a national cap regime. The tables also present, for comparison, the results of the main text referring to the carbon tax and the cap-and-trade regimes.

Figure 9: Dynamic Response to a 1% TFP Shock under Carbon Tax and National Cap - Macroeconomic Variables



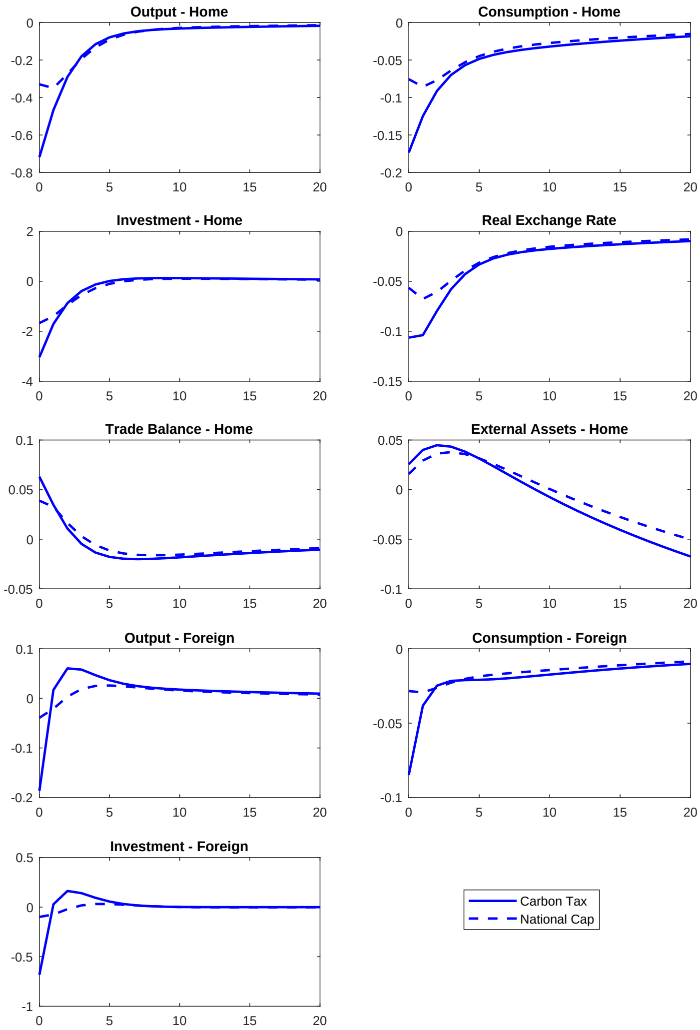
Note: the figure plots the impulse responses to a positive shock on TFP for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state with the exception of the trade balance and of the external asset position that are reported in percentage points from the zero steady state.

Figure 10: Dynamic Response to a 1% TFP Shock under Carbon Tax and National Cap - Environmental Variables



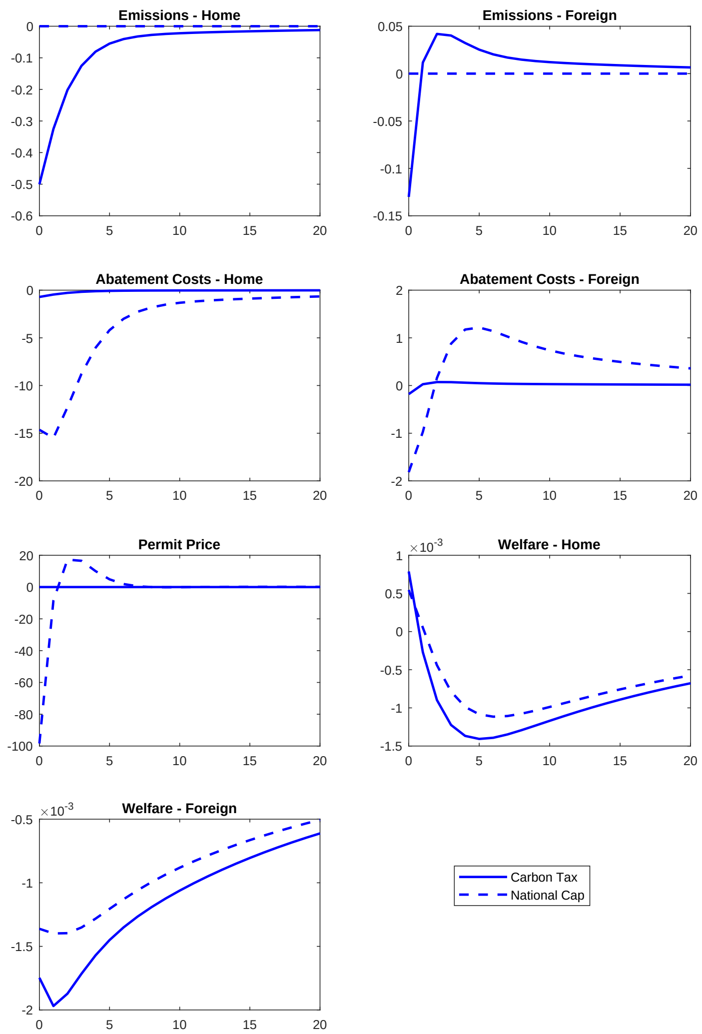
Note: the figure plots the impulse responses to a positive shock on TFP for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state.

Figure 11: Dynamic Response to a 0.5% Monetary Policy Shock under Carbon Tax and National Cap - Macroeconomic Variables



Note: the figure plots the impulse responses to a positive shock on the risk-free interest rate for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state with the exception of the trade balance and of the external asset position that are reported in percentage points from the zero steady state.

Figure 12: Dynamic Response to a 0.5% Monetary Policy Shock under Carbon Tax and National Cap - Environmental Variables



Note: the figure plots the impulse responses to a positive shock on the risk-free interest rate for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state.

Table 2: International Transmission of Shocks - Carbon Tax, National Cap, and Cap-and-Trade (%)

	σ_{Y^D}	$\sigma_{Y^{D*}}/\sigma_{Y^D}$	$\rho(Y^D, Y^{D^*})$
Carbon Tax			
TFP shock	4.6635	17.9785	-15.2519
Monetary shock	5.2153	20.2770	46.0468
Capital quality shock	8.2905	37.8146	-20.2500
Cap-and-Trade			
TFP shock	4.2043	16.1028	-96.8690
Monetary shock	4.3958	10.0523	-95.7749
Capital quality shock	7.6847	35.2991	-54.4854
National Cap			
TFP shock	3.7843	12.2377	-30.3332
Monetary shock	3.7982	12.8768	55.2032
Capital quality shock	7.1932	35.2544	-35.3639
No Policy			
TFP shock	4.6761	17.9641	-15.4019
Monetary shock	5.2189	20.2539	46.0691
Capital quality shock	8.3020	37.8615	-20.3997

Note: the table reports moments generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%.

Adjustment costs on abatement

Table 3 reports simulation results under the assumption that firms are able to freely choose the level of environmental efficiency of their technology, i.e. we set $\gamma_\mu = 0$ so that $\Gamma_{\mu_t}(\mu_t) = 0$. Table 4 reports simulation results under the assumption that firms face symmetric adjustment costs when changing the level of environmental efficiency of their technology μ, i.e. we set $\psi_\mu = 0$ so that $\Gamma_{\mu_t}(\mu_t) = \frac{\gamma_\mu}{2}\left(\frac{\mu_t}{\mu_{t-1}} - 1\right)^2$.

Table 3: International Transmission of Shocks - No Adjustment Costs on Abatement (%)

	σ_{Y^D}	$\sigma_{Y^{D*}}/\sigma_{Y^D}$	$\rho(Y^D, Y^{D^*})$
Carbon Tax			
TFP shock	4.6637	17.9899	-15.3102
Monetary shock	5.2153	20.2816	46.0240
Capital quality shock	8.2917	37.8275	-20.3422
Cap-and-Trade			
TFP shock	4.5775	18.5293	-25.4343
Monetary shock	5.1418	19.8351	40.4153
Capital quality shock	8.1324	38.4456	-25.9909
No Policy			
TFP shock	4.6761	17.9641	-15.4019
Monetary shock	5.2189	20.2539	46.0691
Capital quality shock	8.3020	37.8615	-20.3997

Note: the table reports moments generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%.

Table 4: International Transmission of Shocks - Symmetric Adjustment Costs on Abatement (%)

	σ_{Y^D}	$\sigma_{Y^{D*}}/\sigma_{Y^D}$	$\rho(Y^D, Y^{D^*})$
Carbon Tax			
TFP shock	4.6637	17.9901	-15.3107
Monetary shock	6.9743	19.3162	21.8797
Capital quality shock	9.4931	34.1522	-18.6359
Cap-and-Trade			
TFP shock	4.2043	16.1028	-96.8690
Monetary shock	6.0514	13.1904	-94.0298
Capital quality shock	8.7428	31.8529	-57.2250
No Policy			
TFP shock	4.6761	17.9641	-15.4019
Monetary shock	5.2189	20.2539	46.0691
Capital quality shock	8.3020	37.8615	-20.3997

Note: the table reports moments generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%.