

Climate Actions, Market Beliefs, and Monetary Policy*

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Abstract

This paper studies the role of expectations and monetary policy in the economy’s response to climate actions. We show that in a stochastic environment, without the standard assumption of the perfect rationality of agents, there is more uncertainty regarding the path and the economic impact of a climate policy, with a potential threat to the ability of central banks to maintain price stability. Market beliefs and behavioral agents increase the trade-offs inherent to the chosen mitigation tool, with a carbon tax entailing more emissions uncertainty than in a rational expectations model and a cap-and-trade scheme implying a more pronounced pressure on allowance prices and inflation. The impact on price stability is worsened by delays in the implementation of stringent climate policies, the lack of confidence in the ability of central banks to keep inflation under control, and the adoption of monetary rules tied to expectations rather than current macroeconomic conditions. Central banks can implement successful stabilization policies that reduce the uncertainty surrounding the impact of climate actions and support the greening process while remaining within their mandate.

Keywords: Monetary policy; climate policy; expectations; inflation; market sentiments; business cycle.

JEL Codes: D83, Q50, E32, E71.

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1 Introduction

The following questions may arise when analyzing the economic impact of climate policies in a stochastic environment. Suppose a mitigation plan is implemented through a quantity or a price regulation on emissions. How will the economy's response change if we remove the standard assumption of rational expectations? What is the role of market beliefs in driving or hindering the mitigation process? Finally, on a policy level, can monetary policy tame the irrational exuberance or the doom and gloom of the markets rendering the greening policy more effective while maintaining price stability?

To address these issues, we start our analysis with the simplest version of the canonical New Keynesian model augmented to include a negative environmental externality and agents who lack the cognitive abilities necessary to form rational expectations. The paper highlights the role of expectations in the transmission of climate policies over the business cycle and studies how monetary policy can facilitate the achievement of a predetermined mitigation target while keeping inflation under control.

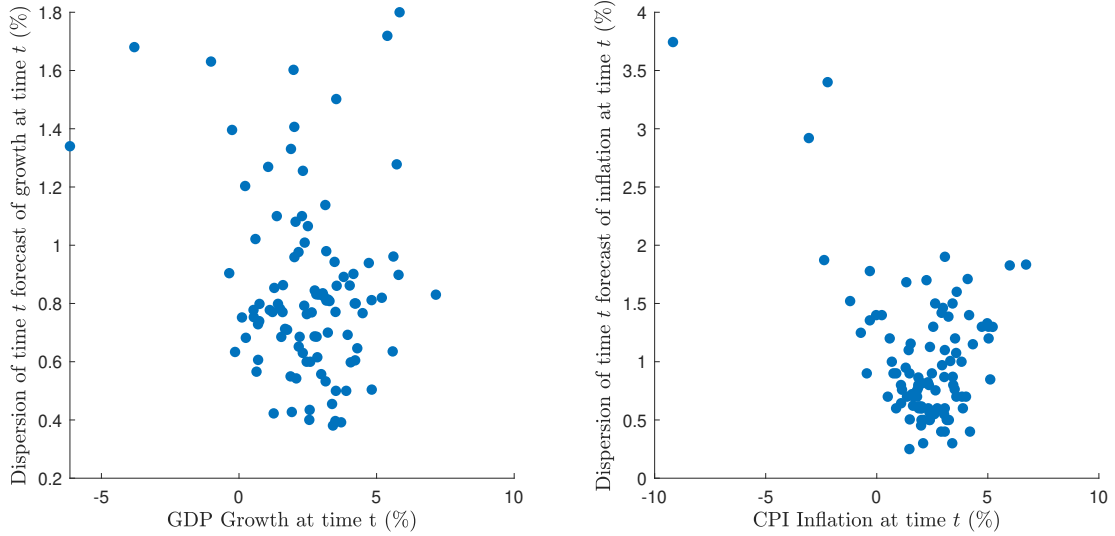
According to standard economic theory, when economic agents make their decisions, they consider all the options available to them, anticipate the possible outcome, and know how the economy works and the probability distributions of future events. Put another way, agents formulate rational expectations. However, do agents do that? Do agents have a sufficient ability to understand economic variables and formulate fully-model consistent expectations? Since the seminal contributions of [Tversky and Kahneman \(1974\)](#), [Grether and Plott \(1979\)](#), and [Thaler \(1980\)](#), an increasing number of studies have taken into account the fact that agents are not perfectly rational when making their decisions and that their actions are subject to errors and cognitive biases. Moreover, in a world where knowledge is bounded and time is pressing, the decision process can be extremely complicated and costly.¹ To overcome these cognitive limits and to the extent that economic forecasting is costly, agents, in need of making quick choices, formulate expectations and make decisions based on simple rules, the so-called heuristics or rules of thumb.² However, agents using heuristics learn from their mistakes and stand ready to choose the rule that exhibits the best performance, generating endogenous dynamics that, in turn, may give rise to short-term macroeconomic fluctuations.

There is substantial literature, based on survey data, rejecting the rational expectations hypothesis and emphasizing the considerable heterogeneity of private sector forecasts of macroeconomic variables. For a complete survey, see [Pesaran and Weale \(2006\)](#). Figure 1 shows some evidence of substantial heterogeneity in expectation formation by showing the dispersion of expected GDP growth and CPI inflation in the US Survey of Professional Forecasters. Data refer

¹On this issue see the early paper of [Evans and Ramey \(1992\)](#).

²Forming rational expectations can be expensive (one needs to do research and be very smart), while observing just current data/variables is cheaper. On the notion of heuristics see, e.g. [Gigerenzer and Selten \(2002\)](#) and [Gigerenzer and Gaissmaier \(2011\)](#).

Figure 1: Cross-Sectional Dispersion of Growth and Inflation Expectations



Note: the figure plots the cross-sectional difference between the 75th percentile and the 25th percentile of projections of real GDP growth and CPI inflation against actual data. All variables are expressed in annualized percentage points. Data are from the US Survey of Professional Forecasters for the period 1995Q1-2022Q4.

to forecasts made in the relevant quarter for the period 1995Q1-2022Q4. The figure documents substantial disagreement among professional economists about both variables. Disagreement tends to be more prominent when growth and inflation are away from their long-run averages.

It is then reasonable to deduce that the presence of heterogeneous agents who change the way they formulate expectations and, therefore, their behavior over the business cycle, adds a further complication and an additional layer of uncertainty also when it comes to the macroeconomic impact of carbon pricing policies.³ In the context of climate policy, agents already operate under the uncertainty inherent to the selected environmental regulatory instrument. While a carbon tax (i.e., price regulation) entails emission uncertainty, a cap-and-trade mechanism (i.e., quantity regulation) introduces short-term volatility of allowance prices. The uncertainty induced by climate policy is directly reflected in the dynamics of output and inflation. Via the expectation channel, agents shape the response of the economy to mitigation policies and may play an enabling or a hampering role in the transition process, depending on how they forecast future economic variables.

The economy's response to climate policies and the effects on macroeconomic stability, particularly on inflation dynamics, are at the center of the current policy debate. While there is a

³Along this line, see [Alessi et al. \(2021\)](#), who study the reaction of financial investors to the Paris Agreement and then to the subsequent withdrawal of the US from it, finding evidence of a strong heterogeneity of reactions across different categories of investors. In particular, after the US withdrawal, the behavior of households is shown to be more sentiment driven, in contrast to that of regulated financial institutions.

broad consensus on the need for policy actions to comply with the goals established by the Paris Agreement and meet climate targets (e.g., [IPCC, 2018](#)), the short-medium-run macroeconomic implications of climate policies are not yet well understood. This poses a challenge, in particular, for the conduct of monetary policy, whose conventional policy horizon is typically around two or three years. The process of reducing emissions is likely to have a significant impact on the economy, with potential repercussions on macroeconomic and price stability, conditioning the environment in which central banks operate and, thus, the conduct of their policies (e.g. [NGFS, 2019](#) and [Schoenmaker, 2021](#)).

Several central banks around the world have joined the Network for Greening the Financial System (NGFS) and are currently evaluating how climate policies can influence their mandates and what role they can play in the fight against climate change (e.g., [Carney, 2015](#), [Rudebusch et al., 2019](#), [Lagarde, 2021](#), [Villeroy de Galhau, 2021](#)). Recently, the European Central Bank presented an action plan to include climate change considerations in its monetary policy strategy (see [ECB 2021](#)). We can expect climate and monetary policy to move closer in the future. Even if governments remain primarily responsible for facilitating an orderly low-carbon transition and undertaking the main policy interventions, there are several areas in which central banks can contribute to support climate actions, simply acting in the perimeter of their mandates.

This paper explicitly contributes to this debate by discussing the role of monetary policy in the face of climate actions when considering (i) short-run uncertainty and (ii) agents that are not fully rational. This allows us to evaluate the performance of climate policy and its interaction with monetary policy in an economy hit by shocks and subject to market beliefs. Bounded rationality and behavioral biases, coupled with business cycle fluctuations, can prevent agents from fully internalizing the impact of climate policies, conditioning the policy effectiveness and the achievement of climate targets. In this context, active monetary policy can anchor expectations and support the greening process of the economy.

The underlying view of our approach is that it is crucial to understand how the stabilization of the price levels by central banks affects the nature of the business cycle whose fluctuations are dominated by movements of ‘animal spirits’, and to understand if the stabilization efforts, aimed at reducing the intensity of booms and busts, might be beneficial also in reducing the uncertainty surrounding the underlying climate policy and in mitigating its short-run costs.

Our results show that without the standard assumption of perfect rationality, there is more uncertainty surrounding the time path, the effectiveness, and the impact of climate policies, opening up to a nontrivial interplay between climate and monetary policies. Specifically, our key findings are as follows. First, the presence of market beliefs and behavioral agents drives and amplifies business cycle fluctuations, making the adjustment dynamics during the transition highly unpredictable. Second, under price regulation, the time needed to achieve an emission-reduction target can be longer in a behavioral model, whereas under a cap-and-trade scheme, there may be a severe threat to price stability. Third, a monetary policy sufficiently reactive to

the output gap or inflation can dampen emission volatility, reducing the uncertainty surrounding the achievement of climate targets and stabilizing inflation, thus reducing the pressure on prices introduced by an increasingly stringent climate policy, regardless of the environmental regime adopted. Fourth, delays in the implementation of the mitigation plan, lack of credibility regarding the ability of the central bank to keep inflation under control, and the adoption of monetary policy rules reacting to market expectations, rather than to fundamentals, are all factors that can amplify fluctuations and worsen the impact of climate actions on price stability.⁴ Overall, our results suggest a critical role for central banks in the fight against climate change within the remit of their mandates.

To the best of our knowledge, this paper is the first attempt to study the transmission of climate actions and the role of monetary policy in the low-carbon transition in a behavioral New Keynesian model, where agents are not fully rational and subject to market beliefs. For the formalization of behavioral agents in the context of the canonical New Keynesian model, this work follows quite closely the contributions by [De Grauwe \(2011, 2012a\)](#), [Kurz et al. \(2013\)](#), [De Grauwe and Macchiarelli \(2015\)](#), [De Grauwe and Gerba \(2018\)](#), [Hommes and Lustenhouwer \(2019\)](#), [Hommes et al. \(2019\)](#) and [De Grauwe and Ji \(2020\)](#).⁵ This approach allows us to capture movements in output and prices observed in the data more closely, while keeping the model relatively simple and preserving transparency in interpreting the main transmission mechanisms.

The building blocks of the model consist of the three main equations used in the simple textbook New Keynesian model. The difference from the standard model is in the way agents use the information to make forecasts and in the modeling of a climate change externality. Nevertheless, unlike standard New Keynesian models, which produce movements of output and prices that are normally distributed and fail to provide an explanation of booms and busts, the behavioral model presented here reproduces output movements that show fat tails and are not normally distributed, consistently with the data.⁶

The interactions between monetary and climate policy in New Keynesian models have been explicitly investigated by [Annicchiarico and Di Dio \(2015, 2017\)](#), [Economides and Xepapadeas \(2018\)](#) and [Chan \(2020\)](#) in a closed economy, and [Annicchiarico and Diluiso \(2019\)](#), and [Pagliari](#)

⁴In the current paper we focus on central bank's credibility, but governments' credibility and commitment around future climate policies are equally important and remain critical issues for the achievement of climate targets ([Rogelj et al., 2023](#)). [Campiglio et al. \(2023\)](#) study firms' beliefs on future carbon prices and show that a weak policy commitment can trap the economy into a vicious circle of credibility loss, carbon-intensive investments, and increasing risk perceptions, leading to a failure of the transition.

⁵Other variants with non-rational agents include the near rationality hypothesis as in [Woodford \(2010\)](#), the learning mechanism as in [Bullard and Mitra \(2002\)](#) and [Evans and Honkapohja \(2012\)](#), and the approach of [Gabaix \(2020\)](#) who introduces the notion of *cognitive discounting* by which non-rational agents discount future events relatively more than rational agents when the forecasting horizon is more distant in the future.

⁶For empirical validation of the behavioral approach adopted in this paper, see, e.g., [De Grauwe \(2012b\)](#). For alternative variants of the New Keynesian model that deviate from the rational expectation hypothesis and introduce heterogeneous market beliefs, see [Branch \(2009\)](#), [Branch and McGough \(2010\)](#), [Levine et al. \(2012\)](#), [Kurz et al. \(2013\)](#), [Massaro \(2013\)](#), and [Annicchiarico et al. \(2019\)](#), among others.

and Ferrari (2021), in an open economy.⁷ Recently, in a New Keynesian model with financial frictions Diluiso et al. (2021) explore the monetary policy’s role during a credible and gradual medium-term mitigation plan, showing how a monetary authority pursuing a strong inflation targeting can limit output losses, while jointly safeguarding financial and price stability. A similar exercise can be found in Olovsson and Vestin (2023) who show that during the green transition, it is optimal for monetary policy to see through the increasing energy prices and focus on core inflation. Finally, Del Negro et al. (2023) argue that, depending on the type of climate policy implemented and the degree of price stickiness in different sectors, the transition can generate a trade-off between the central bank’s objectives for inflation and real activity. However, in all these papers, the economy is not perturbed by any shocks during the greening process, and the agents are assumed to be fully rational. Dietrich et al. (2021) focus on the implications that climate-change-related disaster expectations can have on the conduct of monetary policy and the emergence of cyclical fluctuations. Our paper contributes to this literature and provides a different perspective on the heated debate on the role central banks can play during the transition to a low-carbon economy and the impact that the fight against climate change can have on the price stability objectives (see, e.g., NGFS, 2020a,b).

Finally, by explicitly distinguishing between carbon taxes and cap-and-trade schemes, this paper also contributes to the literature on price versus quantity regulations that since the seminal contribution of Weitzman (1974) has animated the debate among economists, policy analysts and practitioners.⁸ We show that in the presence of uncertainty and non-rational agents, the close connection between these two modes of environmental control becomes more problematic.

The paper is structured as follows. Section 2 presents the behavioral New Keynesian model with environmental externality. Section 3 describes the baseline parametrization. Section 4 looks at the dynamic response of the economy to a mitigation policy and explores the role of monetary policy during the green transition. Section 5 undertakes some sensitivity analysis to examine the role of market beliefs, considers different hypotheses about how agents form their expectations, studies the effects of lack of credibility regarding the ability of the central bank to maintain inflation at its target, and analyzes the potential implications on public debt sustainability of the green transition. Section 6 concludes and discusses some potential avenues for future research.

⁷For an overview of the literature on business cycles and environmental policy, see e.g. Annicchiarico et al. (2021).

⁸See Karp and Traeger (2018) and Stavins (2020) for a comprehensive discussion on this debate and on the related policy implications.

2 A Behavioral New Keynesian Environmental Model

We consider a behavioral variant of the prototypical New Keynesian dynamic stochastic general equilibrium model with imperfect price adjustment *à la* Calvo (1983), including pollutant emissions and climate policy. Private agents formulate expectations by endogenously selecting the forecasting rules based on their relative past performance. In what follows, we present our setup where we adapt the microfoundations of the New Keynesian model with heterogeneous expectations of Kurz et al. (2013), also used in Hommes and Lustenhouwer (2019) and Hommes et al. (2019), among others.

The economy is populated by three types of agents: (i) a continuum of households who consume, supply labor, own firms, and formulate expectations according to simple heuristics; (ii) a continuum of monopolistically competitive polluting firms, facing nominal rigidities and using labor and a fossil resource as production inputs; (iii) a public sector conducting monetary policy through an interest rate rule of the Taylor type and setting climate policy by either controlling the price of carbon or by setting a cap on emissions.

2.1 Households

On the demand side, there is a continuum of mass one of infinitely lived households. The representative household of type i has preferences represented by:

$$\tilde{E}_{i,0} \sum_{t=0}^{\infty} (e^{v_t}) \beta^t \left[\frac{(C_{i,t} - hC_{t-1})^{1-\gamma} - 1}{1-\gamma} - \chi \frac{N_{i,t}^{1+\varphi}}{1+\varphi} - \frac{\varrho}{2} \left(\frac{B_{i,t}}{P_t} \right)^2 \right], \quad (1)$$

where $\tilde{E}_{i,0}$ denotes the subjective expectation operator in the period 0, the variable v_t is an exogenous shock distorting the household discount factor (i.e., an intertemporal preference shock), $\beta \in (0, 1)$ is the discount factor, $C_{i,t}$ is consumption, $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution, $h \in [0, 1)$ measures habit persistence,⁹ C_{t-1} is the lagged value of the average aggregate consumption, taken as given by each atomistic household (external habit), $N_{i,t}$ denotes hours of work, $\chi > 0$ measures the disutility of labor, and $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply. Finally, P_t is the aggregate price level of the economy, $B_{i,t}$ denotes the quantity of one-period nominal bonds purchased in t and $\varrho > 0$. As in Kurz et al. (2013) the last term in (1) introduces a penalty for excessive lending at individual levels and replaces transversality conditions.¹⁰ It should be noted that households

⁹Habit persistence has been shown to improve the empirical performance of dynamic stochastic general equilibrium models. Habit formation, for example, helps to match the interest rate dynamics to several features of asset prices (as shown in Christiano et al. 2001), but it is also useful to have an empirically relevant propagation of monetary shocks in consumption (see Christiano et al. 2005). In this model, a certain degree of habit persistence is necessary to capture the high first-order auto-correlation of output observed in the data. See Section 3 for more details.

¹⁰When the economy is out of its steady state, in fact, heterogeneous agents lend and borrow from each other. See Annicchiarico et al. (2019) where the dynamics of debt and wealth are made explicit.

are heterogeneous as they formulate different expectations about future income and inflation. In Section 2.4 we will see how expectations are formulated and how agents endogenously select their expectation rule.

Each household faces a flow budget constraint of the form:

$$P_t C_{i,t} + B_{i,t} = W_t N_{i,t} + R_{t-1} B_{i,t-1} + \Xi_{i,t}, \quad (2)$$

where W_t is the nominal wage, R_{t-1} is the nominal (risk-free) interest factor and $\Xi_{i,t}$ represents the lump-sum income component, including government transfers and dividends from the ownership of firms. The representative household of type i in period t chooses $C_{i,t}$, $B_{i,t}$ and $N_{i,t}$, so as to maximize (1), subject to (2).

2.2 Production

As in the baseline New Keynesian model, we assume there is a perfectly competitive final good sector assembling differentiated intermediate goods to produce a single final good, Y_t , according to a constant elasticity of substitution (CES) technology:

$$Y_t = \left[\int_0^1 (Y_{i,t})^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)}, \quad (3)$$

where $\sigma > 1$ is the elasticity of substitution between intermediate goods and $Y_{i,t}$ is the intermediate good of generic type i . At the optimum, the demand equation for the generic variety i is $Y_{i,t} = (P_{i,t}/P_t)^{-\sigma} Y_t$, where $P_t = \int_0^1 (P_{i,t}^{1-\sigma})^{1/(1-\sigma)} di$ is the aggregate price index of the economy, such that $P_t Y_t = \int_0^1 P_{i,t} Y_{i,t} di$.

Monopolistic competitive firms produce intermediate goods. Households have equal ownership shares in all firms, but it is assumed that each household manages only one firm. Note that we use the same index i used for types of households in indexing producers, given the assumption made regarding firm management.

The intermediate good producer i uses labor inputs $N_{i,t}$ and a fossil resource $Z_{i,t}$ as a polluting source of energy. In line with the energy economics literature, we assume that the production function is a constant return-to-scale technology of the CES type¹¹:

$$Y_{i,t} = \Delta_t \left[\zeta Z_{i,t}^{\frac{\varkappa-1}{\varkappa}} + (1 - \zeta) (A_t N_{i,t})^{\frac{\varkappa-1}{\varkappa}} \right]^{\frac{\varkappa}{\varkappa-1}}, \quad (4)$$

where $Y_{i,t}$ is production, Δ_t , captures a negative environmental externality impacting all pro-

¹¹The CES-type production function has been commonly used in large scale multi-sectors models, such as EPPA (Jacoby et al. 2006) and WITCH (Bosetti and Frankel 2009), for assessing how the interactions across different sectors affect the propagation of an energy shock. As we take a broad macroeconomic perspective, we pay a ‘cost’ in terms of a relatively stylized formalization of the interaction across sectors. Despite that, we still adopt the CES production function since it has been found to provide a good empirical fit to real-world data in various industries and sectors. See, for example, Kemfert (1998).

ducers in the same way, A_t is an exogenous process measuring labor productivity, $\zeta \in (0, 1)$ is the energy quasi-share parameter and $\varkappa > 0$ is the elasticity of substitution between energy and labor inputs. The CES structure of the production function implies that factor cost shares are allowed to vary over the business cycle. We assume $\varkappa \in (0, 1)$, so to capture a certain complementarity between the two factor inputs. Let $P_{Z,t}$ denote the nominal price of the fossil resource, then the real marginal cost of production can be written as:

$$MC_{i,t} = MC_t = \Delta_t^{-1} \left[\zeta^\varkappa \left(\frac{P_{Z,t}}{P_t} \right)^{1-\varkappa} + (1 - \zeta)^\varkappa A_t^{\varkappa-1} \left(\frac{W_t}{P_t} \right)^{1-\varkappa} \right]^{\frac{1}{1-\varkappa}}, \quad (5)$$

where we have dropped the i subscript for the marginal costs since they are symmetric across firms.

Following Calvo (1983), each producer may reset its price only with probability $1 - \varpi$. The typical firm capable of re-optimizing in period t will choose the *optimal* price, say $P_{i,t}^*$, to maximize the current market value of the expected profits generated while that price will stay put. We further assume that in the periods between price re-optimization, firms will be able to mechanically adjust their prices according to a simple indexation rule:

$$P_{i,t+s} = P_{i,t+s-1} \Pi^\kappa \Pi_{t+s-1}^\kappa, \quad s = 1, \dots, n, \quad (6)$$

where $\Pi_{t+1} = P_{t+1}/P_t$ is the gross inflation rate at the time $t+1$ and Π is its steady-state value, while $\kappa \in [0, 1]$ measures the degree of indexation to past inflation. Since there is a continuum of firms of mass one, in each period a fraction $1 - \varpi$ of firms will be able to re-optimize and a fraction ϖ will change their price according to the indexation rule (6). The solution to the price-setting problem is deferred to the appendix.

2.3 Public Sector, Pollution Stock, and Equilibrium

We assume that the polluting energy input is extracted with no cost by the government, which sells it to the intermediate-goods producers and distributes the proceeds as lump-sum transfers to the households. The government's budget is then balanced at all times. By either setting the price or the supply of the fossil resource, the government can control emissions.¹² In the first case, the government sets the real price per unit of emissions, and this price can be interpreted as a carbon tax; in the second case, the government sets a cap on the overall emissions generated by the economy. We limit our attention to these specific pollution policies since they are two instruments frequently contrasted in the literature and in policy debate.

Let $Z_t = \int_0^1 Z_{i,t} di$ denote the aggregate flow of emissions, then, following Golosov et al.

¹²Analogously one could assume that households own the fossil resource and that the public sector levies a tax on its use or imposes a quantity restriction as a way to price carbon.

(2014), the cumulative emissions in the atmosphere, M_t , evolves as:

$$M_t - \bar{M} = Z_t + (1 - \delta_M) (M_{t-1} - \bar{M}) + Z_t^{RoW}, \quad (7)$$

where \bar{M} denotes the pre-industrial concentration of pollutant, $\delta_M \in (0, 1)$ measures the natural rate at which the atmosphere recovers, and Z_t^{RoW} is an exogenous process capturing the rest-of-the-world emissions. In what follows, we will keep Z_t^{RoW} constant.

Finally, we include a damage channel in the model, namely a level impact channel affecting firms' productivity via the damage factor Δ_t . Following Golosov et al. (2014), who simplify the approach of Nordhaus (2008, 2017), the damage evolves as follows:

$$\Delta_t = \exp(-\eta(M_t - \bar{M})), \eta > 0, \quad (8)$$

where $\eta > 0$ is a scaling coefficient measuring the intensity of the negative externality on production. $1 - \Delta_t$ is then the fraction of production lost due to climate change. This is a parsimonious way of introducing the negative environmental externality by which pollutant concentrations affect productivity.¹³ From the functioning of the climate system, it is easy to understand how changes in emissions in a limited period and implemented by one and only economy do not substantially change cumulative emissions in the atmosphere. This implies that the marginal benefits of unilateral mitigation policies are negligible at business cycle frequency, while the marginal costs are substantial.

Monetary policy is set according to a Taylor-type interest rate rule specified as follows:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\iota_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\iota_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\iota_y}\right]^{1-\iota_R} \exp u_t, \quad (9)$$

where R denotes the steady-state value of the nominal interest rate, Π is the steady-state inflation, Y_t^* is the natural level of output (i.e., the output that would prevail if prices were fully flexible), $\iota_R \in [0, 1)$ is the smoothing parameter, measuring the degree of persistence of the rule, $\iota_\pi > 0$ and $\iota_y > 0$ capture the responsiveness of nominal interest rate to movements in inflation and output gap, while u_t represents an exogenous monetary policy shock.

Finally, since we have assumed that the natural resource is produced at no cost, the resource constraint of the economy is given by:

$$Y_t = \int_0^1 C_{i,t} di. \quad (10)$$

¹³In fully-fledged integrated assessment models, carbon concentration affects global mean temperature and then changes in temperature negatively impact productivity. See Golosov et al. (2014) for a discussion on how the exponential damage function specified here approximates the current state-of-the-art damage function given, e.g., by Nordhaus (2007).

2.4 The Aggregate Model and Expectations

In this section, we first summarize the aggregate equations of the model described in the previous section and then introduce the modeling of expectations. The equilibrium conditions describing the behavior of heterogeneous agents have been log-linearized around a zero-inflation steady state and then aggregated. The aggregate model is reported in Table 1, where y_t , mc_t , z_t , $p_{z,t}$, m_t , y_t^* and δ_t denote output, marginal costs, emissions flow, the relative price of carbon, stock of pollutant, natural output, and environmental damage. All these variables are expressed as natural log deviations from their steady-state values; on the contrary, the inflation rate, π_t , and the nominal interest rates, r_t , are expressed as deviations from their respective steady-state levels. In detail, the first equation of Table 1 describes the aggregate demand (IS curve); the second equation determines the time path of marginal costs, where the last term measures the impact of environmental damage on production; the third equation describes the dynamic of emissions that decrease in the carbon price; the fourth equation is the behavioral analog of the New Keynesian Phillips curve that relates inflation to marginal costs and agents' beliefs about future inflation; the fifth equation describes the accumulation of the pollution stock; the sixth equation refers to the environmental damage factor; finally, the last equation is the interest rate rule, where we have set ι_r at zero. The equations of the aggregate model will come in handy in interpreting the results of our numerical experiments. For the complete derivation of these equations, see Appendix.

An increase in the price of carbon, p_Z , reduces the level of emissions, z , implying, in turn, an increase in marginal costs, mc . The increase in mc would then translate into higher inflation via the New Keynesian Phillips Curve. Therefore, a more stringent climate policy drives inflation upward through its positive effects on marginal costs, similar to a negative productivity shock. The climate action then negatively affects the natural level of output, as it happens in response to an adverse technological shock. This explains why, as we show in our policy experiments, the output gap temporarily increases during the green transition and why there is no policy trade-off between inflation and output gap stabilization.¹⁴

We are now ready to describe how agents formulate expectations about future variables. As anticipated, agents are assumed to have cognitive limitations, therefore, they use simple rules (i.e., heuristics) to forecast future income and inflation. In the words of [Gigerenzer and Gaissmaier \(2011, p. 454\)](#), a heuristic is “a strategy that ignores part of the information to make decisions more quickly and frugally than more complex methods”. The assumption is that agents adopt precise rules of thumb in their decision-making to overcome their cognitive limitations.

Following the heterogeneous expectations framework of [Brock and Hommes \(1997\)](#), the forecasting rules are described by an endogenous selection mechanism by which agents switch

¹⁴Note that while the target level of output for monetary policy, that is the natural level of output, changes with the policy, the inflation target is constant and equal to its deterministic level.

Table 1: The Aggregate Log-Linearized Model

Equation	Description
$y_t(1+h) = \tilde{E}_t y_{t+1} + h y_{t-1} - \frac{1-h}{\gamma} (r_t - \tilde{E}_t \pi_{t+1}) + v_t$	IS
$mc_t = mc_y y_t - mc_\delta \delta_t - mc_z z_t - mc_a a_t - mc_{y-1} y_{t-1}$	Marginal Cost
$z_t = \varkappa mc_t + (\varkappa - 1) \delta_t + y_t - \varkappa p_{Z,t}$	Emissions
$\pi_t = \frac{1-\varpi\beta}{1+\beta\kappa} \frac{1-\varpi}{\varpi} mc_t + \frac{\beta}{1+\beta\kappa} \tilde{E}_t \pi_{t+1} + \frac{\kappa}{1+\beta\kappa} \pi_{t-1}$	New Keynesian Phillips Curve
$m_t = \delta_M \frac{Z}{Z+Z^{RoW}} z_t + (1 - \delta_M) m_{t-1} + \delta_M \frac{Z^{RoW}}{Z+Z^{RoW}} z_t^{RoW}$	Pollution Stock
$\delta_t = -\eta (M - \bar{M}) m_t$	Environmental Damage
$r_t = \iota_\pi \pi_t + \iota_y (y_t - y_t^*) + u_t$	Taylor Rule

Note: the model is log-linearized around a zero steady-state inflation. In the second equation the coefficients are all positive and depend on a complex fashion on the deep parameters of the model. See the Appendix. Variables v_t , a_t and u_t are exogenous stochastic processes driving economic fluctuations.

from one rule to another based on their past forecasting performances. For simplicity, we assume only two types of forecasting rules. Consistently with the approach of [De Grauwe \(2011, 2012a,b\)](#) we consider an ‘extrapolative’ rule and a ‘fundamentalist’ rule.¹⁵

The extrapolative prediction rule, labeled e , is a random walk rule by which agents use the previously observed value of a variable as a forecast. This is a myopic rule according to which agents are insensitive to other information about the functioning of the economy. The fundamentalist rule, labeled f , is more sophisticated. Agents expect future output to be equal to the expected value of its fundamental level, natural output, while inflation in the next period is simply expected to return to its long-run value target set by the central bank. In formulating their expectation regarding the natural level of output, these agents use all the information set available at the time t , that is, they formulate rational expectations, but on the wrong variable since they neglect imperfect price adjustments. The judgment on future output is also based on the (wrong) belief that monetary policy is neutral. Put another way, fundamentalists expect that the next period’s output gap will be equal to zero.¹⁶

According to these forecasting rules, expectations on output and inflation are such that:

$$\tilde{E}_{e,t}(y_{t+1}) = y_{t-1}, \quad \tilde{E}_{e,t}(\pi_{t+1}) = \pi_{t-1}, \quad (11)$$

$$\tilde{E}_{f,t}(y_{t+1}) = E_t y_{t+1}^*, \quad \tilde{E}_{f,t}(\pi_{t+1}) = 0, \quad (12)$$

¹⁵We have opted to discipline how agents formulate expectations consistently with these previous contributions since in this way the model is able to reproduce quite well some empirical features of inflation and output. Notably, when one abandons the rational expectations hypothesis, there is the risk of venturing into the ‘wilderness’ of bounded rationality ([Sims 1980](#)) where there are many ways of modeling the behavior of non-rational agents.

¹⁶Note that this is equivalent to the formalization presented by [De Grauwe \(2012b\)](#) in which under a fundamentalist rule agents estimate the steady-state value of the output gap (which is 0) and use this to forecast the future output gap.

where expectations are formulated at the beginning of the period t before the realization of the shocks. Let $\alpha_{y,t}^e$ ($\alpha_{y,t}^f$) denote the share of agents opting for an extrapolative (fundamentalist) rule for output forecast, and $\alpha_{\pi,t}^e$ ($\alpha_{\pi,t}^f$) the share of agents opting for an extrapolative (fundamentalist) rule for inflation forecast, the market forecasting rules for output and inflation immediately follow:

$$\tilde{E}_t(y_{t+1}) = \alpha_{y,t}^f \tilde{E}_{f,t}(y_{t+1}) + \alpha_{y,t}^e \tilde{E}_{e,t}(y_{t+1}), \quad (13)$$

$$\tilde{E}_t(\pi_{t+1}) = \alpha_{\pi,t}^f \tilde{E}_{f,t}(\pi_{t+1}) + \alpha_{\pi,t}^e \tilde{E}_{e,t}(\pi_{t+1}), \quad (14)$$

where $\alpha_{y,t}^f + \alpha_{y,t}^e = 1$ and $\alpha_{\pi,t}^f + \alpha_{\pi,t}^e = 1$.

Following [Brock and Hommes \(1997\)](#), agents can switch between these rules on the basis of their forecasting performances. Put differently, agents are aware that their predictions may be biased; they then learn from their mistakes and switch to the best performing rule. From this point of view, agents can be seen as *rational* since they continuously evaluate the forecast performance of a given rule.¹⁷

Let U_x^i denote the fitness criterion of the rule $i \in \{f, e\}$ for the generic variable $x \in \{y, \pi\}$. This criterion of success is simply defined as the negative of the weighted mean squared forecasting errors of the forecasting rule:

$$U_{x,t}^i = - \sum_{k=0}^{\infty} \gamma_k \left(x_{t-k-1} - \tilde{E}_{i,t-k-2} x_{t-k-1} \right)^2, \quad (15)$$

where γ_k denote geometrically declining weights measuring the weight attributed by agents to past forecast errors. We assume that agents tend to forget, so they attach relatively higher importance to recent errors than those made far in the past. To capture this tendency to forget in a parsimonious way, we assume $\gamma_k = (1 - \rho)\rho^k$ with $0 \leq \rho \leq 1$, then (15) can be re-written as:

$$U_{x,t}^i = \rho U_{x,t-1}^i - (1 - \rho) \left(x_{t-1} - \tilde{E}_{i,t-2} x_{t-1} \right)^2, \quad (16)$$

where the parameter ρ is a measure of agent memory. In particular, when $\rho = 1$ agents have infinite memory and assign the same weights to all past mistakes; when $\rho = 0$, instead, agents have no memory and only the forecast error of the last period matters. In the latter case, we will see that the economy is more volatile.

In addition, agents may be unpredictably affected by their state of mind when choosing between the two rules, or they may face a measurement error in calculating forecast errors. To capture these factors that may affect decisions, we assume that the comparison between the two values of the metrics chosen as fitness criterion of rules $i \in \{f, e\}$ is based on the following probability P :

$$\alpha_{x,t}^f = P[U_{x,t}^f + \epsilon_{x,t}^f > U_{x,t}^e + \epsilon_{x,t}^e], \quad (17)$$

¹⁷Agents then spend some mental energy in evaluating the performance of a given heuristic. We explicitly account for this possibility in Section 5.3.3.

where now $\alpha_{x,t}^f$ can be interpreted as the probability of opting for a fundamentalist rule, while $\epsilon_{x,t}^f$ and $\epsilon_{x,t}^e$ are random variables catching all the unpredictable factors that may affect agents when choosing between alternatives.

As in the discrete choice model of [Brock and Hommes \(1997\)](#) and based on the work of [Manski and McFadden \(1981\)](#) and [Anderson et al. \(1992\)](#), these random variables are assumed to be logistically distributed. Under the assumption that all agents can simultaneously update the forecasting rule they use, then the fraction of agents opting for rule i in each period will be given by:

$$\alpha_{x,t}^i = \frac{e^{\theta U_{x,t}^i}}{\sum_i e^{\theta U_{x,t}^i}}, \quad (18)$$

where the parameter θ referred to as ‘learning parameter’ or ‘intensity of choice’, reflects the tendency of agents to select the best-performing rule.¹⁸ The parameter θ then measures how fast agents tend to switch to the more successful forecasting rule. The size of this parameter is related to the variance of the random components $\epsilon_{x,t}^f$ and $\epsilon_{x,t}^e$ in (17), and therefore it will depend on the scale of the model. In particular, if this variance tends to infinity, then $\theta \rightarrow 0$ and agents cannot observe any difference in fitness between the two rules or simply they do not exhibit any willingness to learn from past mistakes. In this case, agents flip a coin to make their choice, so that $\alpha_{x,t}^f$ and $\alpha_{x,t}^e$ will be equal to 0.5. When the variance of the random components tends to zero, $\theta \rightarrow \infty$ and agents select the best-performing rule, the probability of opting for one rule can be either 1 or 0. We will see that for a higher θ market beliefs tend to amplify disturbances.¹⁹

Given these assumptions, the economy features four types of agents according to their way of formulating expectations: (i) agents who formulate expectations according to the extrapolative rule for both output and inflation, (ii) agents who formulate expectations according to the fundamentalist rule for both output and inflation, (iii) agents who opt for the extrapolative rule for output and the fundamentalist rule for inflation, (iv) agents who opt for the extrapolative rule for inflation and the fundamentalist rule for output.

¹⁸According to (18) we are considering the case of synchronous updating, where all agents switch to better rules in each period. We will remove this assumption in Section 5.2, where we introduce the possibility of asynchronous updating.

¹⁹It is worth noticing that in the present version the coefficient θ reflects the marginal effect of observable factors (the learning process) relative to the standard deviation of unobserved variables. This means that it is a combination of both the behavioral parameter reflecting the intensity of choice and the scale parameter reflecting the variance of the unobserved portion of utility. See [Train \(2009, chap. 2\)](#) for more details.

3 Parameterization

In this section we describe the parameterization of the model. To this end, we start by assuming that all the exogenous components v_t , a_t and u_t follow an AR(1) process:

$$v_t = \rho_v v_{t-1} + \xi_{v,t}, \quad (19)$$

$$a_t = \rho_a a_{t-1} + \xi_{a,t}, \quad (20)$$

$$u_t = \rho_u u_{t-1} + \xi_{u,t}, \quad (21)$$

where $\rho_v, \rho_a, \rho_u \in [0, 1)$ and $\xi_{v,t}, \xi_{a,t}, \xi_{u,t}$ are normally and independently distributed innovations with mean zero and standard deviations σ_v, σ_a and σ_u , respectively.

To parameterize the model, we proceed in two steps. First, we fix a set of parameters to match the standard macroeconomic and environmental targets in steady state. Second, given the values of those parameters, we choose the remaining ones to match some moments (standard deviation, autocorrelation, and kurtosis) in the data. A model period corresponds to one quarter.

Fixed Parameters. Table 2 lists the choice of parameter values that we fix. In particular, the parameters related to the New Keynesian structure of the model are standard and taken from the current literature. Consumption preferences are assumed to be logarithmic ($\gamma = 1$) and the inverse of the Frisch elasticity φ is set to 1, an intermediate value between micro and macro data estimates. The discount factor β is equal to 0.99, consistent with a real interest rate of 4% per year. The elasticity of substitution between energy and labor inputs, \varkappa , is fixed at 0.3, implying that the two production factors are imperfect complements. The quasi-share parameter measuring the contribution of polluting input to CES production, ζ , is calibrated starting from the share of income spent on energy in the US that in 2018 was around 6% of GDP according to [EIA \(2020\)](#). The probability that prices remain unchanged in each quarter, ϖ , is fixed at 0.75. In the baseline calibration, the policy parameters of the Taylor rule are also standard, that is $\iota_\pi = 1.5$, $\iota_y = 0.125$ and $\iota_r = 0$.

To parameterize the environmental part of the model, we proceed as follows. We start by considering the world's total emissions in 2020 according to the business-as-usual scenario of the DICE model, which is 41.685 giga-tons of carbon dioxide per year.²⁰ The quarterly rate at which the atmosphere recovers, δ_M , is 0.0021 consistently with [Reilly and Richards \(1993\)](#), implying a half-life of carbon in the atmosphere of about 83 years. Knowing that the pre-industrial atmospheric concentration of carbon, \bar{M} , is about 581 giga-tons (see, e.g., [Golosov et al. 2014](#)), we can obtain the steady-state value for M which approximately corresponds to the atmospheric concentration of carbon observed in the DICE model in 2080. According to the DICE simulations, at this pollutant concentration level, the fraction of the output lost

²⁰For details on the DICE model, see [Nordhaus \(2017, 2018\)](#).

for damage is around 0.026. From this assumption, we can retrieve the damage parameter η . Finally, to set the coefficient $Z/(Z^{RoW} + Z)$ in the pollution stock equation of Table 1, we use World Bank data for 2018 and observe that the share of worldwide GHG emissions ascribed to the US is around 13%.

Table 2: Fixed Parameter Values

Parameter	Value	Description
β	0.99	Discount rate
γ	1	Risk aversion coefficient
φ	1	Inverse Frisch elasticity
ζ	0.1724	Energy quasi-share parameter
\varkappa	0.3	Elasticity of substitution between energy and labor inputs
ω	0.75	Calvo's price parameter
δ_M	0.0021	Emissions decay rate
$\eta(M - \bar{M})$	0.0263	Impact damage coefficient
ι_r	0	Smoothing parameter of the Taylor rule
ι_y	0.125	Output gap coefficient of the Taylor rule
ι_π	1.5	Inflation coefficient of the Taylor rule

Fitted Parameters. We choose the remaining parameters, listed in Table 3, in order to match the empirical targets in Table 4. The model-implied targets are computed from simulated moments at the third-order approximation. With this technique, the parameters are estimated so that the squared difference between the simulated and empirical moments is minimized.²¹ Although the model is nonlinear and under-identified, with seven moments determining ten parameters, it fits the moments in Table 4 fairly well, especially for output.

The fitted parameters predict a coefficient of price indexation κ close to zero, in order to capture the low amount of autocorrelation of inflation detected in the data. On the contrary, the habit persistence coefficient h is approximately 0.20 in an attempt to capture the higher amount of output autocorrelation. In particular, both of those fitted values are lower than the counterparts typically found in rational expectation models, since persistence is already significantly generated by backward-looking expectations of extrapolators.²²

The parameter that measures agent memory ρ , and the autoregressive coefficients and standard deviations of the three shocks are broadly comparable to previous findings in the literature. The intensity of choice θ is estimated around 4685 incorporating information about the

²¹In more details, we generate a quasi-random (quasi-Montecarlo) sequence of initial values to create a grid of pseudo-parameters eligible for the solution of the minimization problem. We then use this grid to run several trials and compute the quadratic deviation of the associated moments from the empirical ones. We then select the parameters associated with the trial that have the smallest residual.

²²Milani (2007) shows that, in a model where learning replaces rational expectations, the estimated degrees of habit formation in consumption and inflation indexation drop closer to zero, suggesting that persistence arises mainly from expectations and learning. Our model fitting points in a similar direction.

behavioral parameter (the learning parameter) being normalised by a scale factor capturing the standard deviation of the unobserved factors.²³

In the baseline parameterization under a constant carbon tax policy, the model can reasonably match the moments of output observed in the US quarterly data for the period 1990Q1-2019Q4.²⁴ In particular, the model is able to reproduce the standard deviation and the first auto-correlation of output very well, while the inflation moments are more challenging to capture in such a simple model. Furthermore, the model can reproduce the excess of kurtosis found in the data, which is evidence of fat tails in the output distribution. Also, the positive correlation between output and inflation is roughly met.²⁵

Table 3: Fitted Parameter Values

Parameter	Value	Description
h	0.20	Habit parameter
κ	0.00	Coefficient of price indexation
θ	4685	Learning or intensity of choice parameter
ρ	0.41	Memory of agents
ρ_a	0.80	Technology shock persistence
ρ_v	0.78	Preference shock persistence
ρ_u	0.50	Monetary policy shock persistence
σ_a	0.0097	Standard deviation of the technology shock
σ_v	0.0051	Standard deviation of the preference shock
σ_u	0.0046	Standard deviation of the monetary policy shock

4 Greening the Economy: The Role of Market Beliefs and Monetary Policy

This section shows how expectations and market beliefs interact with different environmental policies and seeks to understand the role monetary policy could play in reducing the trade-offs at stake, controlling potential inflationary pressures, and helping to reach the climate targets. In Section 4.1 we explore the implications of removing the standard assumption of complete rationality for the conduct of environmental policy. Section 4.2 analyzes how the presence

²³To be more explicit, $\theta = \theta^*/\sigma$, where θ^* is the behavioral parameter and σ is the scale factor and is of the same order of magnitude of the shocks hitting the economy. Given that θ^* and σ are not separately identified, we estimate this ratio. See footnote 19 for more details.

²⁴Note that in Table 3 the intensity of choice has not been scaled down by the size of the shocks in (17).

²⁵It can be shown that under a quantity restriction (i.e., a cap on emissions) output is less volatile, consistently with previous findings (e.g. Fischer and Springborn 2011 and Annicchiarico and Di Dio 2015) and less leptokurtic, while inflation is slightly more volatile than under a tax, because of the uncertainty surrounding emission prices over the business cycle.

Table 4: Moments for Output and Inflation - Data and Model under the Baseline Parametrization

	Data	Model
Standard deviation y	0.0102	0.0114
First-order autocorrelation y	0.8463	0.8358
Kurtosis y	3.4601	3.3063
Standard deviation π	0.1743	0.2576
First-order autocorrelation π	0.2821	0.8500
Kurtosis π	3.8955	4.7372
Correlation y, π	0.3412	0.3965

Note: the table reports moments generated by the model under the baseline parameterization for 100 replications of shock sequences of size 1,000 and those of the US data over the period 1990Q1-2019Q4, retrieved from FRED, Federal Reserve Bank of St.Louis. Series used: Real Gross Domestic Product - GDPC1 (HP Filtered series) and Gross Domestic Product: Implicit Price Deflator - GDPDEF (HP Filtered series). The standard deviation of (quarterly) inflation is expressed in percentage points. Skewness of output is 0.0269 (data) and 0.0148 (model); skewness of inflation is -0.6170 (data) and 0.0358 (model).

of behavioral agents and business cycle fluctuations shape the economy’s response to price and quantity-based mitigation scenarios and explores what the implications for price stability are. Finally, Section 4.3 studies the role of monetary policy during the greening process under different underlying environmental regimes, different degrees of monetary policy stringency, and different interest-rate rules.

4.1 Market Beliefs and Environmental Policy: A Simple Example

Here, we analyze the impact of a mitigation policy under different expectation formations. We solve the model both under the rational expectations hypothesis (the orthodox model) and in the case where agents are assumed to formulate expectations according to heuristics, as described in the previous sections. To better elucidate the mechanism of transmission of environmental policy under a different formalization of expectations, we start our analysis with an illustrative example. We consider a permanent increase in the carbon price, p_z , able to induce a 10% reduction in emissions. To achieve this target, the carbon price p_z must increase by 28.6%.

Figure 2 illustrates the dynamic effects of this greening policy in the two variants of the model economy (behavioral vs. rational expectations). The economy’s response to this policy shock abstracts from the presence of business cycle fluctuations, that is, we assume that the economy is in its steady-state equilibrium when the carbon pricing shock hits it. In the behavioral model we assume that the fraction of agents using an extrapolative rule for both inflation and output is initially equal to 0.5.

We observe that under the same policy stringency in the presence of bounded rationality, the time required to achieve the mitigation target almost doubles compared to the rational expectations case. Note that the 10% reduction is reached in period 3 in the rational expectations model and approximately in period 6 in the behavioral one. This delay implies a lower cumulative reduction in emissions of about 2.27% in the latter scenario.

Moreover, different assumptions about expectation formation strongly alter the dynamic behavior of the relevant macro variables. As expected, output decreases in both model configurations in response to the increase in the carbon price. This is due to the rise in marginal costs driven by the higher price of the energy input. However, under the rational expectations hypothesis, agents can fully internalize the effects of the policy and react immediately, thus reducing emissions and production promptly. In this case, the recessionary effects of the policy fully materialize since the beginning. Rational expectations agents are aware that climate policy, by permanently changing the supply-side conditions, will affect their permanent income, therefore, they push down consumption. Conversely, the agents' reaction is more conservative in the behavioral model, implying a slower adjustment of real macroeconomic variables. However, the aggregate dynamics masks striking differences in the underlying adjustment between extrapolators and fundamentalists. On the one hand, fundamentalists expect a return of the economy to its natural level, which is negatively affected by pollution policy. These agents do not account for short-term deviations of output from its natural level and therefore do not have a precise perception of the time path of output during the adjustment process.²⁶ On the other hand, extrapolators are purely backward-looking and initially perceive the climate action as a temporary shock. As a result, these agents slowly adjust their consumption choices, sustaining aggregate demand and production during the mitigation period and slowing down the transition toward a greener economy. Progressively, households start realizing that the shock is permanent and the share of extrapolators decreases over time.²⁷

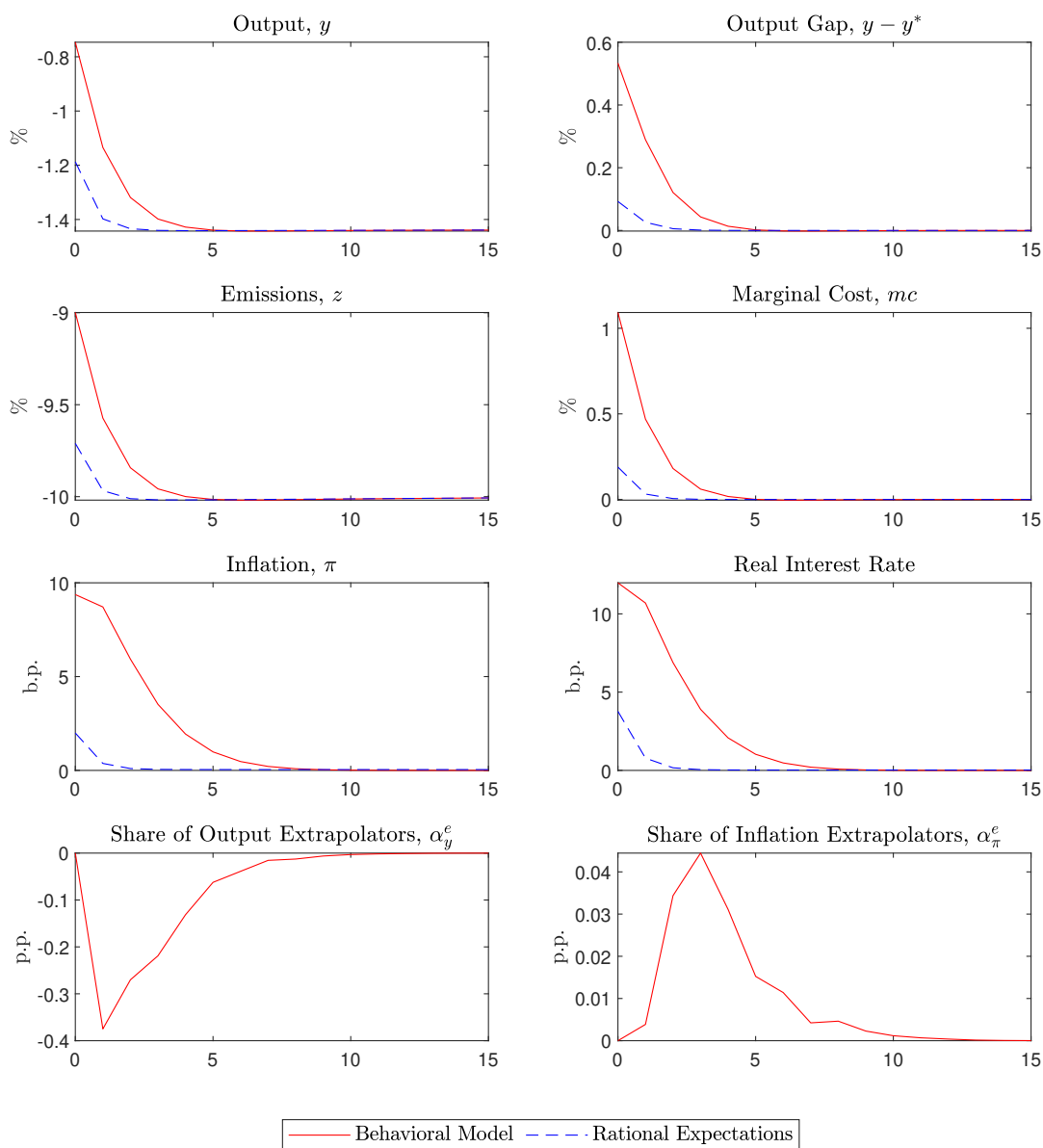
Looking at the behavior of inflation and interest rates, we note what kinds of interaction effects are in place between monetary and climate policies. The carbon pricing policy propagates in the economy through its effects on the marginal cost and natural output, creating an upsurge in inflation and a drop in output. The output gap is positive: price rigidities dampen the decrease of output which falls less than in the case of a fully flexible price economy.²⁸ The increase in inflation leads the central bank to raise the nominal interest rate more than

²⁶Recall that here the output gap is expressed as the difference between the output arising under sticky prices and the natural level of output, meant as the output prevailing in the case of fully flexible prices.

²⁷It is possible to show that under a higher degree of consumption habit the adjustment in output takes longer, partially validating the expectations of the extrapolators, whose share increases, further slowing down the adjustment process. Therefore, the endogenous persistence generated by the presence of behavioral agents could be further augmented, in this model, by the presence of habit formation in consumption that would lead to an increase in the share of backward-looking agents.

²⁸For this reason, a more stringent climate policy cannot be considered, strictly speaking, a cost-push shock since cost-push shocks are interpreted as shocks that directly affect inflation, leaving unchanged the natural level of output, and therefore being able to generate a policy trade-off between inflation and output gap stabilization.

Figure 2: Increase in the Carbon Price under Rational and Behavioral Expectations



Note: the figure plots the response of the economy to a permanent increase in the carbon price aimed at permanently reducing emissions by 10%. All variables are expressed in percentage deviations from their respective business-as-usual value, with the exceptions of the inflation and the real interest rate, expressed in quarterly basis points (b.p.) deviations, and the shares of extrapolators, expressed in percentage points (p.p.) deviations.

proportionally to bring inflation back to its target. The adjustment process is slowed down by non-rational agents, and inflation remains above the target more persistently. The share of inflation extrapolators increases, contributing to making the inflationary pressures of this shock more pronounced. In this case, preserving price stability in response to climate actions looks more challenging.²⁹ Therefore, compared to the dynamics of the standard New Keynesian model with rational expectations, the presence of agents following different heuristic expectation rules substantially affects the effectiveness of policy interventions and the persistence of the adjustment process, especially in the short run.

In this experiment, we have not considered the role of uncertainty in shaping the economy’s response to a climate policy. In fact, there is significant uncertainty about how a greening policy can affect the economy and the conduct of monetary policy, especially during an ambitious mitigation path. This is especially true in behavioral models, where the economy’s response to policies may entail waves of optimism and pessimism generated by wrong market beliefs and where the results are sensitive to the initial conditions of the economy (i.e., the share of different agents in the economy and the phase of the business cycle).

4.2 Market Beliefs and Mitigation Scenarios: Carbon Tax vs. Cap

We are now ready to consider a more ambitious mitigation scenario and analyze the uncertainty surrounding the impact of a greening policy on the main macroeconomic variables in an economy with non-rational agents. Specifically, we start by examining a mitigation policy implemented through a gradual increase in the carbon tax able to generate a reduction in emissions by 20% in 5 years in a deterministic rational expectations economy, where, in each period, agents are assumed to be surprised by the policy shock. This assumption is made to rule out any anticipation effects.³⁰ The mitigation scenario is in line with the emission reduction targets set by the United States for 2030.³¹

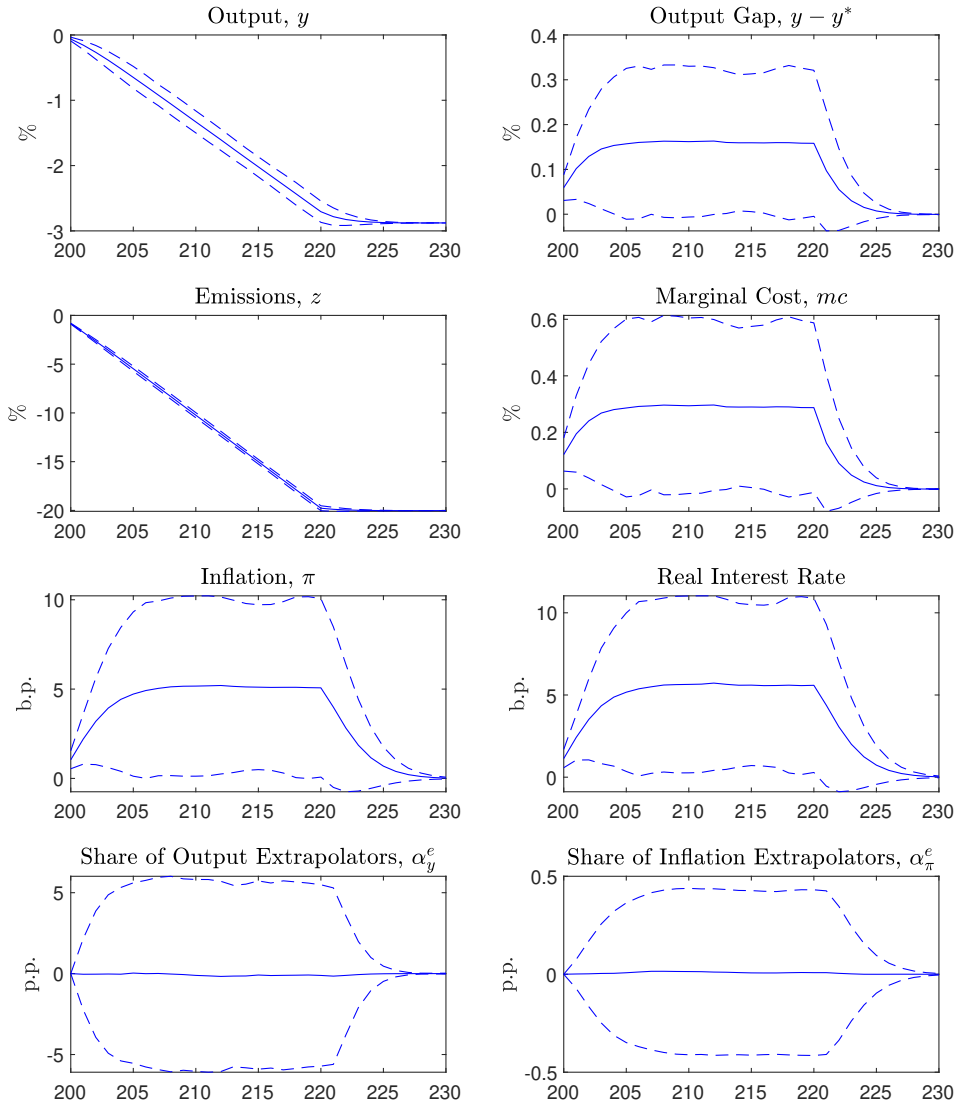
We factor in uncertainty by undertaking two series of simulations based on the behavioral version of the model. The design of the experiment is as follows. In the first baseline simulated series, the economy is hit by exogenous shocks on technology, demand, and interest rate. The

²⁹In the next section, we will see how preserving price stability may be even more difficult under a cap policy prescribing a commensurate quantity restriction on the pollutant.

³⁰In Appendix, we report the dynamics of output and inflation in the benchmark case (business cycle shocks and no policy anticipation), in the case of no policy anticipation and no business cycle shocks, and finally in the case in which the policy is fully anticipated (see Figure A-2). This last scenario can be considered as a case in which government policies are fully credible and there is neither policy uncertainty nor business cycle uncertainty. Clearly, only agents adopting a fundamentalist forecasting rule can exploit this information. We report the results under the carbon tax scenario. Results for the cap-and-trade case are available upon request.

³¹The United States has an economy-wide target of reducing its GHG emissions by 50-52% below 2005 levels by 2030. We use the emission data provided by Crippa et al. (2021) to calculate the reduction achieved so far and the one still needed to reach the target. Consistently with the short-run analyses presented in the paper and the typical horizon of monetary policy, we present here the first five years of the mitigation plan (20% emission reduction compared to current levels).

Figure 3: Timely Mitigation under Behavioral Expectations - Carbon Tax



Note: the figure plots the mean response of the economy to a gradual increase in the carbon price aimed at permanently reducing emissions by 20%. Dashed lines show ± 2 standard deviations from the mean. All variables are expressed in percentage deviations from their respective business-as-usual value, with the exceptions of the inflation and the real interest rate, expressed in quarterly basis points (b.p.) deviations, and the shares of extrapolators, expressed in percentage points (p.p.) deviations.

length of the series is 300 quarters. In the second simulation series, the economy is hit by the same exogenous shocks as in the first simulation series, but it also entails the introduction of a mitigation plan after 200 periods. To compute the response functions of the economy to a mitigation policy introduced in a business-as-usual scenario, we subtract the first simulated series from the second one. Basically, the economy is off the steady state when the carbon pricing policy is implemented. We then replicate this experiment considering 1,000 random shocks' realizations, $\xi_{v,t}$, $\xi_{a,t}$, $\xi_{u,t}$, and compute the mean response functions and the corresponding standard deviations. In other words, we analyze the effects of carbon pricing conditional on the initial state of the economy.

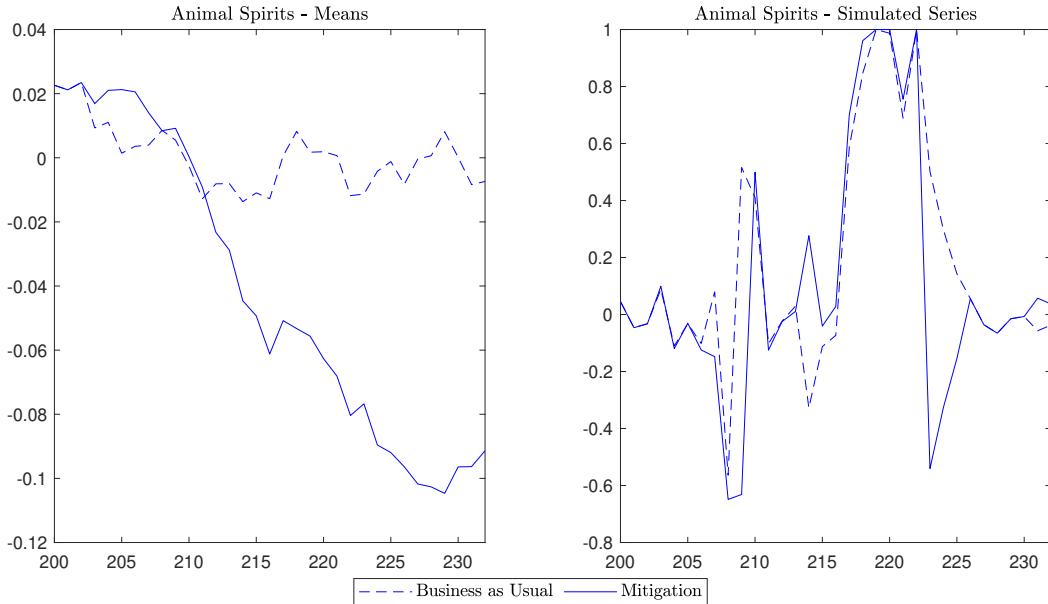
Figure 3 illustrates the mean response (solid lines) and a band of significance of ± 2 standard deviations from the mean (dashed lines). Looking closely at the figure, we can see a wide increase in the uncertainty surrounding the short-term effects of the carbon price. The economy's reaction depends on the initial state of the economy that could be in any phase of the business cycle. The range of variation in the dynamic response to the shock is driven endogenously by self-fulfilling movements of optimism and pessimism that amplify fluctuations and affect how the policy shock is transmitted to the economy. By basing their decisions on biased information, non-rational agents make the economy more prone to fluctuations. In addition, the policy shock itself affects market sentiments, which is why it may take longer to adjust to the new long-term equilibrium.³²

To measure the strength of these market sentiments, in Figure 4 we show the time path of the so-called 'animal spirits', by which we measure the movements in the fraction of optimistic extrapolators, following De Grauwe and Ji (2020).³³ An index close to one suggests that nearly all agents are extrapolators forecasting a positive output, while an index close to minus one indicates that almost all agents are extrapolators predicting a negative output. Figure 4 presents the level of 'animal spirits' for a specific simulation, as well as the mean level across all 1,000 replicated simulations under a carbon tax, differentiating between the business-as-usual case and the mitigation case. It is evident that, on average, the level of 'animal spirits' fluctuates around zero. However, under the green transition scenario, we observe a wave of mild pessimism, leading to an increase in the proportion of agents anticipating a negative output level. When examining the simulated level of 'animal spirits', we notice a significantly higher degree of variability. It is interesting to observe how abruptly the direction of 'animal spirits' can change during the transition compared to the business-as-usual scenario. Such direction changes reflect changes in market sentiments that contribute to increasing the variability in the economy's response to climate policy, as shown in Figure 3. This last result is particularly relevant, since it brings light to an additional layer of uncertainty surrounding the achievement of climate targets.

³²In Appendix we show how the economy evolves under rational expectations. See Figure A-3.

³³The index is equal to $\alpha_{y,t}^e - \alpha_{y,t}^f$ if $y_{t-1} > 0$ and to $-\alpha_{y,t}^e + \alpha_{y,t}^f$ if $y_{t-1} < 0$.

Figure 4: Animal Spirits under a Mitigation Policy - Carbon Tax



Note: the figure plots the mean level and a simulated series for the measure of ‘animal spirits’ under business as usual and under a gradual increase in the carbon price aimed at permanently reducing emissions by 20%.

When we look at the emissions, in a timely and orderly mitigation scenario of Figure 3, they could follow only slightly different trajectories. We observe that the number of quarters needed to reach the objective by 20% emission reduction ranges from 21 to 27 quarters. In the case of a disorderly mitigation scenario and/or in the case of a highly perturbed economy, instead, the emission trajectory can be much more unpredictable, making the adjustment process to the target more or less expensive in terms of cumulative emissions.³⁴ To achieve an equivalent mitigation goal over the same time horizon, but avoid any uncertainty regarding the emission pattern, the government may opt for a quantity-based instrument rather than a price instrument. However, a quantity approach may involve excessive volatility of the emission prices. To understand the uncertainty inherent to the selected instrument, in the first two columns of Table 5, we compare the performance of a carbon tax and an emission cap during the mitigation path considered in Figure 3. We look at the variability of the response of a selection of variables to the climate policy, at the inflation dynamics, and at the welfare loss. In particular, following De Grauwe and Ji (2019), we use the following function as a measure of the welfare loss, denoted as L :

$$L = \vartheta\sigma_{\pi} + (1 - \vartheta)\sigma_{y-y^*}. \quad (22)$$

Here, σ_{π} represents the standard deviation for inflation, and σ_{y-y^*} represents the standard deviation for the output gap. The parameter ϑ measures the preference for price stability. To

³⁴The time path of emissions in a highly perturbed economy is shown in Figures A-4 and A-5 in Appendix.

Table 5: Macroeconomic Volatility over the Mitigation Path

	Timely Mitigation		Delayed Mitigation	
	Tax	Cap	Tax	Cap
σ_z	0.1156	0	0.1442	0
σ_{p_z}	0	0.3208	0	0.4005
σ_{y-y^*}	0.0742	0.0561	0.0926	0.0700
σ_π	2.1385	2.1737	2.6306	2.6739
$E(\pi)$	4.5789	5.2102	5.6848	6.4480
$max(\pi)$	5.2018	5.8658	8.6530	9.7621
Welfare loss L	0.0478	0.0389	0.0594	0.0484

Note: the table reports the standard deviations of the response for a selection of variables along with mean inflation and its maximum observed value over the mitigation time path; the standard deviations σ_{p_z} , σ_z , σ_{y-y^*} are expressed in percentages, σ_π , $E(\pi)$ and $max(\pi)$ in quarterly basis points. For the computation of the welfare loss both standard deviations are expressed in percentages.

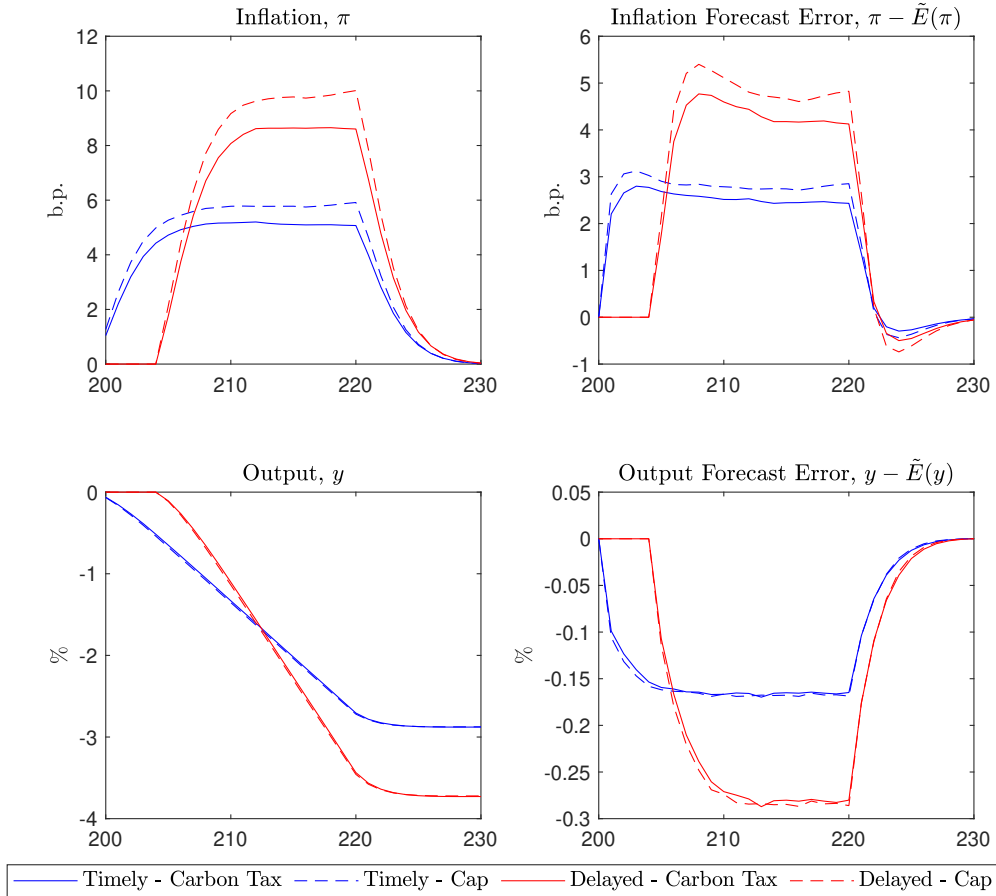
facilitate the interpretation of our results, we assume that ϑ is equal to 0.5, assigning equal weights to inflation and output stabilization objectives. By introducing more uncertainty about the time path of emission prices, a cap policy delivers more inflation volatility in the response of the transition and higher inflation than a tax policy.³⁵ In this respect, maintaining price stability looks more challenging under a quantity restriction than under a carbon tax. However, looking at our measure of welfare losses, a cap scheme seems to perform slightly better. This is due to the fact that a cap-and-trade has a built-in damping effect on short-run fluctuations and output volatility.³⁶ This property prevails because the volatility gap between the two policies is greater for output than for inflation, and, overall, the output gap volatility is higher than inflation volatility under both instruments. Clearly, this welfare result is partially driven by the assumption that the output gap and inflation volatilities have the same weight in the loss function. Different weights would potentially tilt the results: penalizing more inflation volatility would potentially make the tax more attractive in terms of welfare. In the third and fourth columns of the same table, we consider a delayed scenario in which the greening policy is introduced one year later. For comparability, we design this scenario so that after 20 quarters the amount of cumulative emission variation is as in the timely case.

Finally, to better appreciate the dynamics of the economy in the four scenarios of Table 5 in Figure 5 we show the inflation and the output dynamics, along with their market forecast errors. The forecast errors for both variables are substantially more significant in the delayed

³⁵For a similar result see [Santabábara and Suárez-Varela \(2022\)](#) who find that cap-and-trade schemes are associated with greater volatility in headline inflation, while there is no significant effect on inflation when carbon taxes are implemented. Note also that, in the context of business cycles, in a highly perturbed economy, choosing between price and quantity regulations would entail a major policy trade-off between emission certainty and price stability. See Appendix.

³⁶See [Fischer and Springborn \(2011\)](#) and [Annicchiarico and Di Dio \(2015\)](#).

Figure 5: Mitigation Scenarios - Macroeconomic Dynamics and Market Forecast Errors



Note: the figure plots the mean response of the economy to different mitigation scenarios entailing the same cumulative emissions after 20 quarters in the deterministic counterparts. Inflation and its forecast errors are expressed in quarterly basis points (b.p.) deviations, while output and its forecast errors are in percentage deviations.

scenario, whereas a cap is clearly more inflationary than a tax during the adjustment process. Agents tend to undermine inflation and overstate output more intensively under a cap than under a tax, and under a delayed scenario than under a timely mitigation process.³⁷

4.3 The Role of Monetary Policy

In this section, we explore the role of monetary policy in shaping the economy's response to climate policy. In particular, we address the following questions. Can monetary policy reduce the uncertainty regarding the economy's response during the transition? Can monetary policy affect the timing by which a specific mitigation objective is reached?

³⁷In Appendix we show that under rational expectations the forecast errors are driven only by the pollution policy that is phased in as a surprise policy shock. See Figure A-3.

Table 6: Macroeconomic Volatility over the Mitigation Path: Different Reactivity of Monetary Policy

	Tax					Cap				
ι_y	σ_z	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$	σ_{p_z}	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
0	0.1723	0.1107	3.2110	5.8804	6.7769	0.4633	0.0811	3.1604	6.4659	7.3665
0.125	0.1156	0.0742	2.1385	4.5789	5.2018	0.3208	0.0561	2.1737	5.2102	5.8658
0.5	0.0461	0.0295	0.8218	2.7099	2.9989	0.1351	0.0236	0.8849	3.2756	3.6063
1.5	0.0138	0.0087	0.2082	1.3201	1.4267	0.0404	0.0070	0.2215	1.6840	1.8141
3	0.0079	0.0050	0.1043	0.7673	0.8219	0.0220	0.0038	0.1020	1.0002	1.0679
ι_π	σ_z	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$	σ_{p_z}	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
1.1	0.1638	0.1053	3.0726	5.7439	6.6586	0.4906	0.0859	3.3831	6.7725	7.8080
1.5	0.1156	0.0742	2.1385	4.5789	5.2018	0.3208	0.0561	2.1737	5.2102	5.8658
3	0.0443	0.0283	0.7623	2.5624	2.7858	0.1072	0.0186	0.6531	2.7800	2.9888
4	0.0291	0.0185	0.4708	1.9845	2.1274	0.0690	0.0119	0.3846	2.1361	2.2661
5	0.0216	0.0137	0.3278	1.6250	1.7266	0.0514	0.0088	0.2632	1.7429	1.8339
ι_r	σ_z	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$	σ_{p_z}	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
0	0.1156	0.0742	2.1385	4.5789	5.2018	0.3208	0.0561	2.1737	5.2102	5.8658
0.5	0.2028	0.1302	3.6835	5.8150	6.6671	0.5944	0.1039	3.9093	6.5830	7.5058
0.7	0.3769	0.2423	6.9035	8.0499	10.0071	1.1723	0.2053	7.7489	9.1649	11.6168
0.9	1.7284	1.1139	33.1626	25.9815	36.4236	6.1450	1.0791	42.5458	31.6846	47.3159

Note: the table reports the standard deviations of the response for a selection of variables along with mean inflation and its maximum observed value over the mitigation time path; σ_{p_z} , σ_z , σ_{y-y^*} are expressed in percentages, σ_π , $E(\pi)$ and $max(\pi)$ in quarterly basis points.

To address these questions, we consider different values for the interest rate rule parameters, ι_y , ι_π and ι_r , and see how monetary policy interacts with the instrument chosen to achieve the mitigation goal.³⁸ The results are summarized in Table 6. A higher reactivity of the interest rate to the output gap or inflation strongly reduces the average dispersion of the response of the economy to the policy. This is true independently of the underlying environmental regime adopted. The intuition for this result is that a significant stabilization effort of central banks mitigates the intensity of waves of optimism and pessimism triggered by (wrong) market beliefs, thus reducing the uncertainty surrounding the mitigation policy. When reacting more to the output gap or inflation, monetary policy is more restrictive and induces the economy to converge quickly to its new long-run equilibrium. In this case, the fundamentalist rules, envisaging the return of the economy to its natural level, are validated by monetary policy. Under the central bank's more vigorous stabilization effort, there is no longer a trade-off between inflation control and climate policy: more price stabilization can be achieved without leading to more uncertainty

³⁸In the absence of monetary policy uncertainty (i.e, with no shocks on the interest rate rule), macroeconomic volatility over the green transition is reduced and inflationary pressures are strongly limited. See Appendix (Table A-1).

about meeting the climate target.

On the other hand, for an increasing ι_r , the Taylor rule becomes less reactive to current variations of the output gap and inflation. Thus, we observe that macroeconomic volatility goes up. An excessive degree of inertia delivers higher variability because monetary policy cannot stabilize the economy in response to current economic conditions. More importantly, it can be shown that for ι_r set to 0.9, the time needed to reach the mitigation objective ranges from 18 to 38 quarters, so with a potential mitigation delay of more than 4 years.

Overall, from these results, we observe that when the central bank assigns more weight either to inflation or to the output gap, it can align different objectives, namely stabilizing inflation around the inflation target, while facilitating the decarbonization process by avoiding unnecessary volatility and, in the case of the carbon tax, by shortening the time needed to reach a given mitigation target.³⁹ Put another way, conventional monetary policy can work alongside climate policy, reducing the uncertainty surrounding mitigation strategies and at the same time stabilizing both the output gap and inflation. Central banks can then support climate policies without overstressing their competencies. As explained in Section 2, this result stems from the fact that a more stringent climate action operates similarly to an adverse technological shock in this model, leading to a temporary increase in the output gap and to inflationary pressures. As a robustness check, in Appendix, we replace the technological shock with a cost-push shock to test if the absence of any monetary policy trade-off between output gap and inflation stabilization is robust with respect to the nature of the supply-side shock chosen as business cycle driver. We show that, even in the presence of a supply-side shock that shifts inflation directly through the New Keynesian Phillips Curve, leaves untouched the natural level of output, and moves output and inflation in different directions, we do not observe any trade-off for monetary policy along the mitigation path. The reason is that the climate policy shock, which occurs in every period as a surprise shock and whose size increases over time, is able to overcome the effects of shocks moving the output gap and inflation in opposite directions.⁴⁰

4.3.1 Alternative Interest Rate Rules

To further shed light on the role of monetary policy in the mitigation process, we show how our results may change under alternative implementable interest rate rules. In particular, we consider the following forms:

³⁹Note that for this exercise, to help the readability of the table, we do not report the results for welfare, which are straightforward. Since the central bank does not face a trade-off between stabilizing the output gap and stabilizing inflation during the transition, an increase both in ι_π and ι_y reduce the welfare loss. On the contrary, an increase in ι_r , by increasing the overall volatility, leads to an increase in welfare loss.

⁴⁰It is possible to show that, if we remove the climate policy and consider the economy in the business as usual scenario, the standard trade-off between output gap and inflation stabilization arises.

(i) backward-looking interest-rate rule:

$$r_t = \iota_\pi \pi_{t-1} + \iota_y (y_{t-1} - y_{t-1}^*) + u_t, \quad (23)$$

(ii) forward-looking interest-rate rule:

$$r_t = \iota_\pi E_t \pi_{t+1} + \iota_y E_t (y_{t+1} - y_{t+1}^*) + u_t, \quad (24)$$

(iii) market expectations-based interest-rate rule:

$$r_t = \iota_\pi \tilde{E}_t \pi_{t+1} + \iota_y \tilde{E}_t (y_{t+1} - y_{t+1}^*) + u_t, \quad (25)$$

(iv) interest rate rule reacting to output growth:

$$r_t = \iota_\pi \pi_t + \iota_y (y_t - y_{t-1}) + u_t. \quad (26)$$

Rules (i) and (ii) belong to the class of monetary-policy rules that are typically analyzed in the monetary policy literature and require no less information on the part of the central bank than the contemporaneous feedback rule based on the current values of inflation and the output gap.⁴¹ Rule (iii) is a simple implementable expectations-based rule. Here, the assumption is that policymakers can observe the average forecasts made by heterogeneous agents. The rationale of this rule is that monetary policy should react aggressively to market expectations.⁴² Finally, in the feedback rule (iv) the change in interest rate is set as a function of output growth rather than of output gap. This last specification implies that the central bank does not need to know the flexible-price level of aggregate activity. For comparability across monetary rules, we set the policy parameter values as in Table 2. The results are summarized in Table 7.

Clearly, inflation is more stabilized under the baseline interest rate rule and the forward-looking interest rule. Under both a backward monetary policy rule and a rule envisaging a reaction to output growth, the variability of all variables and average inflation tend to increase. In the former case, this can be explained by the fact that by changing the nominal interest in reaction to past events, the central bank is less able to limit the current exuberance of the markets. This result is consistent with those observed in Table 6 for positive values of the persistence parameter ι_r . In the latter case, the monetary rule becomes less stringent by reacting to output variations that during the greening path are negative. However, the worst-performing rule is that based on market expectations. When the interest rate changes in response to private sector expectations, the volatility of all variables is almost twice the one observed under the

⁴¹See e.g. Schmitt-Grohé and Uribe (2007).

⁴²This rule is used in several papers dealing with non-rational agents. See Evans and McGough (2005), Preston (2006) and Branch and McGough (2010) among others. The market expectation $\tilde{E}_t y_{t+1}^*$ is introduced in the model as done for inflation and output in Section 2.4, and depends on the expectation formulated by fundamentalists, $\tilde{E}_{f,t} y_{t+1}^* = E_t y_{t+1}^* = 0$, and on that formulated by extrapolators, $\tilde{E}_{e,t} y_{t+1}^* = y_{t-1}^*$.

Table 7: Macroeconomic Volatility over the Mitigation Path: Alternative Monetary Rules

Tax						
	σ_z	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$	L
$r_t = \iota_\pi \pi_t + \iota_y (y_t - y_t^*)$	0.1156	0.0742	2.1385	4.5789	5.2018	0.0478
$r_t = \iota_\pi \pi_{t-1} + \iota_y (y_{t-1} - y_{t-1}^*)$	0.1600	0.1024	2.7914	5.2023	5.7550	0.0652
$r_t = \iota_\pi \tilde{E}_t \pi_{t+1} + \iota_y \tilde{E}_t (y_{t+1} - y_{t+1}^*)$	0.1270	0.0816	2.4035	5.2965	6.1607	0.0528
$r_t = \iota_\pi \tilde{E}_t \pi_{t+1} + \iota_y \tilde{E}_t (y_{t+1} - y_{t+1}^*)$	0.3111	0.2000	5.8373	8.8525	10.3883	0.1292
$r_t = \iota_\pi \pi_t + \iota_y (y_t - y_{t-1})$	0.1825	0.1173	3.4444	6.6245	7.6843	0.0759
Cap						
	σ_{p_z}	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$	L
$r_t = \iota_\pi \pi_t + \iota_y (y_t - y_t^*)$	0.3208	0.0561	2.1737	5.2102	5.8658	0.0389
$r_t = \iota_\pi \pi_{t-1} + \iota_y (y_{t-1} - y_{t-1}^*)$	0.4743	0.0826	2.9506	5.9210	6.4754	0.0561
$r_t = \iota_\pi \tilde{E}_t \pi_{t+1} + \iota_y \tilde{E}_t (y_{t+1} - y_{t+1}^*)$	0.3518	0.0616	2.4732	6.1488	7.1084	0.0432
$r_t = \iota_\pi \tilde{E}_t \pi_{t+1} + \iota_y \tilde{E}_t (y_{t+1} - y_{t+1}^*)$	1.0267	0.1798	7.0638	10.9348	12.7561	0.1252
$r_t = \iota_\pi \pi_t + \iota_y (y_t - y_{t-1})$	0.4885	0.0856	3.3917	7.2867	8.3503	0.0597

Note: the table reports the standard deviations of the response for a selection of variables along with mean inflation and its maximum observed value over the mitigation time path; σ_{p_z} , σ_z , σ_{y-y^*} are expressed in percentages, σ_π , $E(\pi)$ and $max(\pi)$ in quarterly basis points. For the computation of the welfare loss both standard deviations are expressed in percentages.

contemporaneous baseline rule. This is because monetary policy, instead of limiting divergent behavioral dynamics around the mitigation path, somehow validates the ‘wrong’ expectations that partially ignore the ongoing structural change. This is why an expectations-based rule is potentially destabilizing.

Looking at the welfare results, the first thing to notice is that, independently of the Taylor rule adopted, a cap-and-trade scheme always performs slightly better than a carbon tax, since it leads to lower output gap volatility, which, in this model, prevails on inflation volatility under both policies, both in absolute terms and in terms of volatility gap between the two instruments. In terms of comparison across different Taylor rules, we can see that the welfare ranking directly follows from the volatility ranking with contemporaneous and forward-looking interest rules entailing the lowest welfare losses. The Taylor rule linked to expectations is the worst in terms of welfare, since, as explained above, in this case the economy pays the cost of a much higher volatility environment due to a monetary policy not anchored to fundamentals.

Finally, looking at emission reductions, it can be shown that the uncertainty regarding the time horizon by which the mitigation target is reached changes slightly only in the case of an expectations-based rule, with a time frame ranging from 21 to 30 quarters. Under all the other rules, this time frame stays almost unchanged.

5 Sensitivity Analysis and Extensions

In this section, we carry out a series of checks to assess the robustness of the previous results against changes in the values of the behavioral parameters that might be surrounded by uncertainty and might be particularly relevant in shaping the economy's response to a gradual decarbonization process. We also propose several other extensions of our analysis, allowing for asynchronous updating of the forecasting rules, other expectations rules, and skepticism about the ability of the central bank to keep inflation at its target. We conclude this section by showing the potential impact of the green transition on public debt dynamics.

5.1 Memory and Willingness to Learn

In this section, we look at the role played by the parameters θ , which measures the willingness to learn and ρ , which measures agents' memory. The results are shown in Table 8.

For small values of willingness to learn, agents are less sensitive to the performance of their forecasting rule and tend to decide more randomly. As a result, the initial state of the economy is less relevant for the dynamic adjustments of output, emissions, and inflation, and the uncertainty surrounding the greening path is lower.

Instead, for large values of θ , agents learn from their past mistakes and revise how they formulate expectations based on past performances. We observe a more significant variability of the main macroeconomic variables during the greening plan and a more substantial inflationary pressure, especially under a cap policy. Suppose that the mitigation process starts when the economy is in an expansionary phase. In that case, more agents expect income to stay high in the future based on the extrapolative rule. A higher expected income drives current demand upward, validating initial expectations. In this case, emissions will converge to the new equilibrium following a higher trajectory. On the other hand, if the greening action is taken when the economy is in a recession, the same mechanism will work in the opposite direction. The initial lower level of income implies that more agents expect a lower level of income also for the following period, reducing aggregate demand and thus the current output. Again expectations are self-validating and the economy will converge toward the new long-run equilibrium on a lower trajectory.

When ρ is low, agents learn less from mistakes made in the past and attach a greater weight to the performance of the last period in evaluating a forecast rule. As a result, the economy is more sensitive to the current state of the economy, and the business cycle has greater influence on the mitigation process. On the other hand, for values of ρ closer to 1, agents have more memory and attach a high weight to past mistakes. For this reason, they react relatively less to the forecast error of the last period, and there is less uncertainty surrounding the greening process. However, it can be shown that the emissions slowly converge to their target. This is because agents do not react promptly to the new economic conditions following the increase

Table 8: Macroeconomic Volatility over the Mitigation Path: The Role of Willingness to Learn and Memory

	Tax					Cap				
θ	σ_z	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$	σ_{p_z}	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
500	0.0123	0.0079	0.2282	3.1489	3.4506	0.0342	0.0060	0.2320	3.7563	4.0934
1000	0.0247	0.0158	0.4564	3.3197	3.6598	0.0685	0.0120	0.4639	3.9300	4.3052
2000	0.0494	0.0317	0.9128	3.6614	4.0782	0.1369	0.0239	0.9278	4.2773	4.7286
3000	0.0740	0.0475	1.3692	4.0030	4.4966	0.2054	0.0359	1.3917	4.6247	5.1521
5000	0.1234	0.0792	2.2821	4.6863	5.3334	0.3423	0.0599	2.3196	5.3194	5.9990
ρ	σ_z	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$	σ_{p_z}	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
0	0.1304	0.0835	2.3365	4.6558	5.2626	0.3631	0.0633	2.3732	5.2885	5.9267
0.2	0.1246	0.0799	2.2660	4.6331	5.2497	0.3463	0.0605	2.3025	5.2657	5.9145
0.5	0.1102	0.0708	2.0535	4.5391	5.1611	0.3054	0.0534	2.0874	5.1692	5.8240
0.8	0.0763	0.0491	1.4640	4.2482	4.8106	0.2106	0.0369	1.4878	4.8691	5.4645
0.9	0.0525	0.0338	1.0195	4.0356	4.5285	0.1448	0.0254	1.0351	4.6504	5.1763

Note: the table reports the standard deviations of the response for a selection of variables along with mean inflation and its maximum observed value over the mitigation time path; σ_{p_z} , σ_z , σ_{y-y^*} are expressed in percentages, σ_π , $E(\pi)$ and $max(\pi)$ in quarterly basis points.

in carbon pricing. Since the most recent forecast errors and events play a relatively marginal role in driving the choice between heuristics, agents do not immediately adjust their predictive rule, and the economy will reach the new steady state with some delay.

5.2 Asynchronous Updating

We now introduce the possibility of asynchronous updating by changing how the fraction of agents opting for a specific rule evolves over time.⁴³ In particular, we now replace equation (18) with the following:

$$\alpha_{x,t}^i = \delta_i \alpha_{x,t-1}^i + (1 - \delta_i) \frac{e^{\theta U_{x,t}^i}}{\sum_i e^{\theta U_{x,t}^i}}, \quad 0 \leq \delta_i \leq 1 \quad (27)$$

where the asynchronous updating parameter δ_i captures inertia in the choice of the heuristics. In the extreme case of $\delta_i = 0$ there is synchronous updating and the economy evolves as in our baseline model, where agents stand ready to opt for the best-performing rule, given their state of mind. At the other extreme, for $\delta_i = 1$, agents never update their forecasting rule independently of their performance, that is like saying that agents are stubborn. Table 9 shows the economy's volatility for different values of δ_i . We observe that when agents are more reluctant to switch from one rule to another based on their forecast errors, the economy is less

⁴³This is along the lines of [Diks and Van Der Weide \(2005\)](#), [Hommes et al. \(2005a\)](#) and [Hommes et al. \(2019\)](#) and is consistent with the evidence provided by [Hommes et al. \(2005b\)](#).

Table 9: Macroeconomic Volatility over the Mitigation Path: Asynchronous Updating of Expectations

δ_i	Tax					Cap				
	σ_z	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$	σ_{p_z}	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
0	0.1156	0.0742	2.1385	4.5789	5.2018	0.3190	0.0558	2.1695	5.1945	5.8608
0.2	0.1115	0.0717	2.0786	4.5484	5.1748	0.3078	0.0539	2.1123	5.1638	5.8281
0.4	0.1049	0.0675	1.9735	4.4926	5.1176	0.2900	0.0508	2.0119	5.1067	5.7642
0.6	0.0933	0.0600	1.7765	4.3884	4.9973	0.2591	0.0454	1.8211	4.9989	5.6405
0.8	0.0706	0.0455	1.3646	4.1788	4.7285	0.1973	0.0346	1.4085	4.7810	5.3780
1	0.0121	0.0078	0.2399	3.6158	3.9809	0.0334	0.0059	0.2425	4.2122	4.6147

Note: the table reports the standard deviations of the response for a selection of variables along with mean inflation and its maximum observed value over the mitigation time path; σ_{p_z} , σ_z , σ_{y-y^*} are expressed in percentages, σ_π , $E(\pi)$ and $max(\pi)$ in quarterly basis points.

volatile, and the task of maintaining stable inflation becomes less challenging.

5.3 Credibility Issues and Other Forecasting Rules

In this section, we conduct several experiments addressing credibility issues related to the inflation targeting policy. We also take into account the possibility that extrapolators form less naive expectations about future output, and that fundamentalists are less sophisticated. Additionally, we consider the scenario that we label ‘consistent expectations’, where agents choose the same forecasting rule for both variables.

5.3.1 Credibility Issues on Inflation Targeting and Other Expectations Hypothesis on Inflation

Our analysis has been conducted using elementary forecasting rules for both variables and assuming that only a fraction of agents perceive the central bank’s commitment to maintaining price stability as entirely credible. However, during a greening transition process that is expected to be inflationary, it makes sense to assume that all agents may cast doubt about the credibility of the inflation-targeting regime, reducing the effectiveness of forward guidance. This may be particularly relevant under a delayed scenario.

To address the issue of credibility of the inflation targeting policy, we start by considering two extreme cases. One is to assume that there is 100% skepticism. In this case, all agents are extrapolators when they formulate expectations about inflation. The market inflation expectation is then $\tilde{E}_t(\pi_{t+1}) = \pi_{t-1}$. The other extreme case, i.e., full credibility of the inflation target, assumes that all agents are fundamentalists when forecasting inflation. The market inflation expectation is then $\tilde{E}_t(\pi_{t+1}) = 0$. See Table 10.

Table 10: Macroeconomic Volatility over the Mitigation Path: Credibility Issues, Trend Following Rules, and Adaptive Expectations for Inflation

Tax					
	σ_z	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
Baseline	0.1156	0.0742	2.1385	4.5789	5.2018
Inflation targeting skepticism $\tilde{E}_t(\pi_{t+1}) = \pi_{t-1}$	0.0838	0.0537	4.0277	15.3681	23.3157
Inflation targeting credibility $\tilde{E}_t(\pi_{t+1}) = 0$	0.1377	0.0885	1.4088	2.6535	2.9119
Strong-trend following rule $\tilde{E}_t(\pi_{t+1}) = \pi_{t-1} + 0.9(\pi_{t-1} - \pi_{t-2})$	0.1153	0.0739	11.6851	27.3431	48.9592
Weak-trend following rule $\tilde{E}_t(\pi_{t+1}) = \pi_{t-1} + 0.4(\pi_{t-1} - \pi_{t-2})$	0.0718	0.0460	4.4544	18.4948	24.9987
Adaptive expectations with strong adjustment $\tilde{E}_t(\pi_{t+1}) = \tilde{E}_{t-1}(\pi_t) + 0.9(\pi_t - \tilde{E}_{t-1}(\pi_t))$	0.0181	0.0114	2.4911	19.1216	20.9487
Adaptive expectations with weak adjustment $\tilde{E}_t(\pi_{t+1}) = \tilde{E}_{t-1}(\pi_t) + 0.4(\pi_t - \tilde{E}_{t-1}(\pi_t))$	0.0839	0.0538	3.4157	12.7309	19.7501
Cap					
	σ_{p_z}	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
Baseline	0.3208	0.0561	2.1737	5.2102	5.8658
Inflation targeting skepticism $\tilde{E}_t(\pi_{t+1}) = \pi_{t-1}$	0.2253	0.0393	3.5313	16.3124	23.6720
Inflation targeting credibility $\tilde{E}_t(\pi_{t+1}) = 0$	0.3977	0.0696	1.4222	3.0298	3.2967
Strong-trend following rule $\tilde{E}_t(\pi_{t+1}) = \pi_{t-1} + 0.9(\pi_{t-1} - \pi_{t-2})$	0.3345	0.0584	9.9829	25.9986	47.2227
Weak-trend following rule $\tilde{E}_t(\pi_{t+1}) = \pi_{t-1} + 0.4(\pi_{t-1} - \pi_{t-2})$	0.1916	0.0334	3.7535	19.1302	24.7542
Adaptive expectations with strong adjustment $\tilde{E}_t(\pi_{t+1}) = \tilde{E}_{t-1}(\pi_t) + 0.9(\pi_t - \tilde{E}_{t-1}(\pi_t))$	0.0454	0.0077	1.9934	19.6702	21.2848
Adaptive expectations with weak adjustment $\tilde{E}_t(\pi_{t+1}) = \tilde{E}_{t-1}(\pi_t) + 0.4(\pi_t - \tilde{E}_{t-1}(\pi_t))$	0.2233	0.0390	3.0402	13.7026	20.5551

Note: the table reports the standard deviations of the response for a selection of variables along with mean inflation and its maximum observed value over the mitigation time path; σ_{p_z} , σ_z , σ_{y-y^*} are expressed in percentages, σ_π , $E(\pi)$ and $max(\pi)$ in quarterly basis points.

We also allow for a more complex heuristic considering a trend-adjusted rule for inflation so that the private sector inflation expectation is now $\tilde{E}_t(\pi_{t+1}) = \pi_{t-1} + g_\pi(\pi_{t-1} - \pi_{t-2})$ where $g_\pi > 0$ measures the responsiveness of the expected inflation to the last observed inflation variation. Similarly to [Hommes et al. \(2019\)](#), we assign two possible values to g_π , namely 0.4 and 0.9, so distinguishing between strong- and weak-trending following rules.⁴⁴ Table 10 shows the results. As expected, when all agents maintain skepticism about the credibility of the inflation-targeting policy or adopt trend-following rules to forecast inflation, keeping price stability becomes harder. For a strong-trend following rule, the task becomes even more arduous. In revising their prices, agents expecting persistent deviations of inflation from its target would set too high or too low prices, thus validating their ‘wrong’ expectations and destabilizing the real side of the economy.

Another possibility to be considered is that all agents formulate expectations about future inflation according to an adaptive expectation rule. This rule incorporates past expectations and an error-adjustment term, where current expectations are adjusted based on the gap between actual inflation and previous expectations. Formally, we now have $\tilde{E}_t(\pi_{t+1}) = \tilde{E}_{t-1}(\pi_t) + \lambda_\pi(\pi_t - \tilde{E}_{t-1}(\pi_t))$, where λ_π measures the strength of the error-adjustment term. We consider two values for it, reflecting a milder ($\lambda_\pi = 0.4$) and a stronger ($\lambda_\pi = 0.9$) adjustment to past forecast errors. In this case also, we observe that inflation becomes much more volatile, and more importantly, the level of mean inflation is higher.

Another striking result that emerges from Table 10 is related to the fact that as we transition from a situation of high credibility in inflation targeting to different degrees and forms of skepticism, the volatility of the output gap decreases. In the absence of fundamentalists for inflation, all output extrapolators believe that inflation would be less constrained during the green transition, allowing output to converge quickly towards its natural level. In this sense, there is a trade-off between stabilizing the output gap and controlling inflation, which is linked to the credibility of inflation targeting. The more credible the inflation target, the wider the dispersion of the output gap during the mitigation process.

5.3.2 More Sophisticated Extrapolative Rules for Output

In the baseline case, the output extrapolative rule is very simple. We now consider slightly more sophisticated rules for output extrapolators. In particular, one can reasonably assume that agents may revise their way of formulating expectations about output based on a trend-following rule, or according to adaptive expectations. Again, during a structural change, it makes sense to assume agents may also account for the information provided by the last observed variation in their forecast variable in developing their expectations or they may account for the forecast error made in the previous period to revise their predictions.

⁴⁴Contrary to [Hommes et al. \(2019\)](#), our model, in the present calibration, does not allow values of g_π larger than one, since the equilibrium becomes unstable.

In Table 11 we account for the implications of having a trend-following rule for output by assuming that extrapolators, instead of simply using a random walk rule to predict the next period value for output, formulate their expectations according to a trend-adjusted rule of the form: $\tilde{E}_{e,t}(y_{t+1}) = y_{t-1} + g_y(y_{t-1} - y_{t-2})$, with $g_y > 0$. Fundamentalists act as in the baseline case since, by expecting future output to be equal to its natural counterpart, they already factor in the effects of the ongoing structural change. The last two lines of Table 10 report our findings for the case of weak- and strong-trend following rules for output. When extrapolators react vigorously to the output trend, the inflation rate is strongly stabilized under both regulatory regimes. Under a tax, we note that emissions are much less volatile than in all other cases. Under this rule, extrapolators adjust their expectations for the negative trend, and the economy moves more smoothly towards its new long-run equilibrium.

Finally, for completeness, we also consider the scenario in which the output extrapolators adopt an adaptive expectation rule: $\tilde{E}_{e,t}(y_{t+1}) = \tilde{E}_{e,t-1}(y_t) + \lambda_y(y_t - \tilde{E}_{e,t-1}(y_t))$. When employing an adaptive rule with a strong adjustment to past forecast errors, the economy exhibits reduced volatility. However, with a smaller degree of adjustment, both volatility and mean inflation increase. This outcome is a result of the decline in output levels during the transition. To account for this downward trend, output extrapolators downwardly adjust their expectations for output, aligning them more closely with those of the fundamentalists, particularly when λ_y is higher. On the contrary, with a lower degree of adjustment to past errors, output extrapolators correct their mistakes less, leading to a higher average forecast error and a more volatile economy.

5.3.3 Less Sophisticated Rules for Output and Other Hypotheses on Expectations

We now consider less sophisticated forecast rules for output fundamentalists. In the baseline case, the fundamentalist rule for output consists of a rational expectation rule applied to the wrong variable. However, it is important to note that implementing such a rule requires a certain level of effort from agents, who may be subject to cognitive limitations in computing the future natural level of output.

We begin by examining trend-following rules, similar to those analyzed in the previous section. In these rules, the expected level of output is equal to the current level of natural output adjusted for its trend: $\tilde{E}_{f,t}(y_{t+1}) = y_t^* + g_y(y_t^* - y_{t-1}^*)$, with $g_y > 0$. This is shown in Table 12.

We also consider a rule in which the prediction of output is expected to be the current level of natural output plus an error correction term: $\tilde{E}_{f,t}(y_{t+1}) = y_t^* + e_y(y_t - y_t^*)$, with $e_y > 0$. Under this rule, the output gap is more stabilized around the transition. It is worth noting that the climate policy is introduced as a surprise shock in each period, so if output fundamentalists adjust for the error term, they are doing a better job in aligning output to its natural path compared to the benchmark case. However, as a side effect, inflation becomes more unstable.

Table 11: Macroeconomic Volatility over the Mitigation Path: More Sophisticated Extrapolative Rules for Output

Tax					
	σ_z	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
Baseline	0.1156	0.0742	2.1385	4.5789	5.2018
Strong-trend following $\tilde{E}_{e,t}(y_{t+1}) = y_{t-1} + 0.9(y_{t-1} - y_{t-2})$	0.0544	0.0348	0.9528	1.7082	2.2460
Weak trend following $\tilde{E}_{e,t}(y_{t+1}) = y_{t-1} + 0.4(y_{t-1} - y_{t-2})$	0.0904	0.0580	1.6502	3.4198	3.7424
Adaptive expectations with strong adjustment $\tilde{E}_{e,t}(y_{t+1}) = \tilde{E}_{e,t-1}(y_t) + 0.9(y_t - \tilde{E}_{e,t-1}(y_t))$	0.0387	0.0248	0.7092	1.5658	1.7164
Adaptive expectations with weak adjustment $\tilde{E}_{e,t}(y_{t+1}) = \tilde{E}_{e,t-1}(y_t) + 0.4(y_t - \tilde{E}_{e,t-1}(y_t))$	0.1292	0.0830	2.4069	5.5741	6.7664
Cap					
	σ_{p_z}	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
Baseline	0.3208	0.0561	2.1737	5.2102	5.8658
Strong-trend following $\tilde{E}_{e,t}(y_{t+1}) = y_{t-1} + 0.9(y_{t-1} - y_{t-2})$	0.1481	0.0259	0.9456	1.9407	2.5722
Weak-trend following $\tilde{E}_{e,t}(y_{t+1}) = y_{t-1} + 0.4(y_{t-1} - y_{t-2})$	0.2495	0.0436	1.6671	3.8739	4.2078
Adaptive expectations with strong adjustment $\tilde{E}_{e,t}(y_{t+1}) = \tilde{E}_{e,t-1}(y_t) + 0.9(y_t - \tilde{E}_{e,t-1}(y_t))$	0.1055	0.0185	0.7121	1.7856	1.9463
Adaptive expectations with weak adjustment $\tilde{E}_{e,t}(y_{t+1}) = \tilde{E}_{e,t-1}(y_t) + 0.4(y_t - \tilde{E}_{e,t-1}(y_t))$	0.3600	0.0630	2.4658	6.3943	7.6837

Note: the table reports the standard deviations of the response for a selection of variables along with mean inflation and its maximum observed value over the mitigation time path; σ_{p_z} , σ_z , σ_{y-y^*} are expressed in percentages, σ_π , $E(\pi)$ and $max(\pi)$ in quarterly basis points.

Similarly, under adaptive expectations, the output gap is more stabilized, while inflation is more volatile and, on average, much higher than in the baseline case.

As a final exercise, we return to the original assumption regarding the output fundamentalist rule but with two variations in turn. The first variation considers the existence of a ‘cognitive cost’ associated with obtaining the rational expectation prediction of the natural output. To introduce this cost, denoted as κ , we follow [Jump and Levine \(2019\)](#) and modify Equation 16 for output fundamentalists as follows: $U_{y,t}^f = \rho U_{y,t-1}^i - (1 - \rho) [(y_{t-1} - \tilde{E}_{f,t-2} y_{t-1})^2 + \kappa]$. In Table 12, we present the results for a value of $\kappa = 0.0001$, such that in the deterministic steady state, the share of output fundamentalists is around 0.38 (instead of 0.5 as in the baseline case). As expected, in this scenario, the economy is less stable around the transition path. Since formulating fundamentalist expectations of output incurs a cost, more agents will opt for the extrapolative rule, even though it may be less precise.

Finally, we consider the case of consistent expectations, in which agents formulate expectations for both inflation and output according to either an extrapolative rule or a fundamentalist rule. In this case, the relevant scores that drive agents’ decisions are simply averages of the individual scores for each variable based on forecast errors. As expected, compared to the baseline case, where agents are allowed to have different predictive rules, the economy exhibits lower volatility.

5.4 Public Debt Dynamics Over the Green Transition

So far, we have disregarded any concerns about public debt dynamics during the mitigation process. In all previous experiments, the fiscal effects of the more stringent climate policy were neutralized by assuming that the fiscal authorities adhered to a budget-balanced rule at all times. We now modify our previous simplistic hypothesis and demonstrate how the dynamics of the public debt may evolve as a result of a mitigation policy under different assumptions regarding the ability to implement corrective measures that stabilize debt, the timing of climate policy, and the conduct of monetary policy.

To this end consider the following equation describing the evolution of public debt in nominal terms:

$$D_t = R_{t-1}D_{t-1} - P_t T_t + P_t G_t - P_{Z,t} Z_t + P_t T r_t, \quad (28)$$

where D_t represents the level of public debt at the end of period t , T_t denotes tax revenues, G_t is public consumption, and $T r_t$ are transfers to households. To account for the procyclical nature of tax revenues, we assume that T_t increases with Y_t . Additionally, we assume that taxes respond to the level of public debt in order to ensure a non-explosive path for public debt. Consequently, tax revenues are assumed to follow the following law of motion: $T_t = T_0 + \tau_Y Y_t + \tau_D (D_{t-1}/P_{t-1})$, where T_0 is a constant term, and $\tau_Y, \tau_D > 0$. A higher value of τ_D implies stronger corrective action by fiscal authorities in response to an increase in the debt level.

Table 12: Macroeconomic Volatility over the Mitigation Path: Less Sophisticated Fundamental Rules for Output and Consistency in Prediction Behavior

Tax					
	σ_z	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
Baseline	0.1156	0.0742	2.1385	4.5789	5.2018
Strong-trend following $\tilde{E}_{f,t}(y_{t+1}) = y_t^* + 0.9(y_t^* - y_{t-1}^*)$	0.1799	0.1153	3.0707	3.6271	4.1659
Weak trend following $\tilde{E}_{f,t}(y_{t+1}) = y_t^* + 0.4(y_t^* - y_{t-1}^*)$	0.1382	0.0887	2.4687	4.1755	4.7674
Strong error correction $\tilde{E}_{f,t}(y_{t+1}) = y_t^* + 0.9(y_t - y_t^*)$	0.0543	0.0348	1.4957	6.5695	7.4600
Weak error correction $\tilde{E}_{f,t}(y_{t+1}) = y_t^* + 0.4(y_t - y_t^*)$	0.0731	0.0469	1.6050	5.0467	5.7250
Adaptive expectations with strong adjustment $\tilde{E}_{f,t}(y_{t+1}) = \tilde{E}_{f,t-1}(y_t) + 0.9(y_t - \tilde{E}_{f,t-1}(y_t))$	0.0594	0.0382	1.9469	8.3612	9.6814
Adaptive expectations with weak adjustment $\tilde{E}_{f,t}(y_{t+1}) = \tilde{E}_{f,t-1}(y_t) + 0.4(y_t - \tilde{E}_{f,t-1}(y_t))$	0.0374	0.0241	2.6275	14.8100	19.7357
Cognitive costs	0.1902	0.1223	3.6092	7.0019	8.1402
Consistent forecasting rules	0.0553	0.0355	2.0883	4.5008	5.1027
Cap					
	σ_{p_z}	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
Baseline	0.3208	0.0561	2.1737	5.2102	5.8658
Strong-trend following $\tilde{E}_{f,t}(y_{t+1}) = y_t^* + 0.9(y_t^* - y_{t-1}^*)$	0.5126	0.0895	3.0847	3.8385	4.3838
Weak trend following $\tilde{E}_{f,t}(y_{t+1}) = y_t^* + 0.4(y_t^* - y_{t-1}^*)$	0.3879	0.0678	2.4942	4.6286	5.2413
Strong error correction $\tilde{E}_{f,t}(y_{t+1}) = y_t^* + 0.9(y_t - y_t^*)$	0.1683	0.0294	1.8069	7.5965	8.5530
Weak error correction $\tilde{E}_{f,t}(y_{t+1}) = y_t^* + 0.4(y_t - y_t^*)$	0.1884	0.0329	1.6682	5.7895	6.5061
Adaptive expectations with strong adjustment $\tilde{E}_{f,t}(y_{t+1}) = \tilde{E}_{f,t-1}(y_t) + 0.9(y_t - \tilde{E}_{f,t-1}(y_t))$	0.1974	0.0346	2.3715	9.4454	10.7991
Adaptive expectations with weak adjustment $\tilde{E}_{f,t}(y_{t+1}) = \tilde{E}_{f,t-1}(y_t) + 0.4(y_t - \tilde{E}_{f,t-1}(y_t))$	0.1617	0.0284	3.4146	16.8277	21.7243
Cognitive costs	0.5044	0.0884	3.5321	7.6608	8.7948
Consistent forecasting rules	0.1555	0.0272	2.1508	5.1483	5.7865

Note: the table reports the standard deviations of the response for a selection of variables along with mean inflation and its maximum observed value over the mitigation time path; σ_{p_z} , σ_z , σ_{y-y^*} are expressed in percentages, σ_π , $E(\pi)$ and $max(\pi)$ in quarterly basis points.

For simplicity, we assume that both transfers and public consumption remain constant over time. By dividing all terms in (28) by $P_t Y_t$ and log-linearizing around a zero-inflation steady state, we can express it in terms of nominal income as follows:

$$d_{Y,t} = (R - \tau_D) d_{Y,t-1} + r_{t-1} - \pi_t - \tau_Y y_t - \frac{p_Z Z}{Y} (p_{Z,t} + z_t), \quad (29)$$

where $d_{Y,t}$ is the deviation of the debt ratio, $D_t/(P_t Y_t)$, from its steady state level. All the other variables are expressed as described in the rest of the paper. In order for $d_{Y,t}$ to be stable it must be $R - \tau_D < 1$, that is we need a sufficiently corrective measure to prevent an explosive debt ratio.

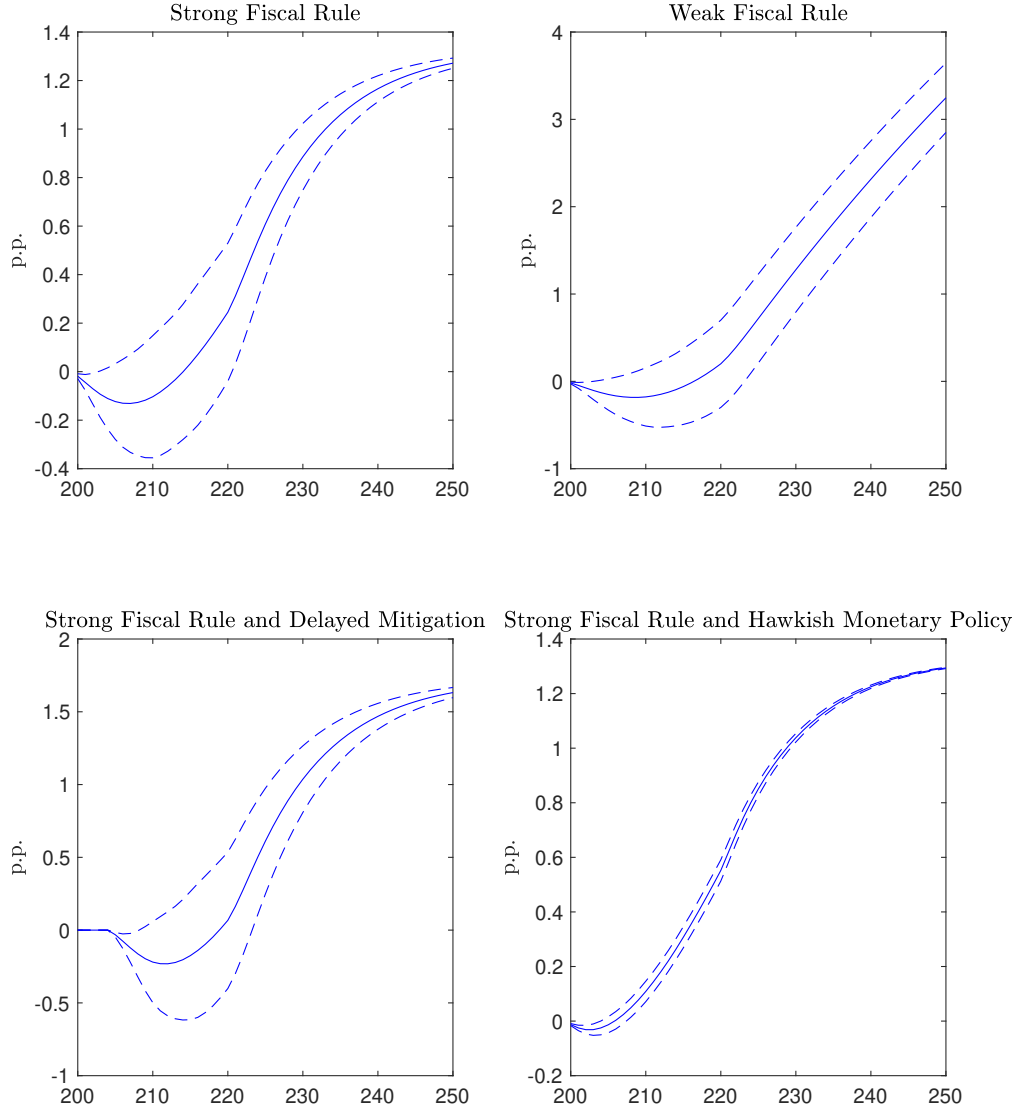
From equation (29), it is evident that a more stringent climate action may impact the debt dynamics through different channels. The first channel pertains to the revenues derived from the environmental policy itself. It involves both positive effects, resulting from the increase in the price of carbon, and negative effects, stemming from the reduced level of emissions. Another channel is related to the effects that climate actions have on income, and therefore on tax revenues. In our model, since an increase in the price of carbon is recessionary, climate actions may significantly jeopardize the stability of public debt. Finally, another channel is associated with the conduct of monetary authorities. A more stringent monetary policy in response to climate action tends to raise borrowing costs, making fiscal stability more challenging. On the other hand, as shown in the previous section, a tighter monetary policy tends to stabilize the economy around the mitigation path, reducing the uncertainty generated by climate action.

In Figure 6 we show the mean response of the debt ratio throughout the transition for the carbon tax policy case. We consider four different scenarios. The first scenario, ‘strong fiscal rule’, involves a high value of τ_D , indicating a vigorous reaction from fiscal authorities in response to an increase in the debt ratio. In the second scenario, ‘weak fiscal rule’, τ_D is low but still such that the debt ratio does not explode. In the third scenario, we show what happens when the implementation of the climate action is delayed by one year, as assumed in Section 4. As a final scenario, we consider the effects of having a hawkish monetary policy throughout the transition, that is, a monetary policy targeting only inflation and with a high ι_π set to 5. The results would suggest that ambitious climate actions can pose challenges to debt stability if sufficiently corrective actions to stabilize debt are not implemented and if there are delays in the mitigation process. On the other hand, a more stringent monetary policy if coupled with a stabilizing fiscal rule, reduces the uncertainty surrounding the response of the debt ratio to the climate policy.

6 Conclusions and Directions for Future Research

There is an ongoing debate among economists and policy analysts about the implications of climate change for monetary policy, and many central banks have already included climate

Figure 6: Public Debt Ratio Over the Mitigation Path



Note: the figure plots the mean response of the public debt ratio to a gradual increase in the carbon price aimed at permanently reducing emissions by 20%. Dashed lines show ± 2 standard deviations from the mean. The public debt ratio is expressed as percentage points (p.p.) deviations from its steady-state level. The results have been obtained under the calibration used for the baseline model; in addition $\tau_Y = 0.3$, $p_Z Z = 0.02$, while τ_D is set to 0.1 (0.02) when the fiscal rule is strong (weak); for the hawkish monetary policy case ι_π is set to 5, while all the other parameters of the interest rate rule are set to zero.

change considerations in their assessments of potential economic and financial risks. This paper shows the relevance of market expectations and business cycle fluctuations on the interaction between monetary and climate policy by focusing on two aspects of this debate. The first aspect refers to the potential implications of different mitigation instruments for the ability of central banks to conduct monetary policy successfully and keep inflation under control. The second aspect concerns the role that central banks themselves can play in supporting the transition process and reducing the macroeconomic uncertainty inherent in the policy tool selected to fight climate change.

The presence of behavioral agents with cognitive limitations amplifies business cycle fluctuations and allows the emergence of waves of optimism and pessimism over the mitigation path, injecting further uncertainty regarding the impact and effectiveness of climate policies. In this context, a green transition is found to pose a more significant threat to the ability of central banks to maintain price stability than in the case of an economy with rational agents. Moreover, the trade-offs between cap-and-trade and carbon tax policies are accentuated, with the two instruments delivering different dynamic adjustments. On the one hand, for price regulation, the time needed to achieve an emission reduction target can be longer than in standard rational expectation models, especially in a highly perturbed economy. On the other hand, while a cap-and-trade scheme allows us to control future emissions levels, it implies significant uncertainty on allowance prices, production costs, and inflation dynamics, posing a major threat to price stability.

Looking at the role of central banks, we find that, under price regulation, a monetary policy more reactive to the output gap or inflation can help stabilize emissions, thus reducing the degree of uncertainty regarding the achievement of climate targets. Under both environmental regimes, a more vigorous response to current fluctuations in macroeconomic variables can help moderate inflation volatility and reduce the pressure on prices due to the more stringent climate policy. Central banks seem then to be able to tame market sentiments and support, in some respect, the green transition.

Delays in the implementation of stringent climate policies, the lack of confidence in the ability of central banks to maintain price stability during the green transition, and the adoption of monetary rules reacting to market expectations rather than to current macroeconomic variables are all factors that can magnify the uncertainty over the mitigation path and worsen the impact on price stability.

The main policy message arising from this paper is that, regardless of adopting new instruments targeted to support the low-carbon transition, central banks can contribute to fighting climate change by primarily acting within the perimeter of their mandate. By implementing successful stabilization policies, central banks can reduce the uncertainty surrounding carbon pricing policies, ensuring better conditions for successful climate actions. On the other hand, a word of caution is needed here, since our results have been conducted in a model that neglects

several important aspects and relies on very specific assumptions.

In this paper, the implications for debt dynamics related to mitigation policies have only been briefly touched upon. In future work, these aspects should be further explored by studying the direct and indirect feedback effects among policy areas.

Future extensions of this work should also test whether the results hold in a model allowing for the possibility of substitution away from fossil fuels. This would be an important channel to explore in the study of the macroeconomic effects of the green transition. The presence of green energy sources would give firms, and potentially households, the possibility of replacing polluting inputs that would become more expensive due to climate policy, with clean sources that will be relatively cheaper. Previous works based on rational expectations models suggest that we would still see a negative effect on aggregate demand and a decline in output, but milder compared to the one we observe in the absence of input substitution. We can expect output decreasing even less in a behavioral model due to the combined effect of re-allocation and backward looking expectations. In addition, under a cap-and-trade system, the price of permits would be lower, putting less pressure on inflation. In the case of a carbon tax, there will be two forces pushing emissions in opposite directions. From one side, the higher output would push emissions up, but the possibility of producing in a clean way would reduce the emission intensity. The balance between these two effects would crucially depend, among others, on the elasticity of substitution between inputs and the degree of price rigidities in different sectors that can limit the speed of adjustment in relative prices. In a behavioral model the higher demand could speed up the re-allocation toward clean sources, potentially creating a virtuous cycle that would accelerate the achievement of climate targets.

Another promising line of future research could be that of relating both monetary policy targets to climate policies and possibly to climate risks, and accounting for the intrinsic uncertainty related to climate-related risks in the solution of the model.

Finally, an interesting aspect to consider would be the strategic interactions between monetary and climate policymakers and how credibility around different policy interventions can shape the expectations along the transition toward a green economy.

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Appendix

Households

In the period t the typical household i chooses $C_{i,t}$, $B_{i,t}$ and $N_{i,t}$, to maximize (1), subject to (2). At the optimum, the following conditions must hold:

$$1 + \varrho \frac{B_{i,t}}{P_t} (C_{i,t} - hC_{t-1})^\gamma = \beta R_t \tilde{E}_{i,t} \left\{ [\exp(\mu_{t+1} - \mu_t)] \left(\frac{C_{i,t} - hC_{t-1}}{C_{i,t+1} - hC_t} \right)^\gamma \frac{1}{\Pi_{t+1}} \right\}, \quad (\text{A-1})$$

$$\chi N_{i,t}^\varphi = \frac{W_t}{P_t} (C_{i,t} - hC_{t-1})^{-\gamma}, \quad (\text{A-2})$$

where equation (A-1) describes the time path of consumption of a household of type i , while equation (A-2) is the labor supply.

Production and Calvo's Pricing Problem with Past Indexation

The typical intermediate-good producer i solves a cost-minimization intratemporal problem given the available technology and taking input prices as given. At the optimum, the demand for labor immediately is

$$\frac{W_t}{P_t} = MC_t \Delta_t^{\frac{\alpha-1}{\alpha}} Y_{i,t}^{\frac{1}{\alpha}} (1-\zeta) (A_t N_{i,t})^{-\frac{1}{\alpha}} A_t, \quad (\text{A-3})$$

while the demand for the energy source is

$$\frac{P_{Z,t}}{P_t} = MC_t \Delta_t^{\frac{\alpha-1}{\alpha}} Y_{i,t}^{\frac{1}{\alpha}} \zeta Z_{i,t}^{-\frac{1}{\alpha}}. \quad (\text{A-4})$$

We now solve the price-setting problem. To make the notation more compact, let $\Psi_t = \Pi_t^{-1} \Pi_{t-1}^\kappa$. Given the price indexation rule, during the time interval in which the typical firm cannot re-set its price, its relative price $p_{i,t+s} = P_{i,t+s}/P_{t+s}$ evolves as:

$$p_{i,t+s} = \left(\prod_{k=1}^s \Psi_{t+k} \right) p_{i,t}^*, \quad (\text{A-5})$$

where $p_{i,t}^* = P_{i,t}^*/P_t$. We have made use of the fact that $\Pi = 1$. Clearly, for $s = 0$, we have $p_{i,t} = p_{i,t}^*$.

Let $Y_{i,t+s|t}$ denote the demand in period $t+s$ faced by a firm i having reset its price in the period t , that is $Y_{i,t+s|t} = p_{i,t+s}^{-\sigma} Y_{t+s}$. Using the result in (A-5) $Y_{i,t+s|t}$ can be expressed as:

$$Y_{i,t+s|t} = \left[\left(\prod_{k=1}^s \Psi_{t+k} \right) p_{i,t}^* \right]^{-\sigma} Y_{t+s}. \quad (\text{A-6})$$

Now consider the case of the firm able to re-optimize in period t . As mentioned above, the representative firm i will choose the price $p_{i,t}^*$ to maximize the current market value of the profits generated while that price remains constant. The optimization problem can be written as:

$$\max_{p_{i,t}^*} \tilde{E}_{i,t} \sum_{t=0}^{\infty} \varpi^s \left[Q_{i,t,t+s} \left(p_{i,t+s} Y_{i,t+s|t} - MC_{t+s} Y_{i,t+s|t} \right) \right], \quad (\text{A-7})$$

subject to (A-6), where $Q_{i,t,t+s} = \beta^s \lambda_{i,t+s}/\lambda_{i,t}$ is the real stochastic discount factor with λ denoting the marginal utility of consumption. The first-order condition for the optimal price is then:

$$p_{i,t}^* = \frac{\sigma}{\sigma-1} \frac{\tilde{E}_{i,t} \sum_{t=0}^{\infty} \varpi^s \beta^s \lambda_{i,t+s} Y_{t+s} \left(\prod_{k=1}^s \Psi_{t+k} \right)^{-\sigma} MC_{t+s}}{\tilde{E}_{i,t} \sum_{t=0}^{\infty} \varpi^s \beta^s \lambda_{i,t+s} Y_{t+s} \left(\prod_{k=1}^s \Psi_{t+k} \right)^{1-\sigma}}. \quad (\text{A-8})$$

Aggregation

In this model agents are heterogeneous because they formulate expectations differently, i.e., they have different market beliefs. To solve the aggregation problem we first need to log-linearize the model around the deterministic steady state.

Equations (A-1) and (A-2) can be easily log-linearized around a zero-inflation steady state to obtain:

$$c_{i,t} = \tilde{E}_{i,t}c_{i,t+1} - h(c_t - c_{t-1}) - \frac{1-h}{\gamma}(r_t - \tilde{E}_{i,t}\pi_{t+1}) + \quad (\text{A-9})$$

$$-\frac{1-h}{\gamma}(\tilde{E}_{i,t}\mu_{t+1} - \mu_t) + \bar{\varrho}b_{i,t},$$

$$\varphi n_{i,t} = w_t - \frac{\gamma}{1-h}c_{i,t} + \frac{\gamma h}{1-h}c_{t-1}, \quad (\text{A-10})$$

where c_i and n_i denote consumption and labor expressed as natural log deviations from their steady-state values, $\bar{\varrho} = (1-h)^{1+\gamma}Y^{1+\gamma}\varrho/\gamma$, $b_{i,t} = B_{i,t}/YP_t$, w_t refers to the natural log deviation of the real wage from its steady-state level, $\pi_t = \Pi_t - 1$ and $r_t = R_t - R$.

Equation (A-9) can be re-written as

$$c_{i,t} = \tilde{E}_{i,t}c_{t+1} + (\tilde{E}_{i,t}c_{i,t+1} - \tilde{E}_{i,t}c_{t+1}) - h(c_t - c_{t-1}) + \quad (\text{A-11})$$

$$-\frac{1-h}{\gamma}(r_t - \tilde{E}_{i,t}\pi_{t+1}) - \frac{1-h}{\gamma}(\tilde{E}_{i,t}\mu_{t+1} - \mu_t) + \bar{\varrho}b_{i,t},$$

where now $\tilde{E}_{i,t}c_{t+1}$ is the subjective expectations of aggregate consumption. Let $c_t = \int_0^1 c_{i,t} di$ denote aggregate consumption, then from the above equation, we have:

$$c_t = \tilde{E}_t c_{t+1} - \frac{1-h}{\gamma}(r_t - \tilde{E}_t \pi_{t+1}) - h c_t + h c_{t-1} + \quad (\text{A-12})$$

$$+ \int_0^1 (\tilde{E}_{i,t} c_{i,t+1} - \tilde{E}_{i,t} c_{t+1}) di + v_t,$$

where $\tilde{E}_t c_{t+1}$ and $\tilde{E}_t \pi_{t+1}$ are the market forecasts for consumption and inflation. Note that we have used the fact that in equilibrium it must be that $\int_0^1 b_{i,t} di = 0$. For simplicity, we assume that $v_t = -\frac{1-h}{\gamma}(\int_0^1 \tilde{E}_{i,t} \mu_{t+1} - \mu_t)$, where v_t follows a first-order autoregressive process.⁴⁵ To facilitate aggregation, following Hommes et al. (2019), we further assume the average expectation of individual consumption is equal to the average expectation of aggregate consumption, that is $\int_0^1 \tilde{E}_{i,t} c_{i,t+1} di = \int_0^1 \tilde{E}_{i,t} c_{t+1} di$.

Aggregate labor supply immediately follows from (A-2):

$$\varphi n_t = w_t - \gamma(1-h)^{-1}c_t + \gamma h(1-h)^{-1}c_{t-1}, \quad (\text{A-13})$$

where w_t is the wage rate expressed as natural log deviation from its steady-state value.

Equations (4), (A-3) and (A-4) can be easily log-linearized to obtain:

$$y_t = \delta_t + \epsilon_Z z_t + \epsilon_N (a_t + n_t), \quad (\text{A-14})$$

$$w_t = m c_t + \frac{\varkappa - 1}{\varkappa} \delta_t + \frac{1}{\varkappa} (y_t - n_t) + \left(1 - \frac{1}{\varkappa}\right) a_t, \quad (\text{A-15})$$

⁴⁵Alternatively, one can also make explicit the expectation rules on the variable v .

$$p_{Z,t} = mc_t + \frac{\varkappa - 1}{\varkappa} \delta_t + \frac{1}{\varkappa} (y_t - z_t), \quad (\text{A-16})$$

where $\epsilon_N \equiv (\Delta AN/Y)^{\frac{\varkappa-1}{\varkappa}} (1 - \zeta)$, $\epsilon_Z \equiv (\Delta Z/Y)^{\frac{\varkappa-1}{\varkappa}} \zeta$, $p_{Z,t}$ is the relative price of emissions expressed in log deviation from its steady state level. All the other variables refer to their capital-letter counterparts, always expressed as natural log deviations from their respective steady-state values.

Log-linearizing (A-8) around the zero-inflation steady state delivers:

$$\widehat{p}_{i,t}^* = (1 - \varpi\beta) mc_t - \varpi\beta \widetilde{E}_{i,t} \psi_{t+1} + \varpi\beta \widetilde{E}_{i,t} \widehat{p}_{i,t+1}^*, \quad (\text{A-17})$$

where $\widehat{p}_{i,t}^* = (p_{i,t}^* - p^*)/p^*$ and $\psi_{t+1} = -\pi_{t+1} + \kappa\pi_t$. Let $\widehat{p}_t^* = \int_0^1 \widehat{p}_{i,t}^* di$. The pricing equation can then be re-written as:

$$\widehat{p}_{i,t}^* = (1 - \varpi\beta) mc_t - \beta\varpi\kappa\pi_t + \varpi\beta \widetilde{E}_{i,t} (\widehat{p}_{t+1}^* + \pi_{t+1}) + \varpi\beta (\widetilde{E}_{i,t} \widehat{p}_{i,t+1}^* - \widetilde{E}_{i,t} \widehat{p}_{t+1}^*), \quad (\text{A-18})$$

where we have used the fact that $\psi_{t+1} = -\pi_{t+1} + \kappa\pi_t$.

Given the definition of aggregate price index $P_t = \int_0^1 (P_{i,t}^{1-\sigma})^{1/(1-\sigma)} di$, in the presence of price stickiness and indexation, we have:

$$P_t^{1-\theta} = \int_0^{1-\varpi} (P_{i,t}^*)^{1-\sigma} di + \int_0^\varpi (P_{i,t-1} \Pi_{t-1}^\kappa)^{1-\sigma} di \quad (\text{A-19})$$

that in log-linear terms can be expressed as:

$$(1 - \varpi) \widehat{p}_t^* = \varpi (\pi_t - \kappa\pi_{t-1}). \quad (\text{A-20})$$

Substituting into equation (A-18) gives:

$$\widehat{p}_{i,t}^* = (1 - \varpi\beta) mc_t - \varpi\beta\kappa\pi_t - \varpi\beta \frac{\varpi}{1 - \varpi} \kappa\pi_t + \frac{\varpi}{1 - \varpi} \beta \widetilde{E}_{i,t} \pi_{t+1} + \varpi\beta (\widetilde{E}_{i,t} \widehat{p}_{i,t+1}^* - \widetilde{E}_{i,t} \widehat{p}_{t+1}^*). \quad (\text{A-21})$$

The above equation can be aggregated over all firms re-setting their price to obtain:

$$\pi_t = \frac{1 - \varpi\beta}{1 + \beta\kappa} \frac{1 - \varpi}{\varpi} mc_t + \frac{\beta}{1 + \beta\kappa} \widetilde{E}_t \pi_{t+1} + \frac{\kappa}{1 + \beta\kappa} \pi_{t-1} + (1 - \varpi)\beta \int_0^1 (\widetilde{E}_{i,t} \widehat{p}_{i,t+1}^* - \widetilde{E}_{i,t} \widehat{p}_{t+1}^*) di. \quad (\text{A-22})$$

Now observe that all firms have access to the same technology, have the same marginal costs, and are subject to the same random shocks. As one above, we then assume that the aggregate expectation on the optimal future price set by each firm manager $\int_0^1 \widetilde{E}_{i,t} \widehat{p}_{i,t+1}^* di$ is equal to the aggregate expectations on the average optimal price of the economy $\int_0^1 \widetilde{E}_{i,t} \widehat{p}_{t+1}^* di$. It follows that (A-22) can be written as:

$$\pi_t = \frac{1 - \varpi\beta}{1 + \beta\kappa} \frac{1 - \varpi}{\varpi} mc_t + \frac{\beta}{1 + \beta\kappa} \widetilde{E}_t \pi_{t+1} + \frac{\kappa}{1 + \beta\kappa} \pi_{t-1}. \quad (\text{A-23})$$

The above equation is the New Keynesian Phillips curve with price indexation and non-rational agents.

Finally, the log-linearized versions of equations (7)-(10) immediately follow:

$$m_t = \delta_M \frac{Z}{Z + Z^{RoW}} z_t + (1 - \delta_M) m_{t-1} + \delta_M \frac{Z^{RoW}}{Z + Z^{RoW}} z_t^{RoW}, \quad (\text{A-24})$$

$$\delta_t = -\eta (M - \bar{M}) m_t, \quad (\text{A-25})$$

$$r_t = \iota_r r_{t-1} + (1 - \iota_r)[\iota_\pi \pi_t + \iota_y (y_t - y_t^*)] + u_t, \quad (\text{A-26})$$

$$c_t = y_t, \quad (\text{A-27})$$

where m_t is the log-deviation of $M_t - \bar{M}$ from its steady state value.

Using the equilibrium condition (A-27) in the aggregate Euler equation (A-12), we obtain:

$$y_t (1 + h) = \tilde{E}_t y_{t+1} + h y_{t-1} - \frac{1-h}{\gamma} (r_t - \tilde{E}_t \pi_{t+1}) + v_t. \quad (\text{A-28})$$

We can then combine (A-13), (A-14), (A-15) with (A-16) to get rid of n_t and w_t , and obtain:

$$m c_t = \left(\frac{\varphi + \frac{1}{\varkappa}}{\epsilon_N} - \frac{1}{\varkappa} + \frac{\gamma}{1-h} \right) y_t - \left(\frac{\varphi + \frac{1}{\varkappa}}{\epsilon_N} + \frac{\varkappa - 1}{\varkappa} \right) \delta_t - \epsilon_Z \frac{\varphi + \frac{1}{\varkappa}}{\epsilon_N} z_t - (1 + \varphi) a_t - \gamma \frac{h}{1-h} y_{t-1}, \quad (\text{A-29})$$

$$p_{Z,t} = m c_t + \frac{\varkappa - 1}{\varkappa} \delta_t + \frac{1}{\varkappa} (y_t - z_t). \quad (\text{A-30})$$

The above two equations, along with (A-23), (A-25)-(A-24) and (A-28) describe the aggregate model summarized in Table 1 where we have assumed that $z_t^{NI} = 0$ and written the coefficients of equation (A-29) in a compact form.

Under flexible prices, the typical firm i will set the price $P_{i,t}$ to maximize profits given the demand schedule $Y_{i,t} = (P_{i,t}/P_t)^{-\sigma} Y_t$. At the optimum $MC = (\sigma - 1)/\sigma$, so that in the log-linearized model $m c_t = 0$. Combining (A-29) with (A-30) and assuming $m c_t = 0$, the natural level of output immediately follows:

$$y_t^* = \frac{\epsilon_N \frac{\varphi+1}{\varphi+\frac{1}{\varkappa}} a_t + \left[1 + \epsilon_N \frac{\varkappa-1}{\varphi+\frac{1}{\varkappa}} + \epsilon_Z (\varkappa - 1) \right] \delta_t^* - \epsilon_Z \varkappa p_{Z,t}^* + \epsilon_N \frac{\gamma h (1-h)^{-1}}{\varphi+\frac{1}{\varkappa}} y_{t-1}^*}{1 - \epsilon_Z \varkappa - \epsilon_N \frac{\frac{1}{\varkappa} - \gamma (1-h)^{-1}}{\varphi+\frac{1}{\varkappa}}}, \quad (\text{A-31})$$

where $\delta_t^* = -\eta (M - \bar{M}) m_t^*$, $m_t^* = \delta_M \frac{Z}{Z+Z^{RoW}} z_t^* + (1 - \delta_M) m_{t-1}^*$ while z_t^* must satisfy the equation below:

$$p_{Z,t}^* = \frac{\varkappa - 1}{\varkappa} \delta_t^* + \frac{1}{\varkappa} (y_t^* - z_t^*). \quad (\text{A-32})$$

From (A-31) we can see that the natural level of output is decreasing in the policy variable $p_{Z,t}^*$. Clearly, if the government sets the price of emissions, then $p_{Z,t}^* = p_{Z,t}$, while in the case of a cap control we will have $z_t^* = z_t$.

Additional Results

In this appendix, we report some additional results. Figure A-1 shows the economy's response to a timely mitigation policy under rational and behavioral expectations. It is interesting to see that, in the case of a gradual emission reduction, the time path of emissions does not differ substantially between the two cases, contrary to what happens to inflation and all the other variables.

Figure A-2 shows the economy’s response to a gradual increase in the carbon price aimed at permanently reducing emissions by 20% under different layers of uncertainty. In the case of anticipated mitigation (dotted lines), the gradual increase of the carbon tax does not come as a surprise. However, only agents adopting a fundamentalist forecasting rule for output exploit this information. As a result, the level of economic activity declines more, and inflationary pressures are mitigated. The second scenario (dashed lines) refers to the case in which the gradual increase of the price of carbon is introduced as a surprise. In this case, regardless of how agents formulate expectations on output, they do not account for the fact that the carbon tax will keep rising until the mitigation goal is reached. As expected the level of economic activity is slightly higher. The third case (continuous lines), which is our benchmark fully illustrated in Figure 3, considers the mean response of the economy to an unanticipated gradual increase in a perturbed economy, that is to say throughout the business cycle when the economy is hit by other shocks.

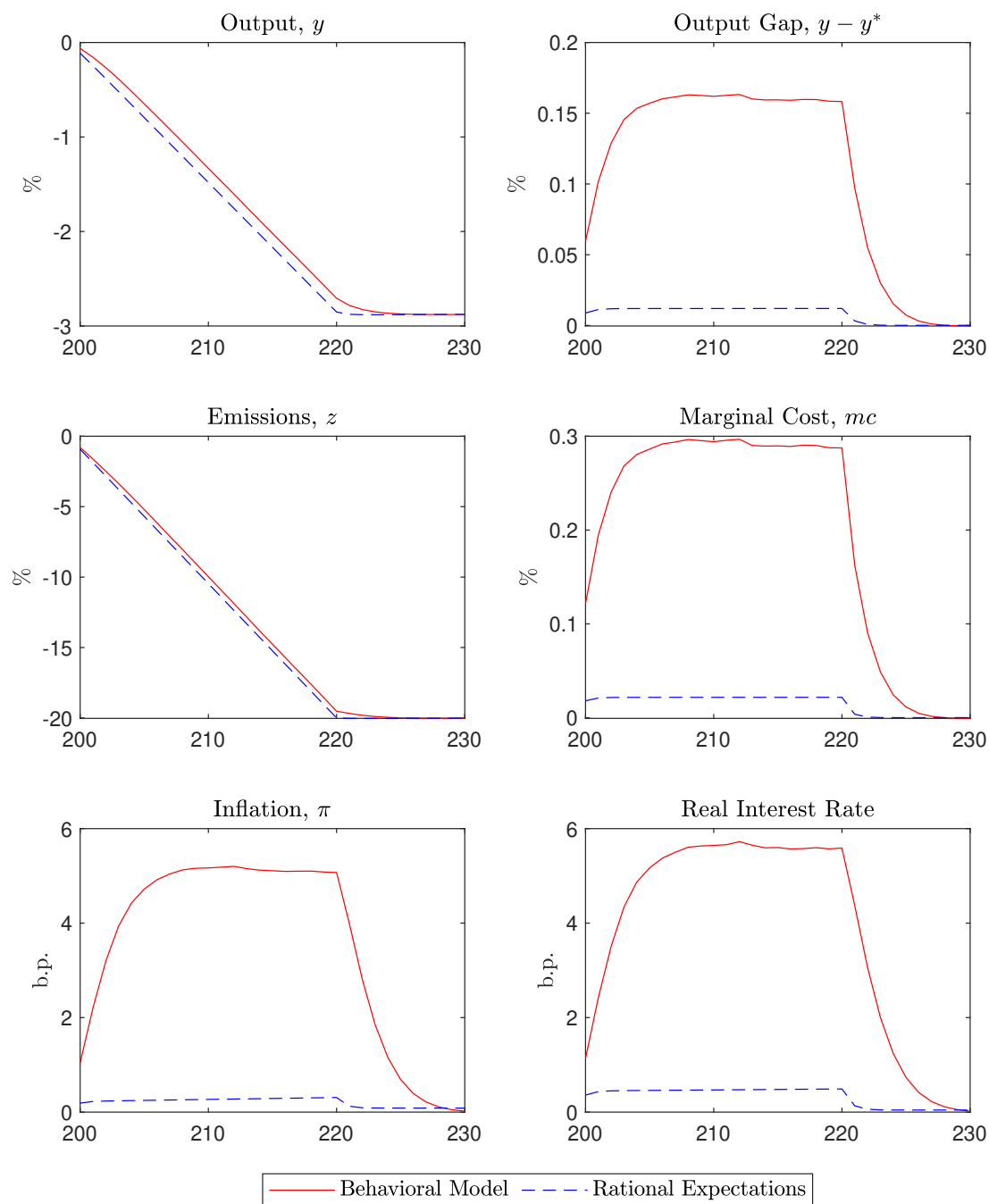
Figure A-3 is the rational expectations counterpart of Figure 5 of the main text. Here, the forecast errors are driven only by the pollution policy phased in as a surprise policy shock.

Figure A-4 plots the response of emissions to a gradual increase in the carbon tax, as in the main scenario of Figure 3, but in a highly perturbed economy, where the standard deviations of shocks driving business cycle fluctuations are three times larger than in the baseline case. In a similar scenario, Figure A-5 shows the response of the emission permits price and the inflation rate to a gradual quantity restrictions of emissions (cap-and-trade scheme). Comparing these two figures makes it clear that the choice between price and quantity regulations poses a significant policy trade-off between emission certainty and price stability in a perturbed economy. Inflation targeting becomes more problematic.

Table A-1 reproduces the results of Table 5 in the absence of monetary policy uncertainty.

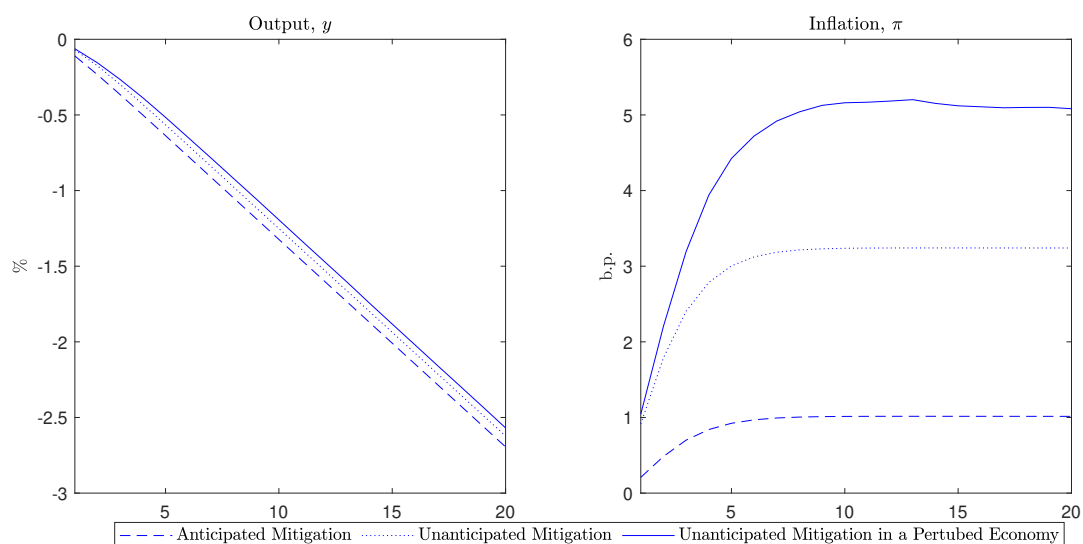
Table A-2 shows how the uncertainty around the mitigation path changes for different values of the monetary policy parameter ι_y under the assumption that the technological shock is replaced by a pure cost-push shock on the New Keynesian Phillips Curve. This exercise is meant to test whether the absence of any monetary policy trade-off between output gap and inflation stabilization is robust with respect to the nature of the supply-side shock chosen as business cycle driver. Table A-2 shows that, also in the presence of a trade off-inducing underlying shock, inflation and output gap volatility move in the same direction. This confirms the relevance of the climate policy shock in driving the volatility results. The nature of this shock, which is not trade-off-inducing, its occurrence in every period, and its size (increasing over time), make it able to overcome the effects of shocks moving output gap and inflation in opposite directions.

Figure A-1: Timely Mitigation under Rational and Behavioral Expectations - Carbon Tax



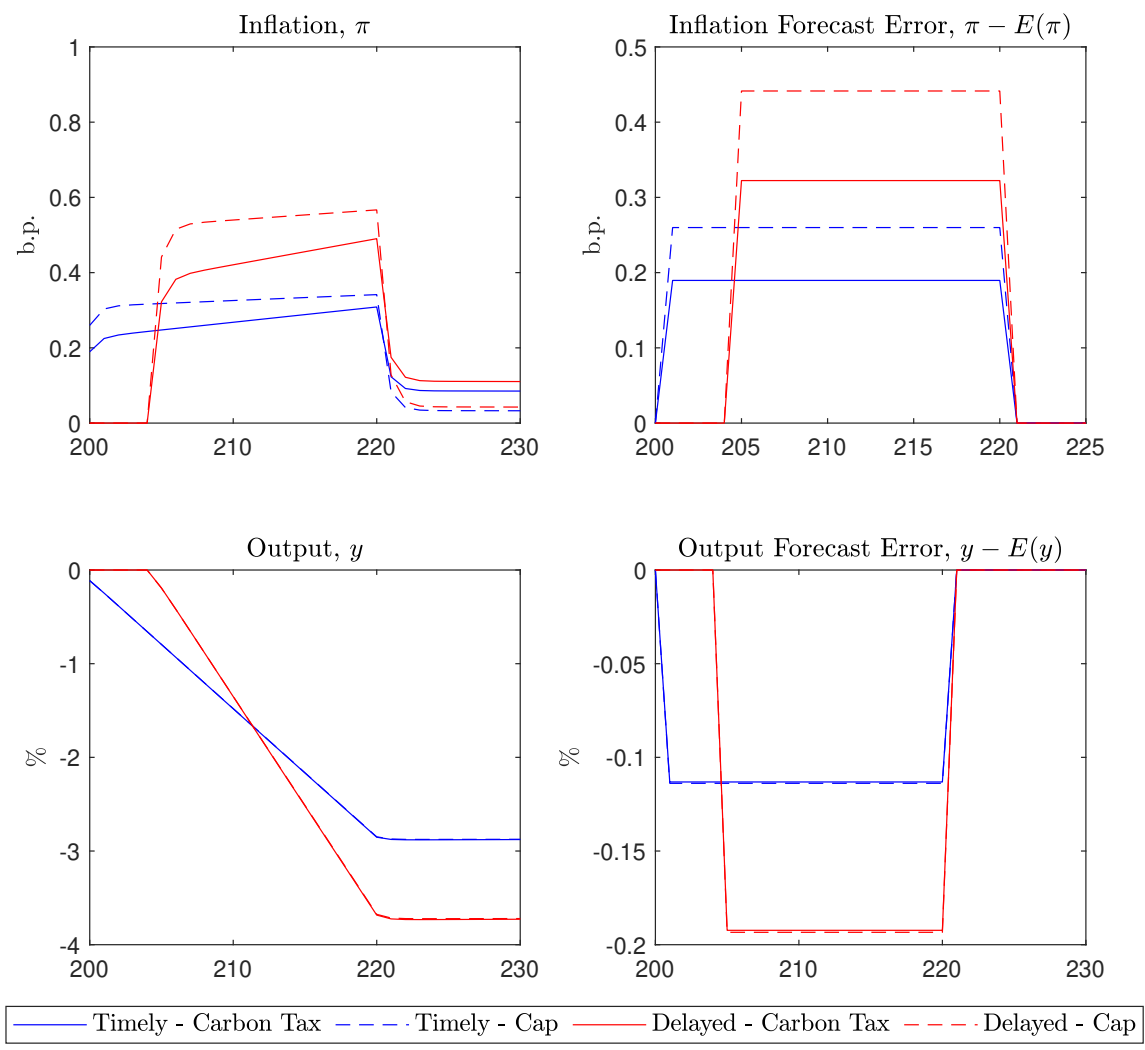
Note: the figure plots the economy's response to a gradual increase in the carbon price aimed at permanently reducing emissions by 20%. All variables are expressed in percentage deviations from their respective business-as-usual value, with the exceptions of inflation and of the real interest rate that are expressed in quarterly basis points (b.p.) deviations.

Figure A-2: Mitigation Scenarios under Different Layers of Uncertainty - Carbon Tax



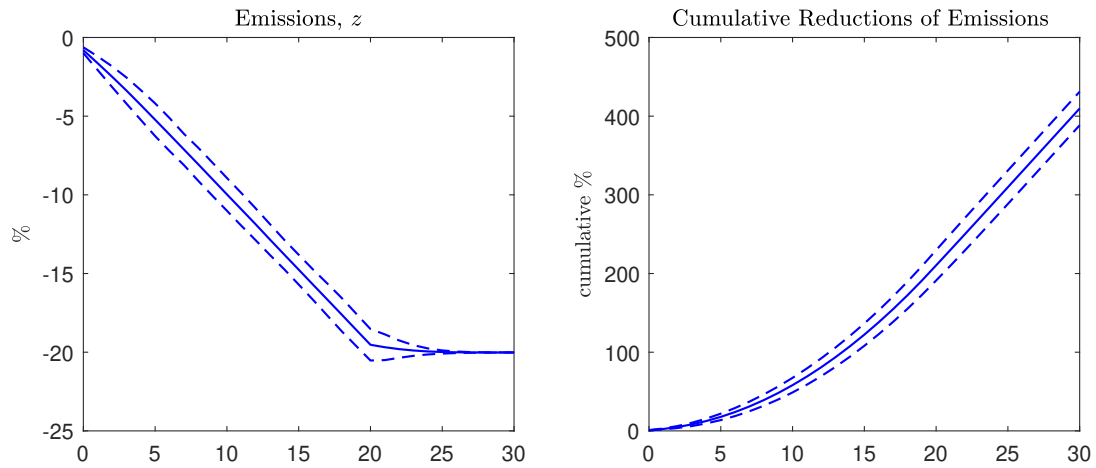
Note: the figure plots the economy's response to a gradual increase in the carbon price aimed at permanently reducing emissions by 20% under different layers of uncertainty. Output is expressed in percentage change relative the steady state; inflation is expressed in quarterly basis points (b.p.) deviations.

Figure A-3: Mitigation Scenarios - Macroeconomic Dynamics and Market Forecast Errors under Rational Expectations



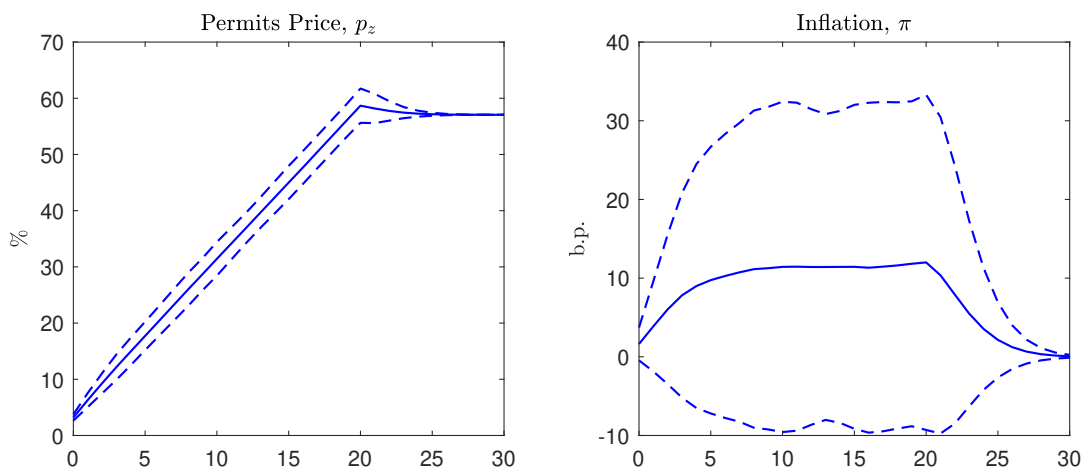
Note: the figure plots the mean response of the economy to different mitigation scenarios entailing the same cumulative emissions after 20 quarters in the deterministic counterparts. Inflation and its forecast errors are expressed in quarterly basis points (b.p.) deviations, while output and its forecast errors are in percentage deviations.

Figure A-4: Emission Dynamics in a Highly Perturbed Economy - Carbon Tax



Note: the figure plots the time path of emissions in a highly perturbed economy. The climate policy consists of a gradual increase in the carbon price aimed at permanently reducing emissions by 20% in a deterministic setting. Cumulative reductions are simply the cumulative variations of emissions.

Figure A-5: Permits Price and Inflation in a Highly Perturbed Economy - Cap



Note: the figure plots the time path of permits price and inflation in a highly perturbed economy. The climate policy consists of a gradual emission restriction.

Table A-1: Macroeconomic Volatility over the Mitigation Path - No Monetary Policy Uncertainty

	Tax	Cap
σ_z	0.1010	0
σ_{p_z}	0	0.2782
σ_{y-y^*}	0.0649	0.0487
σ_π	1.8928	1.9174
$E(\pi)$	4.3355	4.9622
$max(\pi)$	4.9248	5.5854
Welfare loss L	0.0419	0.0339

Note: the table reports the standard deviations of the response for a selection of variables along with mean inflation and its maximum observed value over a timely mitigation time path; the standard deviations σ_{p_z} , σ_z , σ_{y-y^*} are expressed in percentages, σ_π , $E(\pi)$ and $max(\pi)$ in quarterly basis points. For computation of the welfare loss both standard deviations are expressed in percentages.

Table A-2: Macroeconomic Volatility over the Mitigation Path with Cost-Push Shock

l_y	Tax					Cap				
	σ_z	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$	σ_{p_z}	σ_{y-y^*}	σ_π	$E(\pi)$	$max(\pi)$
0	1.4762	0.9506	34.8863	31.9466	38.7695	3.9922	0.7004	33.3397	31.4065	37.7670
0.125	0.9538	0.6140	25.6322	23.7891	28.7003	2.6553	0.4658	25.3562	24.2294	28.9949
0.5	0.3287	0.2114	14.0298	13.2724	15.8926	0.9642	0.1690	14.8841	14.4618	17.2275
1.5	0.0493	0.0316	7.0847	6.6743	7.9917	0.1510	0.0264	8.1623	7.8184	9.3320
3	0.0091	0.0058	4.4913	4.1699	4.9943	0.0311	0.0054	5.4111	5.0735	6.0648

Note: the table reports the standard deviations of the response for a selection of variables along with mean inflation and its maximum observed value over the mitigation time path; σ_{p_z} , σ_z , σ_{y-y^*} are expressed in percentages, σ_π , $E(\pi)$ and $max(\pi)$ in quarterly basis points.