International Transmission of the Business Cycle and Environmental Policy[*]

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Abstract

This paper presents a dynamic stochastic general equilibrium (DSGE) model of environmental policy for a two-country economy and studies the international transmission of asymmetric shocks considering two different economy-wide greenhouse gases (GHG) emission regulations: a carbon tax and a cap-and-trade system allowing for cross-border exchange of emission permits. We find that international spillovers of shocks originated in one country are strongly influenced by the environmental regime put in place. The cross-border reaction to shocks is found to be magnified under a carbon tax. The pattern of trade and the underlying monetary regime influence the cross-border transmission channels interacting with the environmental policy adopted.

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J.E.L. codes: F41, F42, E32, Q58.

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1 Introduction

This paper presents a dynamic stochastic general equilibrium (DSGE) model with two-interdependent economies to highlight the international aspects of environmental policies. In particular, the paper addresses the following fundamental questions. What is the role of different environmental policy regimes in shaping the transmission of shocks in open economies? What is the dynamic behavior of an economy where countries are tied by international trade and by a common environmental policy regime? How does the pattern of trade interact with the underlying environmental policy? What happens if countries share the same currency?

The role played by environmental regulation in open economy and the strategic interactions among countries committed to regulate emissions are topics largely debated in the literature. Computable General Equilibrium (CGE) models and Integrated Assessment Models (IAMs) are at present the main tools used to evaluate costs and benefits of different policies in climate change research.[1] The international dimension of climate policies has also been the object of several studies in the field of international economics. Much of the literature of international trade and environmental regulation has concentrated on inter-industry trade, emphasizing the role of comparative advantages and/or relative factor abundance in determining pollutant emissions.[2]

Only recently, another class of environmental models have been emerging in macroeconomics, where a growing attention is given to the role of uncertainty and the business cycle in influencing the performance of environmental regulation.[3] From a methodological standpoint, this strand of literature is based on dynamic stochastic general equilibrium models that for long time have neglected environmental aspects, as remarked by Arestis and González-Martínez

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[1] A substantial body of the literature, mostly related to CGE models, tackle problems relative to carbon leakage, strategic behaviors (e.g. Burniaux and Martins 2012 and Babiker 2005), and the loss of competitiveness (see Carbone and Rivers, 2017). For a general overview on the relationship between environmental regulation and competitiveness, see e.g. Dechezleprêtre and Sato (2017). Thank to their regional or global structure IAMs are well suited to study the overall costs of different policy instruments accounting for several international interlinkages. For an overview on global scale IAMs, see Weyant (2017).

[2] For a survey of studies focusing specifically on environmental policy analysis in open economy, see e.g. Rauscher (2005). For a comprehensive treatment of the relationships linking trade, economic growth and environment, see Copeland and Taylor (2003)

[3] For an accurate and comprehensive empirical analysis of the cyclical relationship between output and carbon dioxide emissions, see Doda (2014); for an interesting investigation on the behavior of emissions at business cycle frequency in response to different technology shocks, see Khan et al. (2019).

(2015). Notably, this type of models involves the systematic application of intertemporal optimization methods and of the rational expectations hypothesis that determine the behavior of consumption, investment and factor supply for different states of the economy.[4]

As proposed by Khan et al. (2019), we use the acronym E-DSGE to refer to dynamic stochastic general equilibrium models with environmental regulation. Relevant examples of E-DSGE models include Chang et al. (2009), Fischer and Springborn (2011), Heutel (2012), Angelopoulos et al. (2013), Bosetti and Maffezzoli (2014), Annicchiarico and Di Dio (2015) and Dissou and Karnizova (2016). However, so far the international dimension of climate actions has not been investigated in the context of E-DSGE models, therefore the study of the interaction among environmental policy, international trade and economic uncertainty has still remained unexplored. To the best of our knowledge, the only exception in this direction is the contribution by Ganelli and Tervala (2011) who study the international transmission of a unilateral implementation of a more stringent mitigation policy in a New Keynesian E-DSGE model of a global economy. However, neither they consider the international transmission of shocks commonly studied in the business cycle literature, nor the role played by the underlying environmental regime in shaping fluctuations and cross-border spillovers.

The paper aims at filling this gap of the E-DSGE literature, enriching the methodology based upon choice-theoretic stochastic models, by embodying New Keynesian aspects, such as nominal rigidities, imperfect competition and forward-looking price-setting, consistently with Annicchiarico and Di Dio (2015, 2017), and developing the analysis in an open economy model with two interdependent countries, Home and Foreign, engaging in intra-industry trade.[5]

With this model in hand, we are able to explore the international transmission of shocks commonly considered in the business cycle literature, and to study the role played by different environmental regimes in shaping the dynamic response of the economy. In particular, we focus on two policies for controlling emissions: a carbon tax and a cap-and-trade scheme, where emission permits can be traded across countries. In this sense the paper also contributes to

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[4] Dynamic general equilibrium models are also fruitfully used for the study of energy and climate policies in deterministic analyses abstracting from the business cycle. See Conte et al. (2010), Annicchiarico et al. (2017, 2018), and Bartocci and Pisani (2013). This last paper is the only one exploring the international dimension of energy policies analysing the effects of both unilateral and simultaneous interventions throughout the EU.

[5] Intra-industry trade models have been rarely used in the domain of environmental economics. Exceptions include, among others, Rauscher (1997), Benarroch and Weder (2006), Haupt (2006), Lai and Hu (2008).

the price versus quantity literature, started with the seminal paper of Weitzman (1974), by investigating the relative performance of international carbon pricing mechanisms in the face of uncertainty stemming from different sources.[6] Compared to previous price-versus-quantity studies, our set-up accounts for a broader definition of uncertainty, allows for an explicit modeling and calibration of shocks, and adopts a general equilibrium approach in open economy.

We explore the dynamic response of the economy to three shocks hitting only Home, namely (i) technology shocks on total factor productivity (TFP), (ii) shocks on the risk-free interest rate set by the monetary authorities, and (iii) shocks on the quality of capital. The first shock directly affects the supply side of the economy (supply shock), while the second shock influences aggregate demand (demand shock). The shock on the quality of capital, instead, is a hybrid shock, altering directly and simultaneously both the supply and the demand schedules of the economy. This shock transmits through the economy like a financial shock. To further investigate the influence exerted by environmental policies on the international transmission channels of shocks, we also look at the spillover effects under different assumptions regarding the pattern of trade and the underlying monetary regime.

Our main results can be summarized as follows. The international transmission of shocks from one economy to another proves to be affected by the underlying environmental regime. Both the magnitude and the sign of the cross-border spillover effects crucially depend on the source of uncertainty. The adoption of a carbon tax tends to amplify the spillover effects. In particular, we observe major differences between the two regimes in response to monetary policy shocks. In this case cross-border spillovers are still magnified under a carbon tax, however, Home and Foreign outputs are positively correlated under the carbon tax regime, while the correlation turns out to be negative and stronger under the cap-and-trade regime, where the cross-border exchange of emission permits determines a reallocation of production from one country to the other.

When we solve the model assuming a trade pattern such that Home and Foreign goods are imperfect complements, rather than substitutes, the international spillovers tend to be larger, as well as under a higher degree of openness. More interestingly, under both assumptions we

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[6] See, e.g., Pizer (1999, 2002), Quirion (2005) and Jotzo and Pezzey (2007). On the relationship between economic fluctuations and environmental policy, see e.g. Kelly (2005).

observe larger differences across environmental regimes with the carbon tax always giving rise to stronger spillover effects.

We also show how the role played by environmental policies in shaping the international transmission channels of asymmetric shocks changes when the economies share the same currency. In response to a positive TFP shock hitting the domestic economy, the correlation between Home and Foreign output turns out to be positive under a carbon tax, and less negative under a cap and trade.

Finally, we assess, from a welfare perspective, the performance of quantity and price regulations with respect to the no-policy scenario. Overall, we do not observe substantial differences between the two regimes. In the face of TFP shocks the carbon tax performs slightly better than a cap-and-trade regime, while for capital quality shocks the opposite is true. For monetary shocks simulation results are not clear-cut. Our findings are robust to different assumptions regarding the way in which the stock of pollutant affects the economy.

The remainder of the paper is organized as follows. Section 2 describes the two-country model and introduces the various sources of uncertainty giving rise to different dynamic adjustments of the economy. Section 3 summarizes the parametrization used to numerically solve the model. Section 4 presents the dynamic response of macroeconomic and environmental variables to shocks, under the two alternative environmental policy regimes, accounting for the role of international trade and monetary policy in the propagation of disturbances between countries. Section 5 is devoted to the welfare analysis. Section 6 summarizes the main results and concludes.

2 The model

We model an artificial economy with two countries, Home and Foreign, open to international trade and financial capital flows. Home and Foreign are modeled symmetrically, therefore the following description holds for both economies. Foreign variables are denoted by a superscript asterisk. Each country manufactures tradable intermediate goods produced in a number of horizontally differentiated varieties by using labor and physical capital as factor inputs. The intermediate goods sector is characterized by monopolistic competition and price stickiness in the form of quadratic adjustment costs *à la* Rotemberg. Labor and physical capital are immobile between countries. Given the symmetry between Home and Foreign, trade is of intra-industry type and is motivated by the existence of a final good sector technology combining domestic and foreign differentiated intermediate goods.

On the demand side, the economy is populated by households deriving utility from consumption and disutility from labor. Households supply labor and capital to domestic producers, and hold domestic and foreign bonds. The economy features pollutant emissions, which are a by-product of output, and a negative environmental externality on production. Finally, we have a government that sets environmental policy and a central bank making decisions on monetary policy.

2.1 Households

The typical infinitely lived household derives utility from consumption, C_t, and disutility from hours worked, L_t. The lifetime utility U is of the type:

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\varphi_C}}{1-\varphi_C} - \xi_L \frac{L_t^{1+\varphi_L}}{1+\varphi_L} \right), \tag{1}$$

where E_0 is the rational expectations operator, $\beta \in (0,1)$ is the discount factor, $\varphi_C > 0$ is the coefficient of relative risk aversion, $\xi_L > 0$ is a scale parameter measuring the relative disutility of labor, and $\varphi_L > 0$ is the inverse of the Frisch elasticity of labor supply. Households own the stock of physical capital, K_t, and provide it to firms in a perfectly competitive rental market. The accumulated capital stock K_t is subject to a quality shock determining the level of effective capital for use in production. Therefore, the stock of capital held by households evolves according to the following law of motion:

$$K_{t+1} = I_t + (1-\delta)e^{u_{K,t}} K_t, \tag{2}$$

where I_t denotes investments, K_t is physical capital carried over from period $t-1$ and $\delta \in (0,1)$ is the depreciation rate of capital, while u_K is an exogenous process capturing any exogenous variation in the value of installed capital able to trigger sudden variations in its market value and changes in investment expenditure. This shock is meant to mimic an asset price shock, however, to be consistent with the DSGE literature, we will refer to it as a capital quality shock.[7] This shock directly affects the capital in use for production and indirectly influences future investments by changing their expected return, as it is shown below. The process $u_{K,t}$ is such that $u_{K,t} = \rho_K u_{K,t-1} + \varepsilon_{K,t}$, where $0 < \rho_K < 1$ and $\varepsilon_K \sim i.i.d.\ N(0, \sigma_K^2)$.

Investment decisions are subject to convex capital adjustment costs of the type $\Gamma_K(I_t, K_t) \equiv (\gamma_I/2)\left(I_t/K_t - \delta\right)^2 K_t$, $\gamma_I > 0$. Domestic residents have access to a one-period risk free bond, B_t, sold at a price R_t^{-1} and paying one unit of currency in the following period, and to a risk-free asset traded between the two countries, F_t^*, denominated in Foreign currency, sold at a price $(R_t^*)^{-1}$ and paying one unit of foreign currency in the following period. Households receive lump-sum transfers Tr_t from the government, dividends D_t from the ownership of domestic intermediate good-producing firms, and payments for factors they supply to these firms: a nominal capital rental rate $R_{K,t}$ and a nominal wage W_t.

Denoting the final good price index by P_t, the period-by-period budget constraint reads as:

$$\begin{aligned} P_t C_t + P_t I_t + R_t^{-1} B_t + \left(R_t^*\right)^{-1} S_t F_t^* = W_t L_t + R_{K,t} K_t + \\ + B_{t-1} + S_t F_{t-1}^* - P_t \Gamma_K(I_t, K_t) + P_t Tr_t + P_t D_t, \end{aligned} \tag{3}$$

where S_t is the nominal exchange rate expressed as the price of Foreign currency in units of Home currency. The typical household will choose the sequences $\{C_t,\ K_{t+1},\ I_t,\ L_t,\ B_t,\ F_t^*\}_{t=0}^{\infty}$, so as to maximize (1), subject to (2) and (3). See Appendix A for details.

Rewriting the budget constraint in real terms, from the households' utility maximization problem, we obtain the following set of first-order conditions:

$$C_t^{-\varphi_c} = \lambda_t, \tag{4}$$

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[7] In DSGE models, following the finance literature, (see e.g. Merton 1973), this type of shock represents a simple way to introduce an exogenous source of variation in the value of capital and mimic a recession originating from an adverse shock on the asset price. As we will see the shock is designed so as to generate co-movement of consumption, investment, hours and output and to yield a higher spread between the return on capital and the risk free rate. See e.g. Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). To fully represent the dynamics of an economy in response to an asset price shock, the model should include a financial sector. We leave this extension for future research.

$$\begin{aligned} q_t &= \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[r_{K,t+1} + \gamma_I \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} - \frac{\gamma_I}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 \right] \right\} + \\ & +\beta(1-\delta)\mathbb{E}_t \left\{ e^{u_{K,t+1}} \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \right\}, \end{aligned} \tag{5}$$

$$q_t - 1 = \gamma_I \left(\frac{I_t}{K_t} - \delta \right), \tag{6}$$

$$\lambda_t w_t = \xi_L L_t^{\varphi_L}, \tag{7}$$

$$\frac{1}{R_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1}} \right\}, \tag{8}$$

$$\frac{1}{R_t^*} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{S_{t+1}}{\Pi_{t+1} S_t} \right\}, \tag{9}$$

where λ_t denotes the Lagrange multiplier associated to the flow budget constraint (3) expressed in real terms and measures the marginal utility of consumption according to condition (4); $r_{K,t} = R_{K,t}/P_t$; $w_t = W_t/P_t$; q_t is the Tobin's marginal q_t, measuring the ratio of the market value of an additional unit of capital to its replacement cost (here expressed in terms of forgone consumption since households trade off consumption in order to invest); and $\Pi_t = P_t/P_{t-1}$ measures inflation in the final-good sector. Equations (5) and (6) refer to the optimality conditions with respect to capital and investments, where, clearly, if $q_t > 1$ then the level of investments will be higher than the level necessary to replace the depreciated capital. From equation (5), we note how the capital quality shock determines the time path of the Tobin's marginal q_t. A positive shock on $u_{K,t}$ increases the Tobin's q_t and, by condition (6), investments will increase, inducing an accumulation of capital. On the contrary, a negative shock decreases the market value of capital and gives rise to a contraction of investment expenditure and to a decumulation of the installed capital.

Finally, equation (7) describes labor supply, whereas (8) and (9) are the two first-order conditions with respect to domestic and foreign assets, reflecting the optimal choice between current and future consumption, given the return on the two risk-free assets, expected inflation and the expected depreciation of the domestic currency.

2.2 Production

2.2.1 Production of Domestic Intermediate Goods

The intermediate goods producing sector is dominated by a continuum of monopolistically competitive polluting firms indexed by $j \in [0,1]$. Each firm charges the same price at home and abroad and faces a demand function that varies inversely with its output price $P_{j,t}^D$ and directly with aggregate demand Y_t^D for domestic production, that is $Y_{j,t}^D = \left(P_{j,t}^D/P_t^D\right)^{-\sigma} Y_t^D$, where $\sigma > 1$ and P_t^D is an aggregate production price index defined below.

The producer of the variety j hires capital and labor in perfectly competitive factor markets to produce the intermediate good $Y_{j,t}^D$ according to a Cobb-Douglas technology, modified to incorporate a capital-quality shock and the damage from pollution, measured in terms of intermediate output's reduction:

$$Y_{j,t}^D = \Lambda_t A_t \left(e^{u_{K,t}} K_{j,t}\right)^{\alpha} L_{j,t}^{1-\alpha}, \tag{10}$$

where $0 < \alpha < 1$ is the elasticity of output with respect to capital, A_t denotes total factor productivity, and Λ_t is a term capturing the negative externality of pollution on production. As in Gertler and Karadi (2011), we assume that the shock $u_{K,t}$ also changes the utilization of capital used in production. A negative shock will induce a reduction in the utilization of capital so giving rise to an immediate contraction of production.[8]

Referring to Golosov et al. (2014), we adopt the following simplified specification for the damage function Λ_t:

$$\Lambda_t = \exp[-\chi(Z_t - \overline{Z})], \tag{11}$$

where Z_t is the global stock of carbon dioxide in period t, $\overline{Z}$ is the pre-industrial atmospheric CO_2 concentration, and χ is a positive scale parameter measuring the intensity of the negative externality on production or analogously the fraction of production lost for each extra unit of pollutant.[9] The equation describes how economic damages change in function of greenhouse

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[8] In this sense this shock, when negative, could also account for environment-related risk factors that can strand assets, like natural disasters hitting the economy or new environmental regulations that may render obsolete the stock of installed capital.

[9] A similar specification is adopted by Annicchiarico et al. (2017). Both Home and Foreign are equally affected by the negative externality, causing a reduction of the production possibilities of the intermediate

gas concentration in the atmosphere. This kind of formalization is established in the literature and is an exponential version of the well-known Nordhaus damage function introduced in the DICE/RICE family of models (see e.g. Nordhaus 2018). According to Nordhaus, there is a relationship between global temperature increase and income loss. However, whereas Nordhaus explicitly models damages in two steps, the first one mapping carbon concentration into temperature and the second one mapping temperature to damages, Golosov et al. (2014) propose a function directly mapping from the stock of carbon dioxide to economic damages. The damage effects are multiplicative as in the RICE and the DICE models, and the exponential specification represents a good approximation of Nordhaus specifications, as discussed by the authors.[10]

We further assume that productivity A_t is subject to shocks, that is $A_t = Ae^{u_{A,t}}$, where A denotes the steady-state productivity level, while $u_{A,t}$ is assumed to evolve as $u_{A,t} = \rho_A u_{A,t-1} + \varepsilon_{A,t}$, where $0 < \rho_A < 1$ and $\varepsilon_{A,t} \sim i.i.d.\ N(0, \sigma_A^2)$.

Emissions per firm are a by-product of output:

$$E_{j,t} = (1 - \mu_{j,t})\epsilon(Y_{j,t}^D)^{1-\gamma}, \tag{12}$$

where $0 < \mu_t < 1$ is the abatement effort, ϵ is a parameter that we use to scale the emission function and the parameter $0 < \gamma < 1$ determines the elasticity of emissions with respect to output.[11]

Firms are subject to environmental regulation and can choose to purchase emission permits on the market at the price $P_{E,t}$ (or to pay a tax in the case of price regulation), or to incur in abatement costs $AC_{j,t}$ to reduce emissions. When making these decisions, producers are

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sector. Via its effect on the production possibilities of the economy, the environmental externality negatively affects the welfare. Alternative formulations include the damages from pollution directly in the utility function. In a decentralized economy, as the one we consider, the two modelling choices are equivalent and yield similar results. Furthermore, it is worth noting that this paper focuses on the short run. Capturing the effects of pollution on human health, which would be better accomplished by including pollution in the utility function, is therefore beyond the considered time span. In Section 5 we show the welfare results accounting also for the negative effects of pollution on utility.

[10] Although the aforementioned functions represent well-established approaches to formalize climate change damages, considerable uncertainty still remains on the aggregate consequences of pollution. So far, there is no consensus on the form and the parametrization of a general climate damage function. For a discussion on the role of damage modeling in climate change literature, see Bretschger and Pattakou (2019).

[11] Looking at different estimations performed at business cycle frequencies (see e.g. Heutel 2012 and Doda 2014), it is possible to observe that the elasticity is significantly positive and less than one.

assumed to ignore the negative environmental externality. Abatement costs, in turn, depend on firm's output and on abatement effort:

$$AC_{j,t} = \theta_1 \mu_{j,t}^{\theta_2} Y_{j,t}^{D}, \tag{13}$$

where $\theta_1 > 0$ and $\theta_2 > 1$ are technological parameters. Differently from previous E-DSGE models, we assume that firms are not able to freely choose the level of environmental efficiency of their technology, and propose a formalization of the abatement effort more plausible at business cycle frequencies. Previous models assume that firms can change their abatement effort at quarterly frequency, without incurring any additional cost (see e.g. Heutel 2012, Annicchiarico and Di Dio 2015).[12] The ability to fully control abatement effort allows firms to minimize the cost of mitigation policies over the business cycle and to significantly decrease the potential constraint that these policies impose on production and investment. This feature is not realistic since changes in abatement effort and environmental efficiency are typically the result of medium-term efforts that require investments and the adoption of new technologies, while this model focuses on the short-term. Furthermore, considering the nature of investments in abatement, it seems reasonable to assume that a large part of the abatement effort is irreversible.

To account for the fact that improvements in the level of environmental efficiency do not materialize immediately in the short-run, we assume that firms wishing to change their abatement effort incur in adjustment costs. These costs are expressed in units of the aggregate domestic good. We further assume the abatement choice to be partially irreversible implying that firms face limits in their ability to reduce their abatement effort over the business cycle. To introduce these features into our model we assume that the costs of changing the level of effort are represented by a linex function, say $\Gamma_{\mu_t}(\mu_t)$, such that the cost depends on both the magnitude and sign of the effort adjustment. In particular, we assume the following functional

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[12] The attempt to provide a more realistic formalization for the abatement effort follows closely a current debate in IAMs literature. The flexibility in μ is one of the most debated features of the DICE model, which is the reference model for the abatement cost function we adopted in equation (14). The stochastic versions of the DICE, such as the ones proposed by Traeger (2014) and Cai et al. (2012), maintain μ unconstrained as well, due to the inability to solve the model otherwise. Despite the presence of several attempts, to the best of our knowledge no paper has so far addressed the problem directly in the DICE model. In the context of a DSGE model, the flexibility of the abatement effort is even more problematic, due to the focus of the analysis on the short-run.

form:

$$\Gamma_{\mu_t}(\mu_t) = \gamma_\mu \frac{\exp\left(-\psi_\mu\left(\frac{\mu_t}{\mu_{t-1}}-1\right)\right)+\psi_\mu\left(\frac{\mu_t}{\mu_{t-1}}-1\right)-1}{\psi_\mu^2}, \tag{14}$$

where γ_μ and ψ_μ are positive coefficients.[13] The linex function is attractive for two reasons. First, it is differentiable and strictly convex for $\gamma_\mu > 0$. Second, it implies that as μ_t increases the linear term dominates and the costs associated with the abatement effort changes tend to increase linearly. By contrast, as μ_t decreases the exponential term dominates and the costs associated with changes in abatement effort tend to increase exponentially. The higher ψ_μ, the more asymmetric these adjustment costs are. In particular, for $\psi_\mu \to \infty$ downward changes become prohibitive and abatement choices are completely irreversible. For $\psi_\mu \to 0$, instead, (14) boils down to a quadratic form and adjustment costs become symmetric.[14]

Let $p_{E,t} = P_{E,t}/P_t$ and $p_t^D = P_t^D/P_t$, by imposing symmetry across producers, from the solution of firm j's static cost minimization problem, we have the following optimality conditions (see Appendix A):

$$r_{K,t} = \alpha\Psi_t\frac{Y_t^D}{K_t}, \tag{15}$$

$$w_t = (1-\alpha)\Psi_t\frac{Y_t^D}{L_t}, \tag{16}$$

$$\begin{aligned} p_{E,t}(Y_t^D)^{(1-\gamma)} \quad = \quad & \theta_2\theta_1\mu_t^{\theta_2-1}p_t^D Y_t^D - \gamma_\mu\frac{1}{\mu_{t-1}}p_t^D\frac{\exp\left(-\psi_\mu\left(\frac{\mu_t}{\mu_{t-1}}-1\right)\right)-1}{\psi_\mu} + \\ & +\beta\mathbb{E}_t\frac{\lambda_{t+1}}{\lambda_t}\gamma_\mu\frac{\mu_{t+1}}{\mu_t^2}p_{t+1}^D\frac{\exp\left(-\psi_\mu\left(\frac{\mu_{t+1}}{\mu_t}-1\right)\right)-1}{\psi_\mu}, \end{aligned} \tag{17}$$

where equations (15) and (16) are the demands for capital and labor and equation (17) is the optimal abatement choice. Ψ_t is the marginal cost component related to the use of extra units of capital and labor needed to produce an additional unit of output. It can be easily shown that Ψ_t is common to all firms and is equal to $\Psi_t = \left[\alpha^\alpha(1-\alpha)^{1-\alpha}\Lambda_t A\right]^{-1} w_t^{1-\alpha} r_{K,t}^\alpha$. Adding a cost term depending on the rate of change of μ introduces a degree of autocorrelation for the abatement effort.

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[13] The linex specification has been originally proposed by Varian (1974).

[14] Applying twice L'Hôpital's rule on (14), it is possible to show that for $\psi_\mu \to 0$, function $\Gamma_{\mu_t}(\mu_t)$ reduces to $(\gamma_\mu/2)\left(\mu_t/\mu_{t-1}-1\right)^2$.

Consider now the optimal price setting problem of the typical firm j. Acting in a non-competitive setting, firms can choose their price, taking the production price index P_t^D as given, but they face quadratic adjustment costs à la Rotemberg: $\frac{\gamma_P}{2}\left(P_{j,t}^D/P_{j,t-1}^D - 1\right)^2 P_t^D Y_t^D$, where the coefficient $\gamma_p > 0$ measures the degree of price rigidity. Formally, the firm sets the price $P_{j,t}^D$ by maximizing the present discounted value of profits subject to demand constraint $Y_{j,t}^D = \left(P_{j,t}^D/P_t^D\right)^{-\sigma} Y_t^D$. At the optimum we have:

$$\begin{aligned} \left(1-\theta_1\mu_t^{\theta_2}\right)(1-\sigma)+\sigma\left[\frac{p_{E,t}}{p_t^D}(1-\gamma)(1-\mu_t)\epsilon(Y_t^D)^{-\gamma}+\Psi_t\frac{1}{p_t^D}\right]+ \\ -\gamma_P\left(\Pi_t^D-1\right)\Pi_t^D+\beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\gamma_P\left(\Pi_{t+1}^D-1\right)\left(\Pi_{t+1}^D\right)^2\frac{Y_{t+1}^D}{Y_t^D}\frac{1}{\Pi_{t+1}}\right]=0, \end{aligned} \tag{18}$$

where we have imposed symmetry across producers and defined $\Pi_t^D = P_t^D/P_{t-1}^D$.

The above equation is a hybrid New Keynesian Phillips curve, relating current inflation Π_t^D to the expected future inflation Π_{t+1}^D, to the marginal cost Ψ_t related to production inputs, to the marginal cost related to abatement effort $\theta_1\mu_t^{\theta_2}$ and to the environmental regulation marginal cost component $p_{E,t}(1-\gamma)(1-\mu_t)\epsilon\left(Y_t^D\right)^{-\gamma}$. The overall costs sustained by the firm then depend on the available technologies for production and abatement, on the emission function and on the underlying environmental regime. In the absence of any environmental policy regime, our hybrid New Keynesian Phillips curve (18) would collapse into the standard New Keynesian Phillips curve, where the only marginal cost component relevant for inflation dynamics would be that associated with the use of production factor inputs (i.e. Ψ_t).

2.2.2 Domestic Output Index

Each domestic producer supplies goods to the Home and to the Foreign markets. Let $Y_{j,t}^H$ and $X_{j,t}$ denote, respectively, the domestic and the foreign demand for the generic domestic variety j, then $Y_{j,t}^D = Y_{j,t}^H + X_{j,t}$. For simplicity we assume the presence of a perfectly competitive aggregator that combines domestically produced varieties into a composite Home-produced good Y_t^D, according to a CES function $Y_t^D = \left(\int_0^1 \left(Y_{j,t}^D\right)^{(\sigma-1)/\sigma} dj\right)^{\sigma/(\sigma-1)}$. Cost minimization delivers the demand schedule $Y_{j,t}^D = \left(P_{j,t}^D/P_t^D\right)^{-\sigma} Y_t^D$ for each variety. From the zero-profit condition, we obtain the aggregate production price index, $P_t^D = \left(\int_0^1 (P_{j,t}^D)^{(1-\sigma)} dj\right)^{1/(1-\sigma)}$, at which the aggregator sells units of each sectoral output index. Clearly, this output index is allocated in both markets, therefore $Y_t^D = Y_t^H + X_t$, where X_t represents exports of Home to Foreign.

By symmetry, we assume the existence of a perfectly competitive aggregator in the Foreign economy that combines differentiated intermediate goods into a single good to be used for local production of the final good and for exportation.

2.2.3 Production of the Final Good

Competitive firms in the final sector combine a share Y_t^H of the good index Y_t^D produced in the intermediate domestic sector with a share M_t of foreign intermediate production in order to produce the final good Y_t demanded by households for consumption and investment purposes. Therefore, consumption and investment goods are aggregate baskets of domestic and foreign goods sold to households by perfectly competitive firms operating in this sector.

The final good is produced according to the following production function:

$$Y_t = [\kappa^{\frac{1}{\rho}} (Y_t^H)^{\frac{\rho-1}{\rho}} + (1-\kappa)^{\frac{1}{\rho}} (M_t)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}, \tag{19}$$

where κ represents the share of intermediate domestic goods used in the production of final good and $\rho > 0$ is the elasticity of substitution between domestic and foreign intermediate goods. Clearly, $1-\kappa$ represents the degree of openness of the economy.[15]

Final good producing firms sustain the following cost for inputs: $P_t^D Y_t^H + P_t^{D^*} S_t M_t$, where $P_t^{D^*}$ represents the price index of Foreign production expressed in Foreign currency. Taking as given the production price index of the domestic intermediate goods, P_t^D, and the production price index of the imported intermediate goods, $S_t P_t^{D^*}$, firms minimize their cost function

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[15] The form of the production function mirrors the consumption and investment preferences of households for Home and Foreign produced goods, which are considered as imperfect substitutes if $\rho > 1$ (our benchmark case) or as imperfect complements if $0 < \rho < 1$.

choosing the optimal quantities of domestic and imported goods:

$$Y_t^H = \kappa \left(\frac{P_t^D}{P_t}\right)^{-\rho} Y_t, \tag{20}$$

$$M_t = (1-\kappa) \left(\frac{S_t P_t^{D^*}}{P_t}\right)^{-\rho} Y_t. \tag{21}$$

From the zero-profit condition we derive the final good price index:

$$P_t = [\kappa (P_t^D)^{(1-\rho)} + (1-\kappa)(S_t P_t^{D^*})^{(1-\rho)}]^{1/(1-\rho)}. \tag{22}$$

2.3 Public Sector

2.3.1 Environmental Policy

We consider two possible environmental policies: carbon tax and cap-and-trade. Home and Foreign pursue a common environmental policy. Under a carbon tax regime each country imposes a tax rate per unit of emission (i.e. p_E is the same for the two countries and is constant, it can therefore be interpreted as a carbon tax). Under a cap-and-trade regime, Home and Foreign jointly choose the level of cumulative emissions that can be released ($E_t + E_t^* = \bar{E} + \bar{E}^*$). In the intermediate goods sector all firms must hold one permit for each unit of pollution they emit.

We abstract from the existence of a public debt and assume that the fiscal authority runs a balanced budget at all times. The revenues from environmental policy are distributed to domestic households as lump-sum transfers and there is no possibility of sharing revenues between the two countries, that is

$$p_{E,t} E_t = Tr_t, \tag{23}$$

where the term $p_{E,t} E_t$ may refer to the revenues collected from domestic firms through a carbon tax policy or through the government sale of emission permits given the cap, in which case $p_{E,t} \bar{E} = Tr_t$. We rule out the possibility of grandfathering to make the revenue streams from the carbon tax and from the cap-and-trade completely comparable.

Before we proceed, some remarks are in order concerning the different economic implications of the two policies.

A first remark regards the compliance costs imposed by the policy. In choosing abatement, firms must strike a balance between the additional cost related to a major abatement effort and the price they have to pay for each unit of emission. Under a tax this price is constant, so firms will tend to abate less in the face of expansionary shocks. On the contrary, under cap-and-trade the price is endogenous and is determined by the demand of emission permits in the international market, being the supply completely inelastic. It should be noted that, due to the cyclical behavior of abatement costs in the model, which increase during economic booms and decline during economic recessions, a price instrument imposes a lower burden on the economy compared to a quantity instrument.

A second remark is related to the role of uncertainty on the relative welfare performance of the instruments. When uncertainty is introduced, a price instrument is to be preferred to a quantity instrument if the function of abatement marginal benefit is flatter than the abatement marginal cost function, as shown by Weitzman (1974). For stock pollutants as GHGs this tends to be case, especially in a short-run perspective. We will see that a similar mechanism is at work in our model and explains in part the welfare performance of the carbon tax under TFP uncertainty.

The last remark is about the general equilibrium effects that in our analysis of environmental policy performances are important as much as the relative slopes of the marginal benefits and marginal cost curves. Thanks to the general equilibrium set-up, our model provides the possibility to study the interaction between environmental policy and key macro-variables, so that important feedback effects can be observed. In addition, the New Keynesian nature of our model allows us to take into account the distortions induced by imperfectly competitive markets and the frictions due to price adjustment costs and investment adjustment costs.[16]

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[16] On the relevance of price rigidities for welfare properties and the performance of the instruments, see Annicchiarico and Di Dio (2015).

2.3.2 Monetary Policy

The monetary authority manages the short-term nominal interest rate R_t in accordance to the following simple interest-rate rule:

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\iota_\Pi} e^{u_{R,t}}, \tag{24}$$

where R and Π denote the deterministic steady-state of the nominal interest rate and of the inflation rate, $\iota_\Pi > 1$ is a policy parameter and $u_{R,t}$ is an exogenous process capturing the possibility of monetary policy shocks, that is: $u_{R,t} = \rho_R u_{R,t-1} + \varepsilon_{R,t}$, with $0 < \rho_R < 1$ and $\varepsilon_R \sim i.i.d.\ N(0, \sigma_R^2)$. For $\iota_\Pi > 1$, the nominal interest rate varies by more than inflation, implying a restrictive monetary policy in the face of inflation (as the real interest rate would increase) and an accommodative policy in the face of deflation (as the real interest rate would decrease).

2.4 Trade Block, Current Account and Real Exchange Rate

In a two-country setting imports of Home are translated into exports of Foreign, therefore

$$X_t^* = M_t = (1-\kappa)\left(\frac{S_t P_t^{D^*}}{P_t}\right)^{-\rho} Y_t. \tag{25}$$

Likewise, exports of Home are translated into imports of Foreign

$$X_t = M_t^* = (1-\kappa)\left(\frac{P_t^D}{S_t P_t^*}\right)^{-\rho} Y_t^*. \tag{26}$$

The accumulation of Foreign assets for Home is determined by the current account relationship:

$$S_t F_t^* = R_t^* \left(S_t F_{t-1}^* + P_t^D X_t - S_t P_t^{D^*} M_t\right). \tag{27}$$

In the initial steady state F^* is set at zero, thus implying $P_t^D X_t = S_t P_t^{D^*} M_t$. Under a cap-and-trade scheme the current account equation includes a term representing the exchange of emission permits between the two countries, therefore

$$S_t F_t^* = R_t^* \left[S_t F_{t-1}^* + P_t^D X_t - S_t P_t^{D^*} M_t - P_t^E \left(E_t - \bar{E} \right) \right]. \tag{28}$$

The assumption of perfect financial capital mobility between Home and Foreign implies that the nominal exchange rate is determined in the Foreign exchange market as a result of the monetary policy conduct in the two countries.[17] On the other hand, the real exchange rate, defined as $S_t P_t^* / P_t$ (i.e. the ratio between the Foreign price level and the Home price level, where the Foreign price level is converted into domestic currency), is not only influenced by the time path of the nominal exchange rate, but also reflects the response of the final good price indexes to shocks and policy changes.

2.5 Resource Constraint and Stock of Pollution

The resource constraint of the economy can be derived by plugging the government budget constraint, along with the definition of profit of the intermediate sector and the expression for the current account position, into the household budget constraint:

$$P_t^D Y_t^D = P_t C_t + P_t I_t + P_t^D X_t + P_t^D AC_t - S_t P_t^{D^*} M_t + P_t \Gamma_K(I_t, K_t) + P_t^D \Gamma_{\mu_t}(\mu_t) + \frac{\gamma_P}{2} (\Pi_t^D - 1)^2 P_t^D Y_t^D. \tag{29}$$

The stock of pollution Z_t evolves according a natural decay factor $\eta \in (0, 1)$, and on the basis of current period Home emissions E_t, current period Foreign emissions E_t^*, and non-industrial emissions E_t^{NI}:

$$Z_t = \eta Z_{t-1} + E_t + E_t^* + E_t^{NI}. \tag{30}$$

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[17] It can be easily shown that by log-linearizing the two Euler equations (8) and (9) one obtains the familiar uncovered interest parity condition relating the rate of depreciation of Home currency to the nominal interest rate differential, which, in turn, depends on the inflation rates via the interest rate rules adopted by Home and Foreign monetary authorities.

3 Parametrization

The model is calibrated for the world economy and time is measured in quarters. Standard parameters, related to the New Keynesian formalization of the model, follow the existing literature (see e.g. Galí 2015). The discount factor β is set at a value consistent with a real interest rate of 4% per year, that is $\beta = 0.99$. The inverse of the Frisch elasticity of labor supply φ_L is equal to 1. By assuming that the time spent working at the steady state is 0.3, we obtain an implied value for ξ_L, the scale parameter related to the disutility of labor, of 3.8826. The depreciation rate of capital δ is set at 0.025 and the capital share α at 1/3. The degree of price rigidities, the parameter γ_P, is consistent with a Calvo pricing setting with a probability that price will stay unchanged of 0.75 (i.e. average price duration of three quarters), namely $\gamma_P = 58.25$. The inverse of the intertemporal elasticity of substitution φ_C is equal to 1.2, the parameter for capital adjustment costs γ_I is set at 3. Regarding the goods market, we set the elasticity of substitution among intermediate good varieties σ equal to 6 and the intratemporal elasticity of substitution between domestic and foreign intermediate goods ρ equal to 1.5, implying that domestic and foreign varieties are imperfect substitute. In line with the average values of the import/GDP ratio observed for the world economy in period 2010-2015 according to World Bank data, we assume a propensity to import of 0.3, that implies a share of domestic intermediate goods used in the final sector κ equal to 0.7. The steady-state target inflation is equal to zero ($\Pi = 1$), while the relative price of intermediate goods and the real exchange rate, p^D and S^R, are both normalized to 1. Turning to parameter related to monetary policy, we set the interest rate response to inflation, ι_Π, at 1.5.

With regards to the environmental part of the model, we refer to previous environmental DSGE models and Integrated Assessment Models for climate change, in order to obtain plausible values for environmental parameters. We set the elasticity parameter of emissions to output γ at 0.304 and the pollution decay factor η at 0.9979 following Heutel (2012), the parameter of the abatement cost function θ_2 at 2.8 as in Nordhaus (2008), while θ_1 is normalized to 1. The parameter determining the size of the adjustment cost of abatement changes, γ_μ, is set at 1.5, that is half of the value of the parameter determining the size of the capital adjustment costs, while the parameter governing the asymmetry of these costs, ψ_μ, is set at 10. To obtain the steady-state level for emissions, we refer to the policy runs of the RICE-2010 model, specifically to the simulation results for year 2015. We take the level of global carbon emissions, and the level of global industrial emissions, both measured in gigatons of carbon (GtC) per year, then we assume that Home and Foreign contribute in equal way to output and emissions. Through these data we are able to recover the level of global non-industrial emissions, emissions for domestic and foreign country, and the steady state level of output in the intermediate goods sector.

Finally, by looking at the RICE model, we know that abatement costs, measured as fraction of output, are equal to 0.00013. This calibration strategy delivers implicit values for the pollution stock in model units, emission intensity, the scale parameter ϵ and the price of emission permits (or the carbon tax). The calibration is, in fact, done in such a way that the steady-state values of the endogenous variables are the same in the two environmental policy regimes, and the permit price of the cap-and-trade regime is equal to the carbon tax. When the economy is hit by shocks, however, the price of emission permits will change reflecting the changes in the market conditions. Regarding the negative externality on production, we calibrate Λ on the basis of the total damage for year 2015, measured as fraction of output, that amounts to 0.0030. Estimating that the pre-industrial atmospheric CO_2 concentration ($\overline{Z}$) represents 3/4 of the total pollution stock, we obtain a value for the intensity of negative externality on output χ and for the total factor productivity A.

Finally, for the stochastic processes of the model we assume a high degree of autocorrelation for the exogenous shocks by setting ρ_A and ρ_K at 0.85, while ρ_R is set at 0.5. Table 1 lists all the parameters of the model.[18]

4 International Transmission of Shocks and Environmental Policies

In this Section we analyze the international transmission of asymmetric shocks under two alternative environmental regimes. We analyze the effects of three temporary shocks hitting

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[18] The model is solved with Dynare. For details, see http://www.dynare.org/ and Adjemian et al. (2011).

only Home: (i) a positive productivity shock increasing the TFP, (ii) a positive shock on the risk-free interest rate set by the monetary authorities, and (iii) a negative shock on the quality of capital. We focus on a selection of macroeconomic and environmental variables, providing two different representations of the results for each of the three shocks considered. In the first representation, we compare the response of the economy under the two alternative policies starting from a common steady state scenario in which an environmental regulation is already in place. In the second representation, we compare the performance of these instruments with a situation in which no environmental regulation is implemented.[19] With this approach we are not only able to compare the dynamic properties of the policy instruments, but also to observe how the dynamics change compared to a no-policy scenario, based on the environmental regime adopted.[20]

In the last part we solve the model under different assumptions and undertake stochastic simulations to compute some summary statistics.

We can identify three international transmission channels of shocks: i) an "aggregate demand channel" through which an expansion of Home demand affects the demand for foreign goods via trade; ii) a "competitiveness channel" by which relative changes in the cost conditions in Home lead to expenditure shifting effects and to terms-of-trade effects; iii) a "financial channel" that drives movements of the nominal exchange rate and reflects the behavior of the monetary policy.[21] We will see that the strength of the first two channels depends significantly on the environmental regime adopted. Under a cap-and-trade environmental regime, there is an additional channel operating through the cross-border exchange of emission permits. In many occasions this additional channel will be able to change the magnitude and the sign of the international spillovers.

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[19] Using the same parametrization of Section 3, the model is solved setting p_E at zero, so implying zero abatement. In the no-policy case the steady level of all the relevant macroeconomic aggregates (production, consumption and emissions) is higher along with the level of welfare. This is because the model features only a weak negative externality of pollutant on productivity and not the burden of the environmental regulation.

[20] The response to shocks of an economy in the absence of environmental regulation is available in the Supplementary Material.

[21] The competitiveness channel and the implications of changes in the terms of trade have been analyzed extensively in the literature on trade and the environment. More stringent environmental regulations alter the production costs of firms (and so their competitiveness), influences the pattern of trade and triggers terms-of-trade effects that may be positive or negative. In our model with intra-industry trade a more stringent environmental policy generates positive terms-of-trade effects. On this point, see Rauscher (2005).

Before we proceed, a remark is in order. The bulk of economic fluctuations could be interpreted as an equilibrium outcome resulting from the economy's response to shocks. From this perspective, cyclical fluctuations and international spillovers do not necessarily reflect inefficiencies. The response of the economy is the result of the optimal choice made by rational agents that react to exogenous disturbances taking into account the underlying environmental regime and the monetary policy. The behavior of the economy also depends on the size of adjustment costs and on the degree of price stickiness.

4.1 TFP Shock

Figures 1 and 2 show the economy's response to a one percent transitory increase in productivity hitting only Home. Continuous lines refer to the dynamic response of the economy under a carbon tax, while dashed lines report the response under a cap-and-trade scheme.

By looking at Figure 1 it is possible to see that, in response to a positive shock on the TFP, domestic consumption, investment and output immediately increase.[22] The marginal productivity of labor and capital goes up, so firms are induced to expand production. Households' lifetime wealth increases and therefore consumption expands.

We observe that all these positive effects are magnified under a carbon tax regime, because the environmental-related cost component borne by firms tends to increase by less than under a cap-and-trade (see Figure 2) and firms are allowed to pollute more. In addition, adjustment costs on abatement do not allow an immediate adjustment of abatement effort, so further limiting the expansion of production under a cap-and-trade.

The increase in investments and consumption generates a higher demand for imports (the aggregate demand channel). However, as a positive technology shock implies that the Home economy will be more productive than the Foreign one for a while, Home varieties are relatively cheaper and exports increase (the competitiveness channel). The consequent worsening of the Home's terms of trade has income effects that tend to reduce the expansion of aggregate demand. The decline of domestic prices, along with the action of the interest rate rule (24),

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[22] In response to a positive technology shock labor is countercyclical, as usual in New Keynesian models. Nominal rigidities do not allow an immediate adjustment of prices and this has a negative impact on the labor market. On the other hand, under flexible prices the shock is more expansionary. See the Supplementary Material.

yielding a lower nominal interest rate, gives rise to a depreciation of the domestic currency (the financial channel).[23] Concerning the trade balance, regardless of the environmental policy, a typical J-curve effect arises: in the first periods after the shock the price effect dominates, imports are costlier than exports and this deteriorates the trade balance. At later stages quantities adjust: the volume of export starts to rise because of the higher Foreign demand for domestic goods that are relatively low-priced, while after the initial increase imports decline. At the earlier stages of the adjustment we also observe a deterioration in the foreign asset position of Home, followed by a steady increase.[24]

The expenditure switching from Foreign to Home goods, due to the competitiveness channel, explains the impact of this shock on foreign output and investments. However, looking at Foreign investment and output we note that they behave very differently depending on the environmental regime. As remarked above, in fact, under a carbon tax, domestic firms pollute more than under a cap-and-trade and output expands by more, whereas abatement costs do not vary significantly. This is because firms facing a constant carbon tax do not incur a higher marginal cost per unit of emissions when their production expands. The greater expansion of Home output then explains the initial positive spillover effects on Foreign output and investments. However, these positive effects already fade away after two quarters, because of the relatively lower demand for foreign goods.[25]

Under a cap-and-trade regime, instead, both Foreign output and investments decline immediately after the shock and remain under their steady state level all along the simulation period. The initial increase in imports from Home is in fact lower than that observed under a carbon tax, because lower is the expansion of Home income and demand following the positive technology shock. In this case both the competitiveness and the aggregate demand channels are weaker. In addition, the asymmetric shock determines an outflow of emission permits from

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[23] Recall that in Home monetary policy will be accommodative in response to a positive TFP shock. The real interest rate on the risk free asset will go down, further inducing a higher consumption and a shift of saving toward physical capital.

[24] The response of trade balance and of net external asset position of Home crucially depends on the elasticity of substitution ρ between domestic and foreign goods. It can be shown that in the case of imperfect complementarity (i.e. $0 < \rho < 1$), in fact, Home trade balances never improve during the adjustment process, while we observe a stronger depreciation of the domestic currency.

[25] Under flexible prices, Foreign output declines on impact also under a carbon tax. In this case Home prices immediately adjust, reflecting the lower costs, and the expenditure switching toward domestic production will materialize on impact. See the Supplementary Material.

Foreign to Home allowing domestic firms to pollute more, while the price of emission permits, determined in the international market, increases sharply. In Foreign, this outflow of emission permits, along with the sharp increase in the permit price and the expenditure shifting toward Home production, explain the reduction of output and investment as well as the dampened reaction of Foreign consumption.[26] Foreign consumption slightly increases thanks to the improvement in the Foreign terms of trade (that is weaker in this environmental regime) and to the sharp contraction of investments. In Figure 2 the time path of emissions is shaped by the underlying policy and reflects the behavior of output in both countries.

Welfare increases in both countries mainly as a result of the higher consumption. This is due to the fact that the shock is small and temporary, and the environmental damage is small. As expected, and consistently with early studies on prices-versus-quantity regulation under uncertainty, the positive effects are larger under the carbon tax.[27]

In Figure 3, we consider the performance of the two environmental regimes compared to the no-policy scenario for selected variables. For each variable results are reported as percentage point deviation of the dynamic response under environmental policies (represented in Figures 1 and 2) from its dynamic response in the no policy scenario case. The main evidence is that, under a carbon tax, domestic and foreign macroeconomic variables dynamically behave as in the absence of any environmental regulation. The time path for emissions are the same as well. The tax policy allows firms to increase emissions at a constant marginal cost, while the introduction of a cap-and-trade, on the contrary, changes drastically the dynamics of the model, both in sign and magnitude. Under the cap-and-trade, the economy departs from its "natural" trajectory. A quantity policy that pegs the aggregate level of pollutant emissions, forces firms to devote more resources to comply with the environmental regulation (i.e. both investing in abatement and purchasing permits). This diminishes the level of output available for consumption and investment, as shown above, implying a milder expansion of these variables in Home and even a reduction in Foreign.

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[26] If the cap were national, with no possibility of exchanging permits across countries, Foreign output would increase on impact, while the expansion of Home production would be lower. See the Supplementary Material.

[27] This result is fully in line with the findings of Pizer (1999, 2002), Hoel and Karp (2002), and Newell and Pizer (2003) who extend the Weitzman's analysis to show that a tax is likely to be more efficient than a cap-and-trade system in the face of uncertainty.

4.2 Monetary Policy Shock

In Figures 4 and 5 we consider the response of the economy to a monetary policy shock. In detail, we assume an increase of 0.50 percent in the innovation $\varepsilon_{R,t}$, implying a restrictive monetary shock hitting Home. The rise in the interest rate reduces investment in physical capital and consumption, triggering a fall of output in Home. In addition, the domestic currency appreciates, so that we observe a short-lived improvement in the trade balance. The price effect on imports dominates the volume effect on net exports which materializes only at later stages. Consistently, the external asset position first improves and then worsens. It can be shown, in fact, that exports decline more than imports as a result of the appreciation of the domestic currency (the financial channel). The appreciation of the domestic currency then exacerbates the recessionary effects of the monetary policy shock.

The main Home macroeconomic variables show the same patterns in the two regimes, although the immediate response to the shock is different in magnitude. In particular, a carbon tax amplifies the effects induced by the contractionary shock, while a cap-and-trade policy reduces the impact on output, investment and consumption. The intuition for the different result is the following. In the face of this contractionary shock firms spend less on abatement since they pollute less. Under a cap and trade, firms demand less emission permits, so inducing a decrease in their price, while under the carbon tax the unit cost of emission does not change. The decline of price of emission permits then partially alleviates the negative effects on production for the Home firms.

Looking at the Foreign macroeconomic variables the differences generated by the two environmental regimes are remarkable. Under the carbon tax regime the aggregate demand channel depresses Foreign output that follows the decline of Home imports. On impact we observe a negative reaction of Foreign consumption and investment. In the following periods the expenditure switching effect prevails and these variables recover quickly, following the movement of the trade balance. Emissions follow the same pattern of production in both countries, while abatement costs do not change significantly.

Under the cap-and-trade regime, instead, Foreign output increases, and so investments. The tightening of the monetary policy generates a contraction of Home production (although less severe than under a carbon tax) and a decline in the Home demand for permits, magnified by the existence of adjustment costs on abatement and by the quasi-irreversibility of the abatement technology implied by the functional form (14). The price of carbon decreases sharply and we observe a reallocation of permits in favor of Foreign, where the fall in the price of permits makes abatement extremely uncompetitive. Foreign emissions then increase along with a sharp fall in abatement costs. The reduced abatement costs and of permit prices free up resources for firms and translate into higher production and investments for Foreign. Foreign consumption instead slightly declines as a result of the worsening in the terms of trade of this country.[28]

In the two policy regimes the effects on welfare reflect the differences discussed above. The contractionary monetary policy shock is detrimental for both economies, but of course more for Home, while the negative effects are magnified under a carbon tax.

Looking at Figure 6, it is possible to observe that, under a carbon tax, from a purely dynamic point of view the economy behaves like in the no-policy scenario. In the case of a cap-and-trade policy, instead, the results distance themselves more from the no-policy dynamic path, confirming the results found in the case of the TFP shock.

4.3 Quality of Capital Shock

We now focus the attention on the economy's response to a one percent negative shock on the quality of capital. See Figures 7, 8 and 9. For this shock we also plot the spread between the real return on capital and the real interest rate on the risk free asset. The negative shock decreases the capital value, and, at the same time, the effective quantity of capital available for production. This shock simultaneously depresses demand and supply as it implies a reduction of investments and an increase in the marginal cost of firms that suffer a deterioration of their production capacity. Since the shock is temporary, households find it optimal to decrease investment immediately, given the lower marginal product of capital, while consumption follows a U-shaped dynamics. In general, we observe a negative co-movement of the main real variables: consumption, investment and output. In the face of this deflationary shock the central bank will react by decreasing the nominal interest rate by more than proportionally, according to

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[28] If the cap were set at national level, with no trade of permits between countries, Foreign output would fall as well. See the Supplementary Material.

rule (24), so partially offsetting the negative impact of the adverse shock. Nevertheless, as a result of the negative shock on the quality on capital, the spread between the return on capital and the risk free rate increases on impact. The decline of domestic income leads to a lower demand for all goods, both domestic and foreign through the aggregate demand channel.

In Foreign the value of capital is relatively higher and investments increase. Foreign output increases, while consumption decreases following the deterioration of the terms of trade. The sharp increase in Foreign expenditure tends to boost the demand for Home goods. Following the contraction of Home imports, combined with the expansion of its exports, the trade balance improves and stays positive up to the fifth quarter after the shock. As a result the external asset position improves and then worsens, while the domestic currency first depreciates and then appreciates through the financial channel. Home suffers from a strong competitiveness loss that is detrimental for exports.

Considering the dynamic implications of the underlying environmental policy, we note that, under the cap-and-trade regime, the decline of Home production is milder than in the case of carbon tax. As in the case of a recessionary monetary policy shock, we observe a reallocation of permits from Home to Foreign and a fall in the their price. The fall in the emission permits price alleviates the negative effects of the capital quality shock for Home producers. Foreign producers, in turn, take advantage of the lower price of emissions on the market by buying permits, and expand their production. Also in this case, adjustment costs on abatement tend to amplify the fall in the demand for permits and so of their price. On the contrary, under the carbon tax the unit cost of emission is given, and in the face of the negative shock firms will find it optimal to produce less and face lower abatement costs. The major contraction of output gives rise to lower consumption and investments, and to a larger drop of imports. At the same time Home emissions decline by more than under a cap-and-trade, while Foreign emissions increase by less. The response of welfare in the two economies under the two regimes reflect the discussed differences. In both economies welfare decline by more under the carbon tax.

The different dynamic behavior of the economy under the two environmental regimes is even more evident in Figure 9, where we show the response of selected variables in percentage point deviation from the no-policy scenario case. Under a cap-and-trade, on impact, the response of output is about 0.4 p.p. higher than in the no-policy case, while as observed for previous shocks, under the carbon tax the economy mimics the dynamic behavior observed in the no-policy case.

4.4 International Spillovers, Pattern of Trade and Monetary Regime

In this subsection we explore the role played by the pattern of trade and by monetary policy in the transmission of the business cycle across different environmental policy regimes. In particular, we solve the model under three different assumptions in turn: (i) domestic and foreign bundles of goods are imperfect complements, rather than imperfect substitutes, (ii) higher degree of openness to international trade, (iii) currency union. To address these points in a parsimonious way we look at the standard deviations for Home and Foreign output and at their correlation. These statistics, commonly used for policy evaluation in business cycle models, measure volatility and co-movement between variables. The relative standard deviation accounts for the size of the international spillovers, while the correlation accounts for their sign. Both statistics are computed using stochastic simulations considering each shock in turn hitting only Home. In this way we are able to measure the magnitude and the sign of international spillovers under different sources of uncertainty for the two environmental regimes and for the no-policy case.[29]

We start by considering the benchmark case, where the model is solved under the baseline calibration of Table 1. Results are reported in Table 2, where σ_{Y^D} and $\sigma_{Y^{D*}}$ denote the standard deviations of Home and Foreign output, while $\rho(\cdot,\cdot)$ is the coefficient of correlation between variables. We note what follows.

First, the volatility of Home output and the relative standard deviation of Foreign output are found to be larger under a carbon tax regime. The higher volatility of domestic output under a carbon tax is just the result of the fact that under a cap-and-trade the emission permit price is procyclical and therefore tends itself to stabilize output in response to shocks. The finding that a quantity instrument generates less volatility is consistent with previous results provided by closed-economy E-DSGE models (e.g. Fischer and Springborn 2011, Annicchiarico

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[29] Given the optimal decision rules, for each shock we draw 200 realizations of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.001.

and Di Dio 2015, Dissou and Karnizova 2016). In addition, we find that a carbon tax amplifies the magnitude of the international spillovers, in particular when the economy is hit by monetary policy shocks for which we note a much higher relative standard deviation of Foreign output than that observed under a cap-and-trade.

Second, the underlying environmental regime alters the sign of the relationship between output of the two countries in response to a monetary policy shock. On the one hand, we observe that in response to TFP and capital quality shocks, Home and Foreign outputs are negatively correlated, both under a carbon tax and a cap-and-trade. On the other hand, under monetary policy uncertainty the relationship between Home and Foreign outputs is positive under a carbon tax and negative under a cap-and-trade scheme. The intuition is the following. Under a carbon tax, shocks hitting the nominal interest rate are mainly absorbed by the slow price adjustment. The sharp changes on Home output propagate abroad via trade and via the exchange rate movements with no other active counterbalancing forces. Therefore positive shocks translate into positive effects on Foreign output. By contrast, under a cap-and-trade regime monetary policy shocks occurring in Home are absorbed over time by the slow price adjustments, and partially counterbalanced by the countercyclical changes in the permit price that stabilize Home output and reduce the international spillovers via the trade channel. Likewise, in Foreign the effects on output of the changes in aggregate demand driven by Home dynamics are more than counterbalanced by the effects induced by the changes in the price of emission permits. Therefore, under a cap-and-trade regime shocks occurring in Home affect Foreign also through the exchange of emission permits, reverting the sign of the relationship between Foreign and Home output.[30]

Third, the size of the correlation between Home and Foreign output is magnified under an international cap-and-trade regime in response to all the shocks considered. The mechanisms behind the permits market, the increase or decrease of permits price, as well as the allocation of permits from one country to another, reflect strongly on the production of both countries.[31]

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[30] Under a cap with no cross country exchange of emission permits, the volatility of Home output would be lower than for the case of cap-and-trade, while the degree of correlation between Home and Foreign output would be smaller for TFP and capital quality shocks, and positive for the monetary policy shocks. See the Supplementary Material.

[31] In the Supplementary Material we reproduce our results under the assumption that firms are able to fully adjust their abatement effort and under the assumption of symmetric adjustment costs (i.e. reversibility of

Finally, we note that, consistently with the findings of the previous section, results are very similar in the case of carbon tax and in the no-policy scenario. This evidence is true for the benchmark case, but it is also robust to different assumptions regarding the pattern of trade and the monetary policy regime that we discuss in the rest of the section.

Table 3 reports the results assuming that foreign and domestic bundles of goods are imperfect complements rather than imperfect substitutes, in particular, we set the elasticity of substitution ρ in equation (19) at 0.5. The Home economy is now less volatile, but international spillovers are greater. We note in fact that the standard deviation of Home output, σ_{Y^D}, is lower than in the benchmark case, while the relative standard deviation of Foreign output is larger. Therefore, the effects of the shocks are now shared more intensively with Foreign. The only exception is observed for monetary policy shocks under a cap-and-trade regime, where the relative standard deviation of Foreign output is slightly lower compared to the baseline model. As discussed above, under a cap-and-trade, following a monetary policy shock hitting only Home, there will be an inflow or an outflow of emission permits, along with a change in their price, able to generate an opposite reaction of Foreign output from that observed for Home output. In Table 3 however, this cross-border reallocation of production is weakened by the hypothesis of imperfect complementarity.

Table 4 presents the results under the assumption that the share of imported varieties, $1-\kappa$, in the final good production function is equal to 0.5 instead of 0.3. We observe that with a higher degree of openness the relative standard deviation of Foreign output is higher that in the benchmark case, while the volatility of Home output is lower. Intuitively, in more integrated markets the transmission of shocks is intensified. We observe that under a cap-and-trade regime, a higher degree of openness sharply mitigates the (negative) correlation between Home and Foreign output. The propensity to import is now higher, therefore changes in Home income will reflect at a greater extent on import demand and so on Foreign output, partially offsetting the counterbalancing effects derived from the exchange of emission permits.

Finally, Table 5 presents the results under the assumption that Home and Foreign share the

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abatement technology). We note that, while in the first case the difference between regimes are minor than in the benchmark case of Table 2, with costly abatement adjustment the relative standard deviation of Foreign output is always significantly higher under a carbon tax.

same currency, therefore the two countries are subject to the same monetary policy which now responds to an average of the two inflation rates. With this exercise we are so able to sterilize the effects of exchange rate adjustments. We note that in response to the TFP shock under the carbon tax the correlation between Home and Foreign output turns out to be positive and much less negative under a cap and trade. Following a positive TFP shock hitting Home, the monetary authority will react to the price decline of Home by reducing the nominal interest rate in the currency union. This accommodative monetary policy will induce an expansion also in Foreign, compensating the negative expenditure shifting effects induced by the more favorable cost conditions of Home producers. On the other hand, under a cap-and-trade regime, where the possibility of importing emission permits from abroad diminishes the positive spillover effects on Foreign output induced by the common monetary policy, the correlation remains negative, but the intensity of the relationship is weaker.

5 Welfare Analysis

In this Section we compare the cross-country performance of alternative environmental regimes using the welfare as metrics. In particular, the welfare measure we use is the unconditional expectation of the lifetime utility function of households, given by:

$$SW = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\varphi_C}}{1-\varphi_C} - \xi_L \frac{L_t^{1+\varphi_L}}{1+\varphi_L} \right), \tag{31}$$

where $\mathbb{E}$ is the expectation operator. As done in the previous Section, we assume that only Home is hit by shocks. As a benchmark we consider the no-policy case scenario. Let $SW_{no-policy}$ and SW_{policy} denote the values of the welfare measure SW attained under the no-policy case and under an alternative environmental policy regime that can be a carbon tax or a cap-and-trade policy. To evaluate the welfare effects we the consider the percentage deviation of SW_{policy} from $SW_{no-policy}$, considering each source of uncertainty at a time. Table 6 presents the results for the benchmark case, while Appendix B presents some additional findings under different hypotheses regarding the pattern of trade and the monetary regime.

Given our parametrization of the damage function and the assumption that the stock of pollutant affects only productivity, we observe that each environmental policy entails a welfare loss when compared to the no-policy scenario case. This is due to fact that we are not considering the optimal environmental policy.[32]

The relative performance of the policies seems to depend on the source of uncertainty, in accordance with the fact that different types of shocks propagate in different ways and therefore result in different incentives for consumers and firms.[33] The carbon tax performs slightly better than the cap-and-trade policy in the face of the TFP shock, while the opposite is true for the capital quality shock. Even though we observe some differences, both for Home and for Foreign, we are not able to reject the hypothesis that the mean and the median of welfare are equal in the two regimes. Both the t-test and the Wilcoxon rank sum test deliver probability values higher than 25% for all shocks considered. This finding suggests that there are general equilibrium effects that offset the welfare differences that are usually observed in partial equilibrium analyses.

To complete our welfare analysis, we now assume that utility is negatively affected by pollution. In particular, in addition to the damages to TFP, the stock of pollution affects welfare directly. To this end we modify our welfare measure as follows:

$$SW = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\varphi_C}}{1-\varphi_C} - \xi_L\frac{L_t^{1+\varphi_L}}{1+\varphi_L} - \chi\left(Z_t - \overline{Z}\right)\right], \tag{32}$$

where χ is calibrated as in $\Lambda_t = \exp[-\chi(Z_t - \overline{Z})]$. The results are reported in Table 7. In this case, the welfare under environmental policy is higher. Considering the different sources of uncertainty we still observe that a carbon tax delivers higher welfare under TFP shocks and lower under capital quality shocks. However, as before, the differences across regimes are not statistically significant.[34]

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[32] Recall that in steady state welfare is the same under a cap-and-trade and a tax. In our baseline calibration the resulting environmental policies over-internalize environmental externalities.

[33] See e.g. the discussion in Kelly (2005) and Dissou and Karnizova (2016).

[34] In Appendix B we report the welfare analysis under the assumption that there is only a negative environmental externality on utility.

6 Conclusions

Climate change represents one of the most pressing policy issues at stake. A clear understanding of the economic repercussions associated with the implementation of different environmental policies is crucial. For this reason, environmental issues have been recently raising the hurdles also for DSGE modeling. In this respect, the paper presents a stylized but rigorous framework to study the international dimension of climate actions in a two-country fully interdependent economy in the presence of uncertainty. With this tool in hand, we are able to provide some clarifications on the role played by environmental regimes in shaping the propagation of shocks between countries.

Our results show how the international transmission mechanism of uncertainty is influenced by the policy tool chosen to stabilize greenhouse gas concentrations in the atmosphere. Unexpected shocks hitting a country may generate spillover effects, whose sign and intensity depend not only on the nature of uncertainty, but also on the underlying environmental regime. The cross border spillover effects are always magnified under a carbon tax, especially when it comes to monetary shocks. On the contrary, under a cap-and-trade regime, where countries can trade emission allowances, we observe less cross-border pressure on output. This is because under an international cap-and-trade scheme the outflow of permits towards an economy in expansion reduces the positive spillover effects from the international trade channel. Similarly, the inflow of emission permits from an economy in recession lessens the negative cross-border effects from international trade and may revert the sign of the spillover. The degree of openness, the trade pattern and the underlying monetary policy regime are shown to play a non-trivial role in this interplay among economic and environmental policy variables. From a welfare perspective, however, the ranking between the two policies depends on the type of shock considered, even if the differences across regimes are not statistically significant.

The model studied in this paper leaves out a number of features that have been identified as potentially important for understanding the economic implications of climate actions in open economy. First, the model does not allow for international mobility of labor and physical capital. Clearly, this poses a limit to the re-allocation of production activity resulting from asymmetric and persistent shocks. Second, the importance of the pattern of trade in determining the propagation mechanism is only touched upon in this paper and deserves further and deeper investigation in a context where firms structure their production through outsourcing and offshoring of activities within so-called global value chains. Third, in this paper the economy is composed by two identical economies. Similar investigations should be carried out allowing for a certain degree of asymmetry in technology and size between countries. This might also push the analysis toward the study of strategic trade policy and transboundary pollution. Finally, a further step to advance this analysis should regard a thorough analysis of the interaction between stabilization policies and economy-wide emission regulations. We leave these issues for future research.

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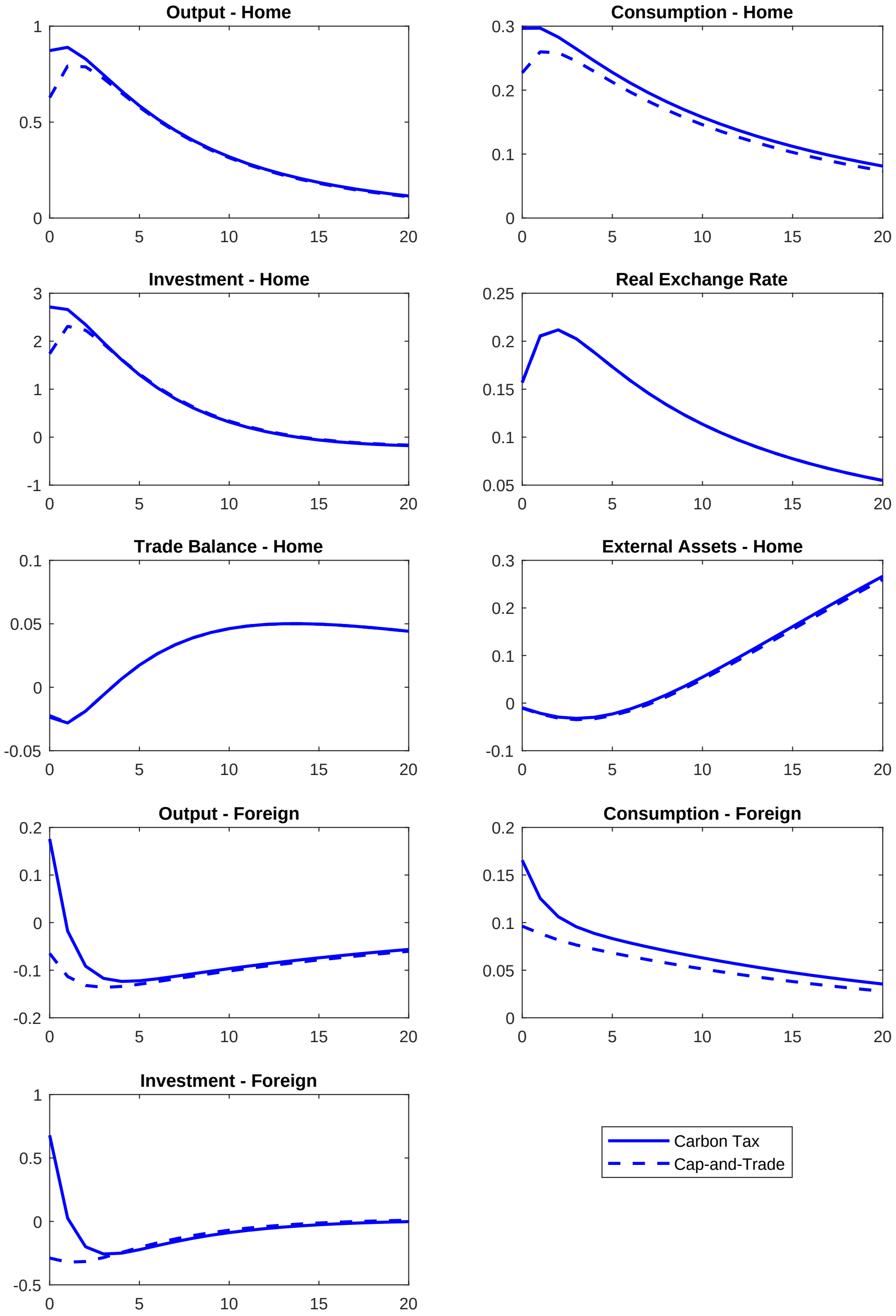
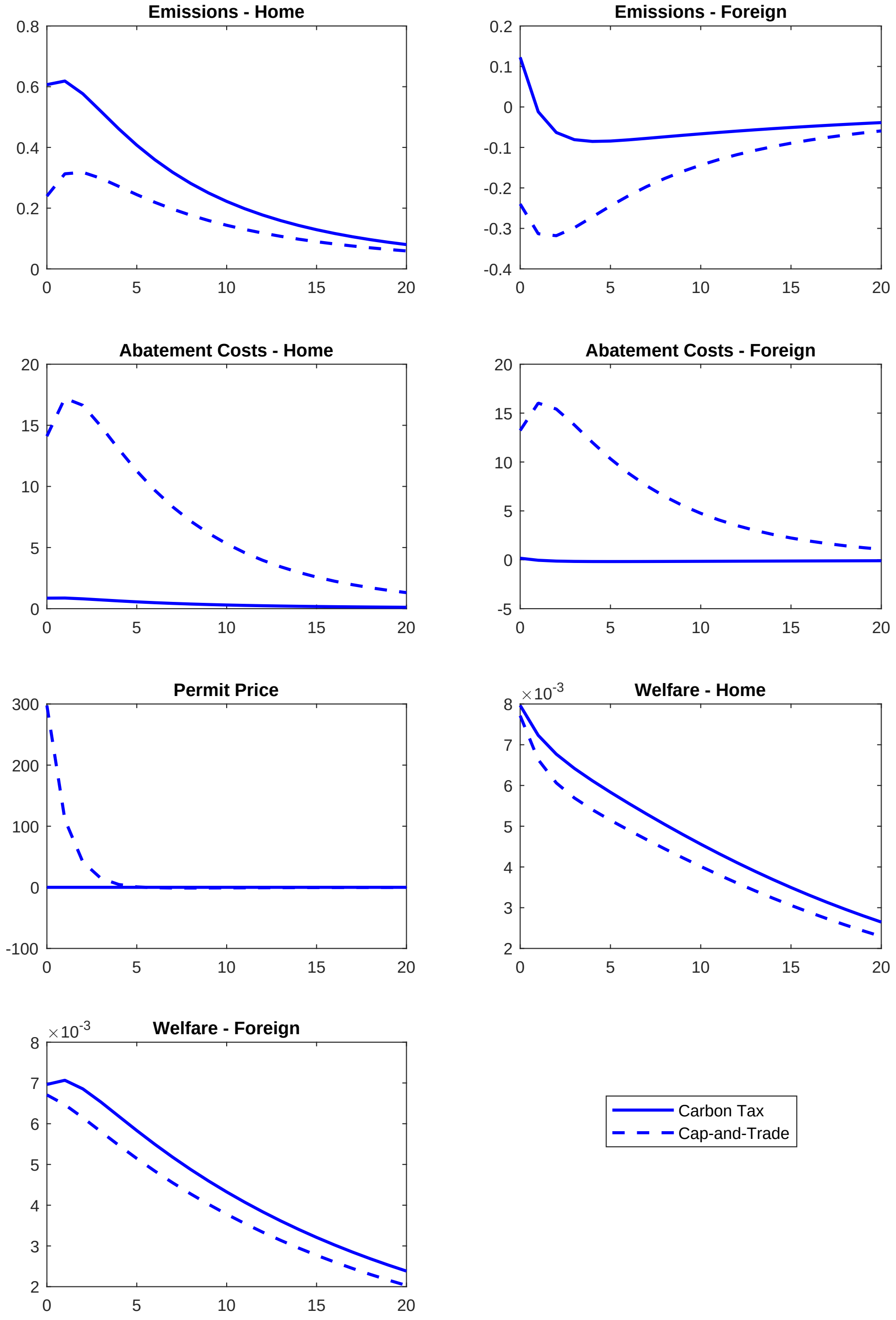
Figure 1: Dynamic Response to a 1% TFP Shock - Macroeconomic Variables



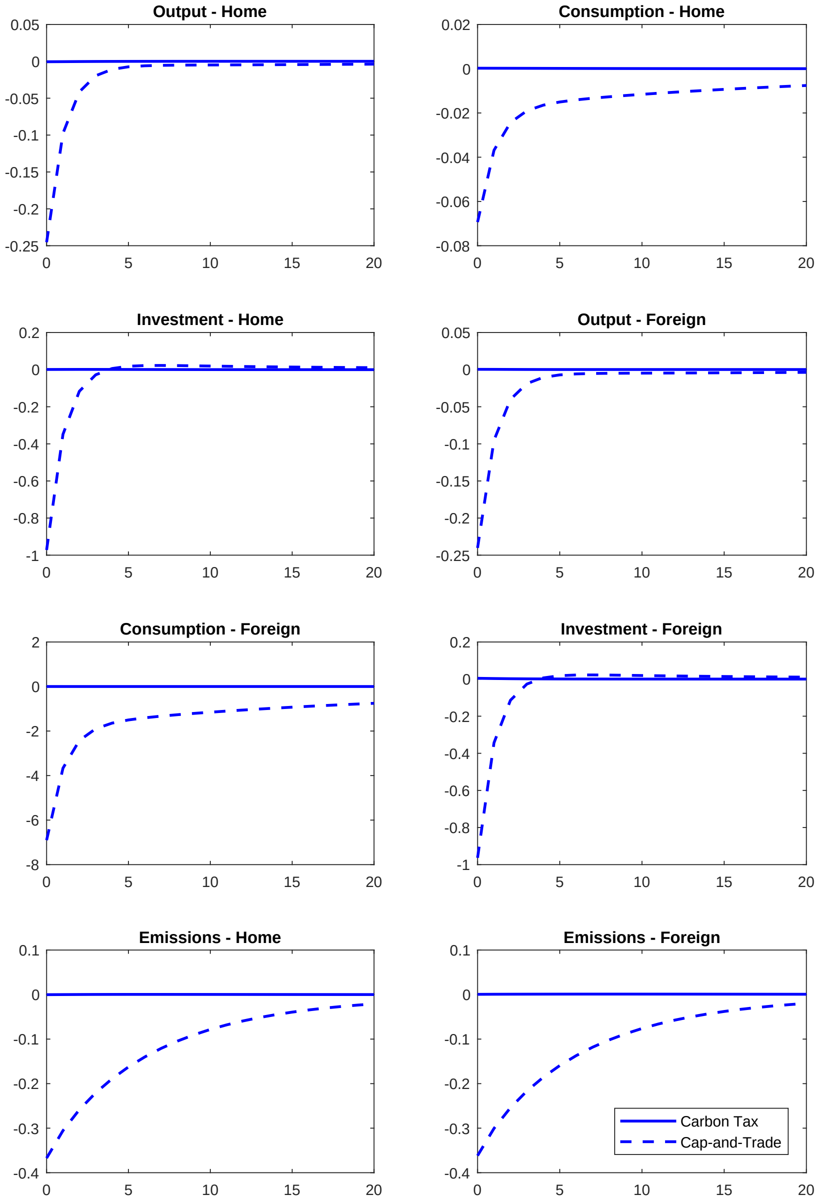
Note: the figure plots the impulse responses to a positive shock on TFP for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state with the exception of the trade balance and of the external asset position that are reported in percentage points from the zero steady state.

Figure 2: Dynamic Response to a 1% TFP Shock - Environmental Variables and Welfare



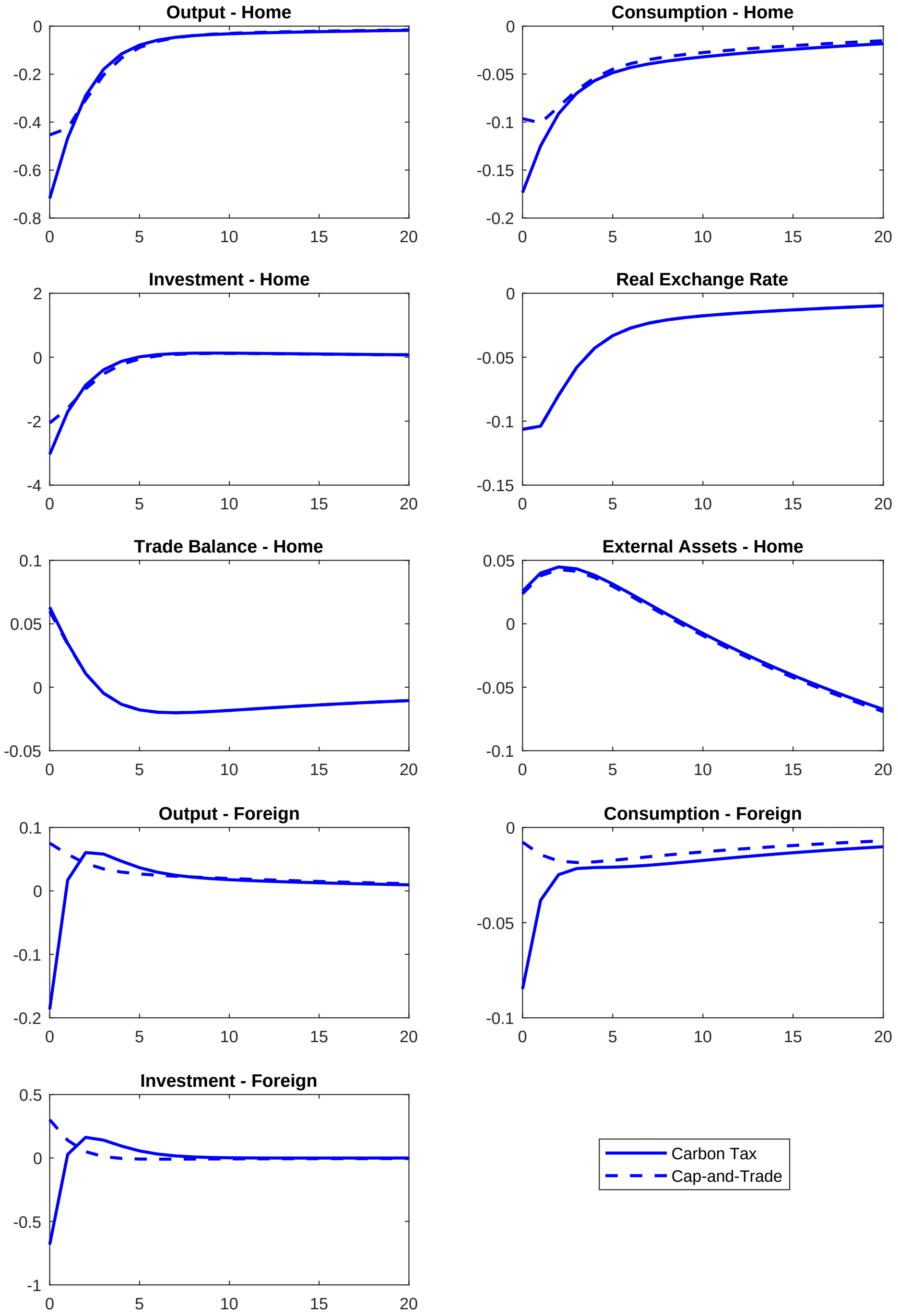
Note: the figure plots the impulse responses to a positive shock on TFP for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state.

Figure 3: Dynamic Response to a 1% TFP Shock - Policy v. No Policy Scenarios



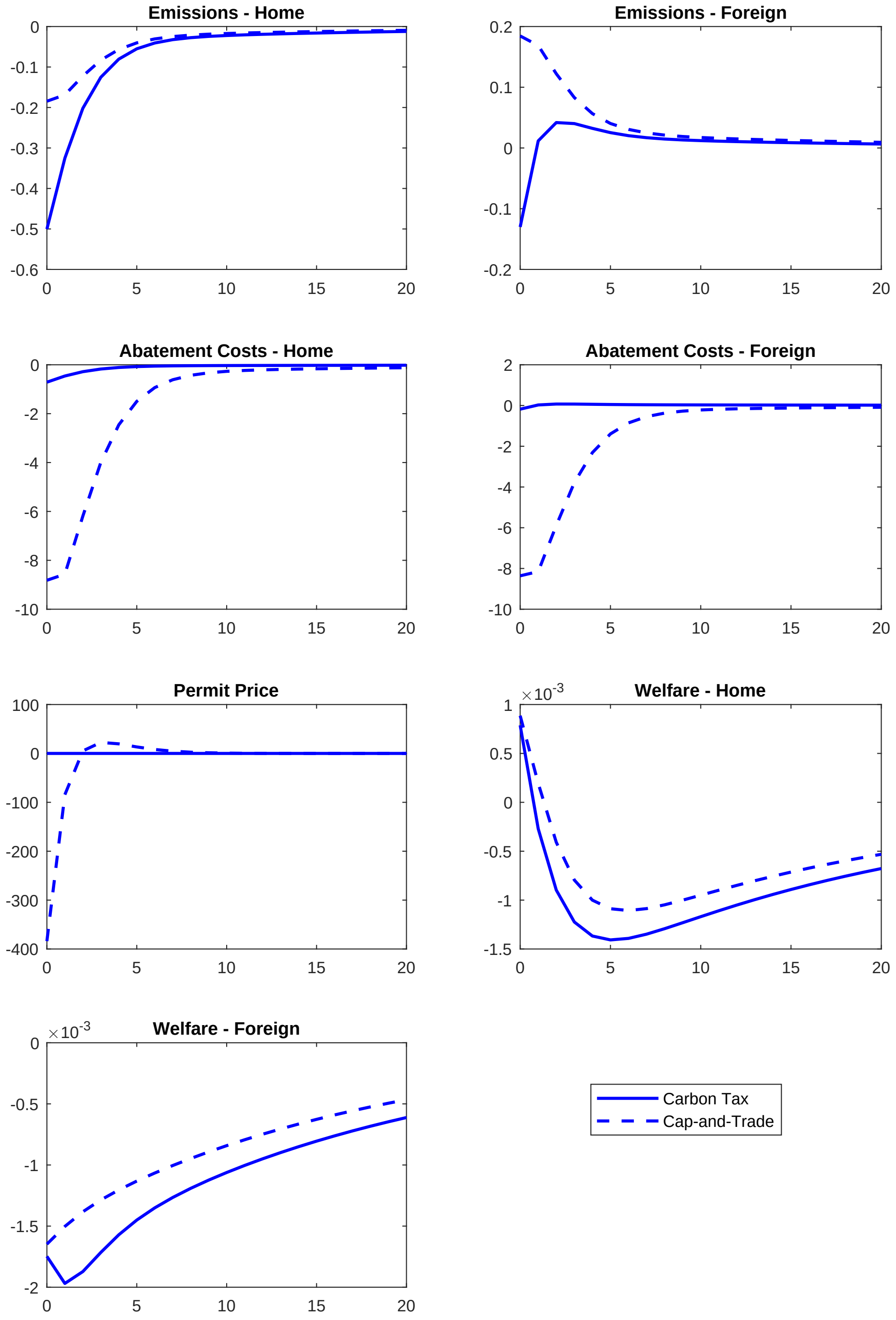
Note: the figure plots the impulse responses to a positive shock on TFP for a 20-quarter time horizon (horizontal axes); for each variable, say X, results are reported as percentage point deviation of the dynamic response under environmental policies, say x_{policy}, (represented in Figures 1 and 2) from its dynamic response in the no-policy scenario case, say $x_{no-policy}$.

Figure 4: Dynamic Response to a 0.5% Monetary Policy Shock- Macroeconomic Variables



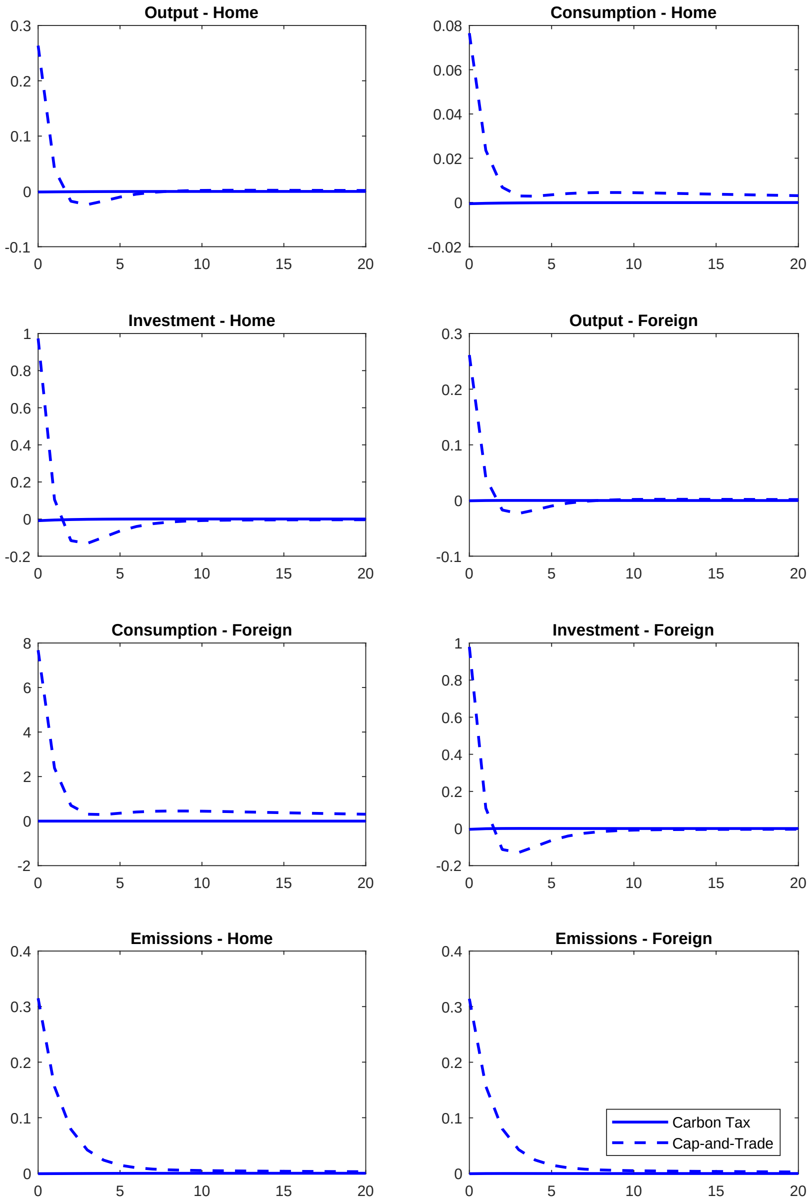
Note: the figure plots the impulse responses to a positive shock on the risk-free interest rate for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state with the exception of the trade balance and of the external asset position that are reported in percentage points from the zero steady state.

Figure 5: Dynamic Response to a 0.5% Monetary Policy Shock - Environmental Variables and Welfare



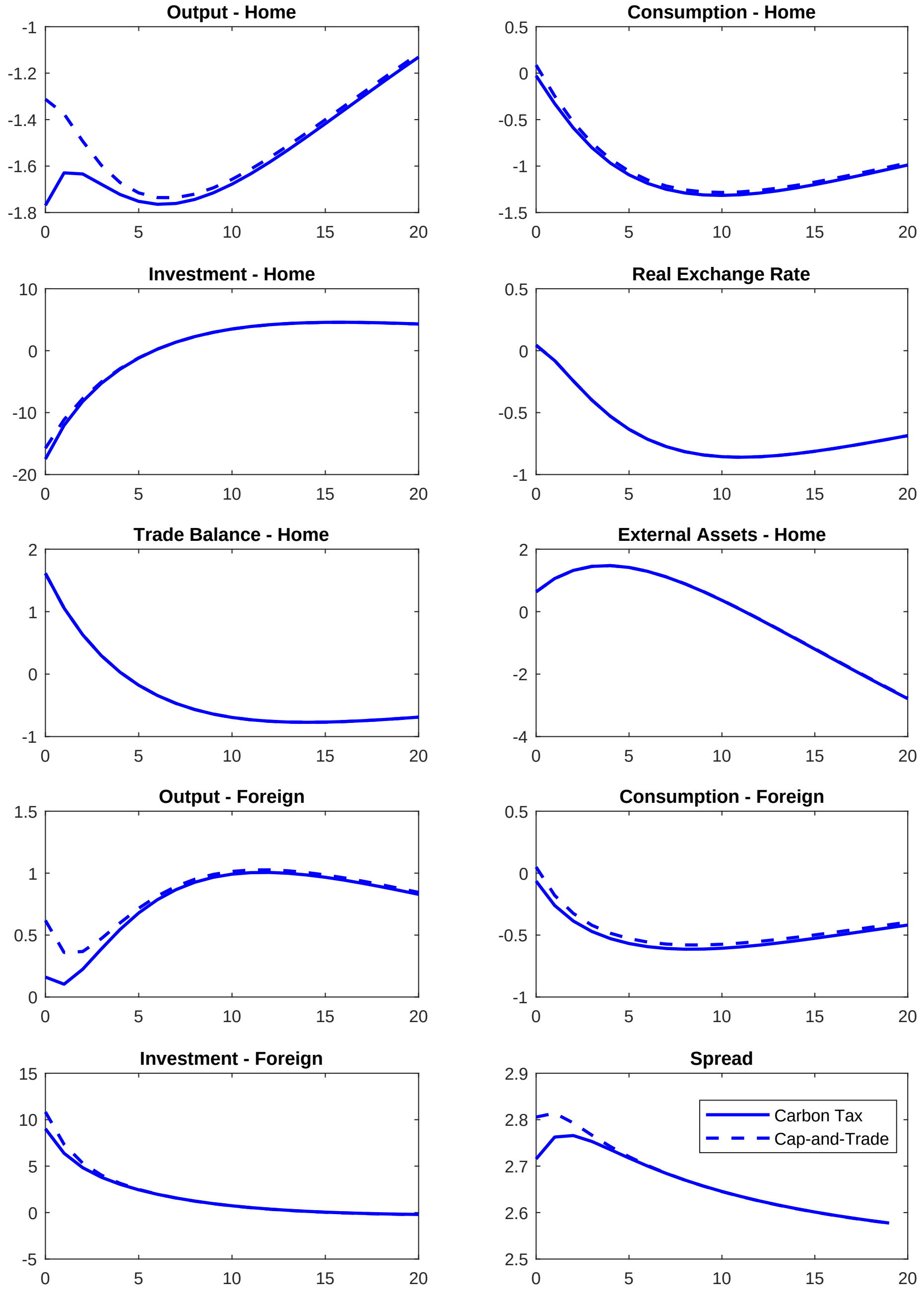
Note: the figure plots the impulse responses to a positive shock on the risk-free interest rate for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state.

Figure 6: Dynamic Response to a 0.5% Monetary Policy Shock - Policy v. No Policy Scenarios



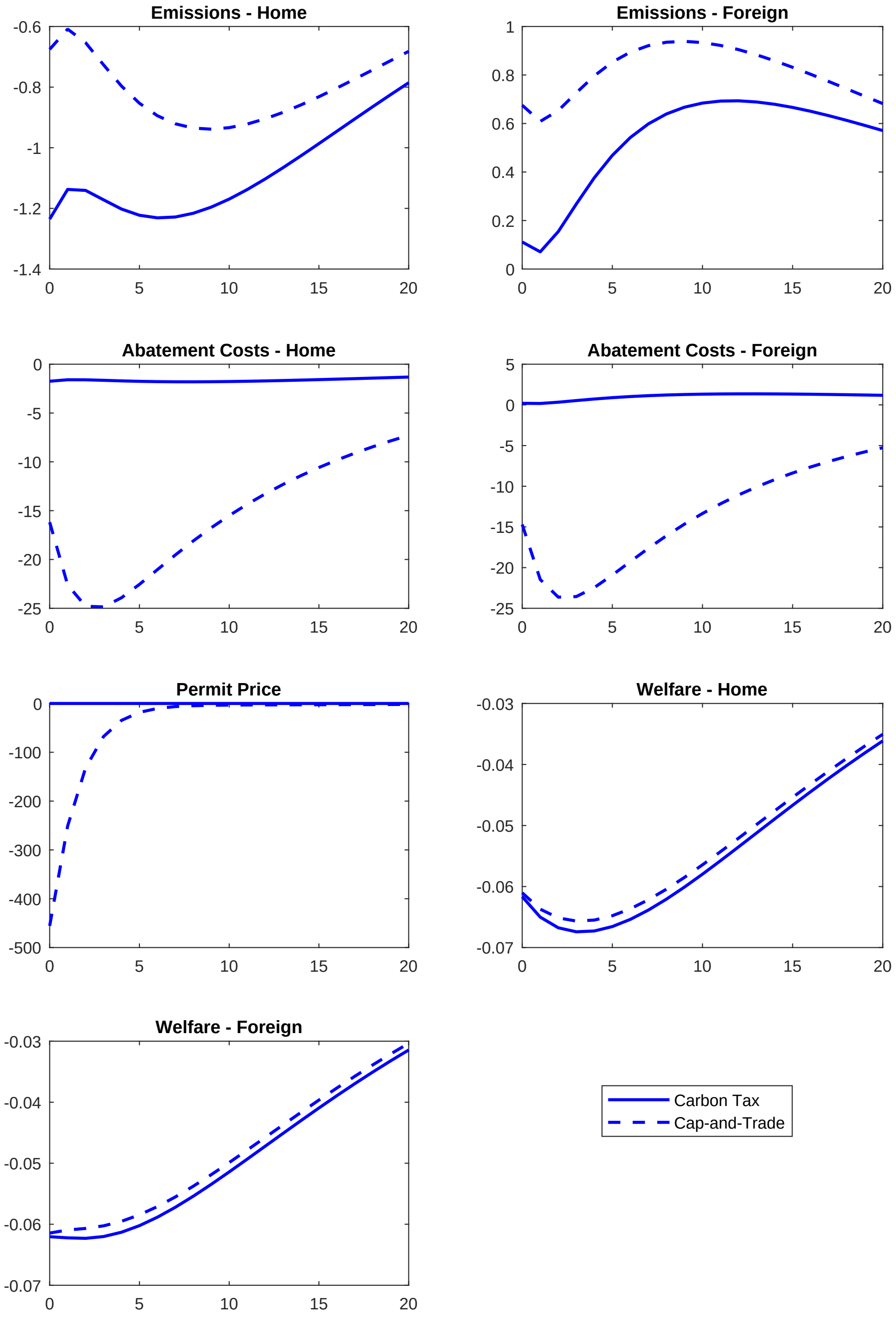
Note: the figure plots the impulse responses to a positive shock on the risk-free interest rate for a 20-quarter time horizon (horizontal axes); for each variable, say X, results are reported as percentage point deviation of the dynamic response under environmental policies, say x_{policy}, (represented in Figures 3 and 4) from its dynamic response in the no-policy scenario case, say $x_{no-policy}$.

Figure 7: Dynamic Response to a -1% Capital Quality Shock - Macroeconomic Variables



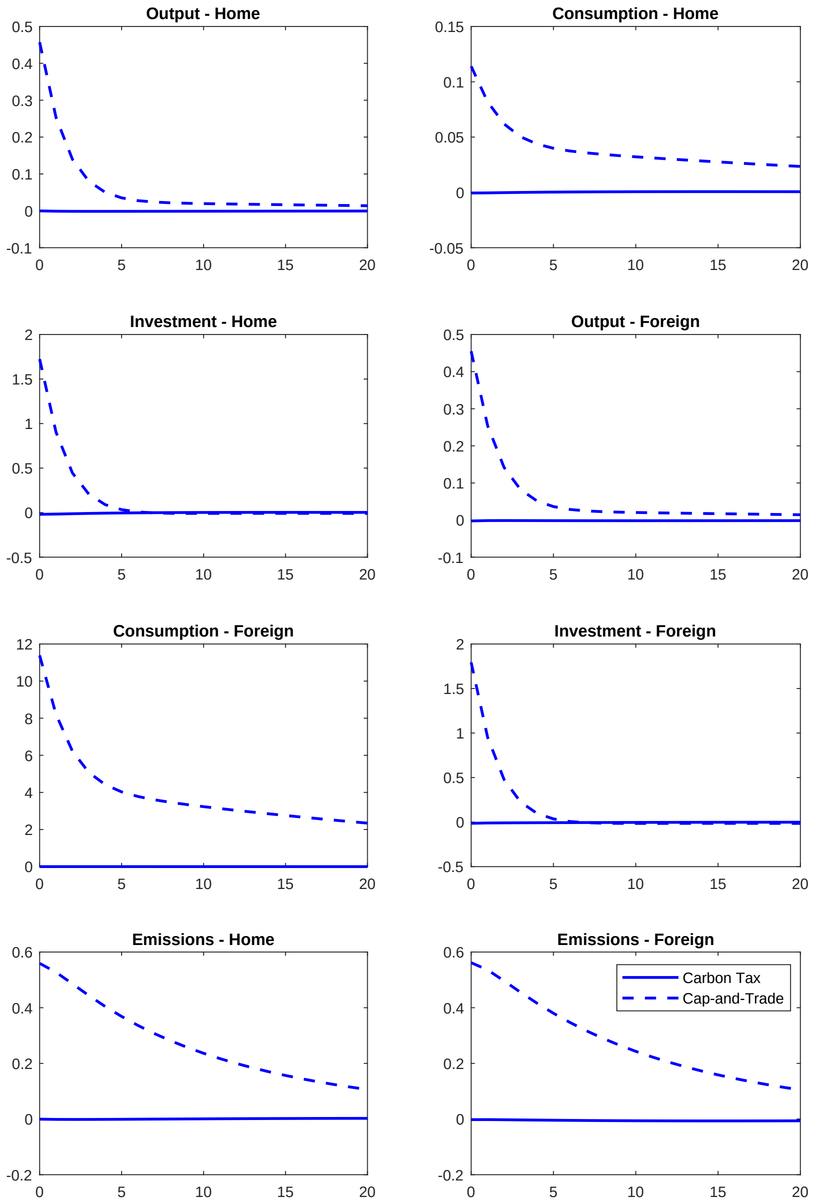
Note: the figure plots the impulse responses to a negative shock on the quality of capital for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state with the exception of the trade balance and of the external asset position that are reported in percentage points from the zero steady state. The spread between the real return on capital and the real nominal interest rate on the risk free asset is expressed in percentage point deviations

Figure 8: Dynamic Response to a -1% Capital Quality Shock - Environmental Variables and Welfare



Note: the figure plots the impulse responses to a negative shock on the quality of capital for a 20-quarter time horizon (horizontal axes); results are reported as percentage deviations from the initial steady state.

Figure 9: Dynamic Response to a -1% Capital Quality Shock - Policy v. No Policy Scenarios



Note: the figure plots the impulse responses to a positive shock on the quality of capital for a 20-quarter time horizon (horizontal axes); for each variable, say X, results are reported as percentage point deviation of the dynamic response under environmental policies, say x_{policy}, (represented in Figures 7 and 8) from its dynamic response in the no-policy scenario case, say $x_{no-policy}$.

Table 1: Parametrization

Parameter	Value	Description
α	1/3	technology parameter
β	0.99	quarterly discount factor
$1-\gamma$	1-0.304	elasticity of emissions to output
γ_I	3	parameter for capital adjustment costs
γ_P	58.25	degree of price rigidities
γ_μ	1.5	parameter for abatement adjustment costs
ψ_μ	10	degree of asymmetry of abatement adjustment costs
δ	0.025	quarterly capital depreciation rate
ϵ	0.3829	emissions scale parameter
η	0.9979	pollution decay rate
θ_1	1	abatement cost function parameter
θ_2	2.8	abatement cost function parameter
ι_Π	1.5	interest rate rule: inflation coefficient
κ	0.7	share of domestic goods used in the final sector
ξ_L	3.8826	disutility of labor parameter
ρ	1.5	elasticity of substitution between Home and Foreign goods
ρ_A	0.85	persistence of productivity shock
ρ_K	0.85	persistence of quality of capital shock
ρ_R	0.5	persistence of monetary policy shock
σ	6	elasticity of substitution between good varieties
φ_C	1.2	coefficient of relative risk aversion
φ_L	1	inverse of the Frisch elasticity of labor supply
χ	2.3069e-06	intensity of negative externality on output
A	13.2581	total factor productivity - TFP

Table 2: International Transmission of Shocks - Benchmark Case (%)

	σ_{Y^D}	$\sigma_{Y^{D*}}/\sigma_{Y^D}$	$\rho(Y^D, Y^{D^*})$
Carbon Tax			
TFP shock	4.6635	17.9785	-15.2519
Monetary shock	5.2153	20.2770	46.0468
Capital quality shock	8.2905	37.8146	-20.2500
Cap-and-Trade			
TFP shock	4.2043	16.1028	-96.8690
Monetary shock	4.3958	10.0523	-95.7749
Capital quality shock	7.6847	35.2991	-54.4854
No Policy			
TFP shock	4.6761	17.9641	-15.4019
Monetary shock	5.2189	20.2539	46.0691
Capital quality shock	8.3020	37.8615	-20.3997

Note: the table reports moments generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%.

Table 3: International Transmission of Shocks - Imperfect Complementarity between Home and Foreign Goods (%)

	σ_{Y^D}	$\sigma_{Y^{D*}}/\sigma_{Y^D}$	$\rho(Y^D, Y^{D^*})$
Carbon Tax			
TFP shock	4.2433	25.1639	-8.0115
Monetary shock	4.7757	27.5328	57.8610
Capital quality shock	7.8128	41.7816	-23.0223
Cap-and-Trade			
TFP shock	3.8434	19.0509	-81.2602
Monetary shock	3.9994	8.4538	-42.9178
Capital quality shock	7.1870	40.4271	-56.1392
No Policy			
TFP shock	4.2555	25.1554	-8.2273
Monetary shock	4.7798	27.5019	57.8244
Capital quality shock	7.8253	41.8567	-23.2197

Note: the table reports moments generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%.

Table 4: International Transmission of Shocks - High Degree of Openness (%)

	σ_{Y^D}	$\sigma_{Y^{D*}}/\sigma_{Y^D}$	$\rho(Y^D, Y^{D^*})$
Carbon Tax			
TFP shock	3.9767	33.8689	-14.4495
Monetary shock	4.3347	38.5245	46.6487
Capital quality shock	6.9548	88.0695	-48.7154
Cap-and-Trade			
TFP shock	3.6416	26.1972	-72.0727
Monetary shock	3.6779	18.3257	-17.0405
Capital quality shock	6.9118	77.9953	-67.8979
No Policy			
TFP shock	3.9879	33.8759	-14.6757
Monetary shock	4.3379	38.4983	46.5999
Capital quality shock	6.9659	88.2237	-48.9490

Note: the table reports moments generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%.

Table 5: International Transmission of Shocks - Currency Union (%)

	σ_{Y^D}	$\sigma_{Y^{D*}}/\sigma_{Y^D}$	$\rho(Y^D, Y^{D^*})$
Carbon Tax			
TFP shock	4.3397	26.6125	22.5172
Monetary shock	6.3641	100.0000	100
Capital quality shock	7.9252	33.7577	-2.1207
Cap-and-Trade			
TFP shock	3.9110	13.8526	-54.8364
Monetary shock	4.4022	100.0000	100
Capital quality shock	7.2226	33.2280	-37.4006
No Policy			
TFP shock	4.3520	26.5718	22.3744
Monetary shock	6.3678	100.0000	100
Capital quality shock	7.9377	33.7796	-2.4077

Note: the table reports moments generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%.

Table 6: Welfare Analysis - Benchmark Case

		TFP shock	Monetary shock	Capital quality shock
No Policy	Welfare H	-266.3476	-266.3824	-267.0655
	Welfare F	-266.4164	-266.3909	-265.8923
	Welfare World	-532.7640	-532.7733	-532.9578
Carbon tax	Welfare H (% dev. from no policy)	-0.01226	-0.01225	-0.01170
	Welfare F (% dev. from no policy)	-0.01223	-0.01226	-0.01312
	Welfare World (% dev. from no policy)	-0.01225	-0.01226	-0.01241
Cap-and-Trade	Welfare H (% dev. from no policy)	-0.01253	-0.01228	-0.00854
	Welfare F (% dev. from no policy)	-0.01227	-0.01225	-0.01118
	Welfare World (% dev. from no policy)	-0.01240	-0.01227	-0.00985

Note: the table reports results based on mean welfare generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%. Steady state welfare in H and F in the no-policy scenario is -266.3872, while under the two policy scenarios is -266.4199, so that the percentage deviation from the no-policy scenario is -0.0123.

Table 7: Welfare Analysis - Pollutant in the Utility Function and Damage Function

		TFP shock	Monetary shock	Capital quality shock
No Policy	Welfare H	-266.6953	-266.7299	-267.4097
	Welfare F	-266.7641	-266.7384	-266.2365
	Welfare World	-533.4594	-533.4683	-533.6462
Carbon tax	Welfare H (% dev. from no policy)	0.00538	0.00539	0.00586
	Welfare F (% dev. from no policy)	0.00541	0.00538	0.00450
	Welfare World (% dev. from no policy)	0.00540	0.00538	0.00518
Cap-and-Trade	Welfare H (% dev. from no policy)	0.00519	0.00537	0.00784
	Welfare F (% dev. from no policy)	0.00544	0.00540	0.00527
	Welfare World (% dev. from no policy)	0.00532	0.00538	0.00656

Note: the table reports results based on mean welfare generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%. Steady state welfare in H and F in the no-policy scenario is -266.7347, while under the two policy scenarios is -266.7203, so that the percentage deviation from the no-policy scenario is 0.0054.

Appendix A

Consumer's Optimization Problem

The representative consumer chooses the sequences $\{C_t,\ K_{t+1},\ I_t,\ L_t,\ B_t,\ F_t^*\}_{t=0}^{\infty}$, so as to maximize (1), subject to (2) and (3). The Lagrangian function associated to the optimization problem of the representative consumer is

$$\mathcal{L}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \left(\frac{C_t^{1-\varphi_C}}{1-\varphi_C} - \xi_L \frac{L_t^{1+\varphi_L}}{1+\varphi_L} \right) + \\ \lambda_t \left[\begin{array}{c} w_t L_t + r_{K,t} K_t + \frac{B_{t-1}}{P_t} + \frac{S_t F_{t-1}^*}{P_t} + Tr_t + D_t + \\ -\frac{\gamma_I}{2} (\frac{I_t}{K_t} - \delta)^2 K_t - C_t - I_t - \frac{R_t^{-1} B_t}{P_t} - \frac{(R_t^*)^{-1} F_t^* S_t}{P_t} \end{array} \right] + \\ +\zeta_t \left[I_t + (1-\delta) e^{u_{K,t}} K_t - K_{t+1} \right]. \end{array} \right\},$$

where λ_t and ζ_t are the Lagrange multipliers associated to budget constraint and to the capital accumulation equation, respectively. The marginal Tobin's q_t is defined as ζ_t/λ_t and measures the relative marginal value of installed capital with respect to consumption. The first-order conditions (4)-(9) immediately follow from the solution to the intertemporal optimization problem.

Intermediate Good Producer's Optimization Problem

Given all the cost components, nominal profits $D_{j,t}^N$, of the typical j firm are given by

$$D_{j,t}^N = P_{j,t}^D Y_{j,t}^D - W_t L_{j,t} - R_{K,t} K_{j,t} - P_{j,t}^D Z_{j,t} - P_{E,t} E_{j,t} - \frac{\gamma_P}{2} \left(\frac{P_{j,t}^D}{P_{j,t-1}^D} - 1 \right)^2 P_t^D Y_t^D - P_t^D \Gamma_{\mu_t}(\mu_{j,t}), \tag{A-1}$$

where adjustment costs are defined in terms of aggregate domestic output Y_t^D.

The typical j firm sets the sequence $\{K_{j,t},\ L_{j,t},\ \mu_{j,t},\ P_{j,t}^D\}_{t=0}^{\infty}$ so as to maximize the expected present discounted value of profits. The Lagrangian function associated to the optimization problem is

$$\mathcal{L}_{i,0} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left\{ \begin{array}{c} \frac{P_{j,t}^D}{P_t} Y_{j,t}^D - w_t L_{j,t} - r_{K,t} K_{j,t} - \frac{P_{j,t}^D}{P_t} AC_{j,t} - \frac{P_{E,t}}{P_t} E_{j,t} + \\ -\frac{\gamma_P}{2} \left(\frac{P_{j,t}^D}{P_{j,t-1}^D} - 1 \right)^2 \frac{P_t^D}{P_t} Y_t^D - \frac{P_t^D}{P_t} \Gamma_{\mu_t}(\mu_{j,t}) + \\ +\Psi_{j,t} \left[\Lambda_t A_t (e^{u_{K,t}} K_{j,t})^{\alpha} L_{j,t}^{1-\alpha} - Y_{j,t}^D \right], \end{array} \right\},$$

where $AC_{j,t} = \theta_1 \mu_{j,t}^{\theta_2} Y_{j,t}^D$, $E_{j,t} = (1-\mu_{j,t}) \epsilon (Y_{j,t}^D)^{1-\gamma}$, $Y_{j,t}^D = \left(P_{j,t}^D / P_t^D \right)^{-\sigma} Y_t^D$ and $\Psi_{j,t}$ is the Lagrange multiplier associated to production function and corresponds to the real marginal cost component related to the manufacturing of an additional unit of good j. Note that abatement costs $AC_{j,t}$ are defined in terms of units of good $Y_{j,t}^D$ forgone to sustain these costs, that is why we use the price $P_{j,t}^D$ instead of the aggregate price index P_t^D. This is the convention used in Annicchiarico and Di Dio (2017). Alternatively, one may opt to define these costs in term of the aggregate domestic good index as done for the adjustment costs, as for instance in Annicchiarico and Di Dio (2015).

The first-order conditions (15)-(17) and the hybrid New Keynesian Phillips curve (18), describing the optimal pricing condition, immediately follow from the solution of the intertemporal optimization problem.

Equilibrium Conditions

Let define $f_t^* = S_t F_t^* / P_t$, $S_t^R = S_t P_t^* / P_t$ and $S_t / S_{t-1} = 1 + s_t$, the following equations describe the decentralized competitive equilibrium of the model. Since we assume that the structure of the Foreign economy is isomorphic to that of the Home, we present only the equations for the Home economy and common equations.

$$C_t^{-\varphi_C} = \lambda_t, \tag{A-1}$$

$$\begin{aligned} q_t \;&=\; \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[r_{K,t+1} + \gamma_I \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} - \frac{\gamma_I}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 \right] \right\} + \\ &\quad + \beta (1-\delta) E_t \left(e^{u_{K,t+1}} \frac{q_{t+1} \lambda_{t+1}}{\lambda_t} \right), \end{aligned} \tag{A-2}$$

$$-\xi_L L_t^{\varphi_L} + \lambda_t w_t = 0, \tag{A-3}$$

$$\gamma_I \left(\frac{I_t}{K_t} - \delta \right) = q_t - 1, \tag{A-4}$$

$$\frac{1}{R_t} = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1}} \right), \tag{A-5}$$

$$K_{t+1} = I_t + (1-\delta) e^{u_{K,t}} K_t, \tag{A-6}$$

$$r_{K,t} = \alpha \Psi_t \frac{Y_t^D}{K_t}, \tag{A-7}$$

$$w_t = (1-\alpha) \Psi_t \frac{Y_t^D}{L_t}, \tag{A-8}$$

$$\begin{aligned} p_{E,t} (Y_t^D)^{(1-\gamma)} \;&=\; \theta_2 \theta_1 \mu_t^{\theta_2 - 1} p_t^D Y_t^D - p_t^D \gamma_\mu \frac{1}{\mu_{t-1}} \frac{\exp\left(-\psi_\mu \left(\frac{\mu_t}{\mu_{t-1}} - 1 \right) \right) - 1}{\psi_\mu} + \\ &\quad + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} p_{t+1}^D \gamma_\mu \frac{\mu_{t+1}}{\mu_t^2} \frac{\exp\left(-\psi_\mu \left(\frac{\mu_{t+1}}{\mu_t} - 1 \right) \right) - 1}{\psi_\mu}, \end{aligned} \tag{A-9}$$

$$Y_t^D = \Lambda_t A_t \left(e^{u_{K,t}} K_t \right)^\alpha L_t^{1-\alpha}, \tag{A-10}$$

$$\begin{aligned} &\left(1 - \theta_1 \mu_t^{\theta_2} \right) (1-\sigma) + \sigma \left[\frac{p_{E,t}}{p_t^D} (1-\gamma)(1-\mu_t) \epsilon (Y_t^D)^{-\gamma} + \Psi_t \frac{1}{p_t^D} \right] + \\ &- \gamma_P \left(\Pi_t^D - 1 \right) \Pi_t^D + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \gamma_P \left(\Pi_{t+1}^D - 1 \right) \left(\Pi_{t+1}^D \right)^2 \frac{Y_{t+1}^D}{Y_t^D} \frac{1}{\Pi_{t+1}} \right] = 0, \end{aligned} \tag{A-11}$$

$$Y_t = [\kappa^{\frac{1}{\rho}} (Y_t^H)^{\frac{\rho-1}{\rho}} + (1-\kappa)^{\frac{1}{\rho}} (M_t)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}, \tag{A-12}$$

$$Y_t^H = \kappa Y_t \left(\frac{1}{p_t^D} \right)^\rho, \tag{A-13}$$

$$Y_t^D = Y_t^H + X_t, \tag{A-14}$$

$$M_t = (1-\kappa)\left(\frac{1}{p_t^{D^*}}\frac{1}{S_t^R}\right)^{\rho} Y_t, \tag{A-15}$$

$$\Pi_t = \frac{p_{t-1}^D}{p_t^D}\Pi_t^D, \tag{A-16}$$

$$Tr_t = p_{E,t}E_t, \tag{A-17}$$

$$X_t = (1-\kappa)\left(\frac{S_t^R}{p_t^D}\right)^{\rho} Y_t^*, \tag{A-18}$$

$$p_t^D Y_t^D = C_t + I_t + p_t^D AC_t + p_t^D X_t - S_t^R p_t^{D^*} M_t + \Gamma_K(I_t, K_t) + p_t^D \Gamma_{\mu_t}(\mu_t) + \frac{\gamma_P}{2}(\Pi_t^D - 1)^2 p_t^D Y_t^D, \tag{A-19}$$

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\iota_\Pi} e^{u_{R,t}}, \tag{A-20}$$

$$E_t = (1-\mu_t)\epsilon(Y_t^D)^{(1-\gamma)}, \tag{A-21}$$

$$AC_t = \theta_1 \mu_t^{\theta_2} Y_t^D, \tag{A-22}$$

$$u_{K,t} = \rho_K u_{K,t-1} + \epsilon_{K,t}, \tag{A-23}$$

$$u_{A,t} = \rho_A u_{A,t-1} + \varepsilon_{A,t}, \tag{A-24}$$

$$u_{R,t} = \rho_R u_{R,t-1} + \epsilon_{R,t}. \tag{A-25}$$

Common equations determine the time path of the depreciation rate of the domestic currency s_t, the net external asset position f_t^*, the real exchange rate S_t^R, the stock of pollution Z_t in the atmosphere and the related damage Λ_t:

$$\frac{1}{R_t^*} = \beta \mathbb{E}_t\left[\frac{\lambda_{t+1}(1+s_{t+1})}{\Pi_{t+1}\lambda_t}\right], \tag{A-26}$$

$$f_t^* = R_t^*\left[\frac{(1+s_t)}{\Pi_t}f_{t-1}^* + p_t^D X_t - S_t^R p_t^{D*} M_t\right], \tag{A-27}$$

or

$$f_t^* = R_t^*\left[\frac{(1+s_t)}{\Pi_t}f_{t-1}^* + p_t^D X_t - S_t^R p_t^{D*} M_t - p_t^E\left(E_t - \bar{E}\right)\right], \tag{A-28}$$

$$S_t^R = S_{t-1}^R(1+s_t)\frac{\Pi_t^*}{\Pi_t}, \tag{A-29}$$

$$Z_t = \eta Z_{t-1} + E_t + E_t^* + E_t^{NI}, \tag{A-30}$$

$$\Lambda_t = \exp[-\chi(Z_t - \overline{Z})]. \tag{A-31}$$

The overall economy is then described by 23 variables related to Home, $\{C_t$, E_t, I_t, K_t, L_t, M_t, p_t^D, $p_{E,t}$, q_t, R_t, $r_{K,t}$, Tr_t, w_t, X_t, Y_t, Y_t^D, Y_t^H, AC_t, λ_t, μ_t, Π_t, Π_t^D, $\Psi_t\}$, 23 variables related to Foreign $\{C_t^*$, E_t^*, I_t^*, K_t^*, L_t^*, M_t^*, $p_t^{D^*}$, $p_{E,t}^*$, q_t^*, R_t^*, $r_{K,t}^*$, Tr_t^*, w_t^*, X_t^*, Y_t^*, Y_t^{D*}, Y_t^{H*}, AC_t^*, λ_t^*, μ_t^*, Π_t^*, Π_t^{D*}, $\Psi_t^*\}$, and 5 common variables, $\{f_t^*$, s_t, S_t^R, $Z_t, \Lambda_t\}$.

For each country we have an additional equation represented by the environmental policy regime in place. Under a carbon tax $p_{E,t} = p_E$ and $p_{E,t}^* = p_E^*$, with $p_E = p_E^*$; under an international cap-and-trade regime, $E_t + E_t^* = \bar{E} + \bar{E}^*$ and $p_{E,t} = p_{E,t}^*$.

Appendix B

Table B-1: Welfare Analysis - Pollutant in the Utility Function

		TFP shock	Monetary shock	Capital quality shock
No Policy	Welfare H	-266.6615	-266.6962	-267.3783
	Welfare F	-266.7302	-266.7047	-266.2052
	Welfare World	-533.3918	-533.4010	-533.5835
Carbon tax	Welfare H (% dev. from no policy)	-0.00724	-0.00723	-0.00674
	Welfare F (% dev. from no policy)	-0.00721	-0.00724	-0.00809
	Welfare World (% dev. from no policy)	-0.00723	-0.00724	-0.00740
Cap-and-Trade	Welfare H (% dev. from no policy)	-0.00748	-0.00726	-0.00391
	Welfare F (% dev. from no policy)	-0.00723	-0.00723	-0.00647
	Welfare World (% dev. from no policy)	-0.00736	-0.00724	-0.00519

Note: the table reports results based on mean welfare generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%. Steady state welfare in H and F in the no-policy scenario is -266.7010, while under the two policy scenarios is -266.7203, so that the percentage deviation from the no-policy scenario is -0.0072.

Table B-2: Welfare Analysis - Imperfect Complementarity between Home and Foreign Goods

		TFP shock	Monetary shock	Capital quality shock
No Policy	Welfare H	-266.4411	-266.3963	-266.5297
	Welfare F	-266.3550	-266.3819	-266.3053
	Welfare World	-532.7962	-532.7782	-532.8350
Carbon tax	Welfare H (% dev. from no policy)	-0.01214	-0.01225	-0.01213
	Welfare F (% dev. from no policy)	-0.01238	-0.01227	-0.01235
	Welfare World (% dev. from no policy)	-0.01226	-0.01226	-0.01229
Cap-and-Trade	Welfare H (% dev. from no policy)	-0.01221	-0.01225	-0.00938
	Welfare F (% dev. from no policy)	-0.01186	-0.01219	-0.01289
	Welfare World (% dev. from no policy)	-0.01204	-0.01222	-0.01113

Note: the table reports results based on mean welfare generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%. Steady state welfare in H and F in the no policy scenario is -266.3872, while under the two policy scearios is -266.4199, so that the percentage deviation from the no policy scenario is -0.0123.

Table B-3: Welfare Analysis - High Degree of Openness

		TFP shock	Monetary shock	Capital quality shock
No Policy	Welfare H	-266.3466	-266.3828	-267.2492
	Welfare F	-266.4155	-266.3904	-265.7906
	Welfare World	-532.7621	-532.7732	-533.0398
Carbon tax	Welfare H (% dev. from no policy)	-0.01228	-0.01226	-0.01120
	Welfare F (% dev. from no policy)	-0.01222	-0.01226	-0.01358
	Welfare World (% dev. from no policy)	-0.01225	-0.01226	-0.01238
Cap-and-Trade	Welfare H (% dev. from no policy)	-0.01258	-0.01229	-0.00688
	Welfare F (% dev. from no policy)	-0.01229	-0.01226	-0.01048
	Welfare World (% dev. from no policy)	-0.01243	-0.01227	-0.00867

Note: the table reports results based on mean welfare generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%. Steady state welfare in H and F in the no-policy scenario is -266.3872, while under the two policy scenarios is -266.4199, so that the percentage deviation from the no-policy scenario is -0.0123.

Table B-4: Welfare Analysis - Currency Union

		TFP shock	Monetary shock	Capital quality shock
No Policy	Welfare H	-266.3500	-266.3875	-267.0579
	Welfare F	-266.4147	-266.3875	-265.8979
	Welfare World	-532.7646	-532.7751	-532.9559
Carbon tax	Welfare H (% dev. from no policy)	-0.01226	-0.01226	-0.01168
	Welfare F (% dev. from no policy)	-0.01223	-0.01226	-0.01314
	Welfare World (% dev. from no policy)	-0.01225	-0.01226	-0.01241
Cap-and-Trade	Welfare H (% dev. from no policy)	-0.01252	-0.01224	-0.00859
	Welfare F (% dev. from no policy)	-0.01227	-0.01224	-0.01118
	Welfare World (% dev. from no policy)	-0.01239	-0.01224	-0.00988

Note: the table reports results based on mean welfare generated by the model for 200 realizations of shock sequences of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.1%. Steady state welfare in H and F in the no-policy scenario is -266.3872, while under the two policy scenarios is -266.4199, so that the percentage deviation from the no-policy scenario is -0.0123.