Technical Appendix to: "GHG Emissions Control and Monetary Policy"

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Calibration and Steady State in the Social Planner Solution

In steady state the equilibrium conditions describing the first-best solution (18)-(20) read as follows

$$M = (1-\delta_M)M + (1-U)\,\varphi\exp(-\chi(M-\widetilde{M}))AL + \tilde{Z}, \tag{T-1}$$

$$\frac{1}{L} - \mu_L L^\eta - \lambda^M (1-U)\,\varphi\exp(-\chi(M-\widetilde{M}))A = 0, \tag{T-2}$$

$$-\frac{\phi_1\phi_2 U^{\phi_2-1}}{1-\phi_1 U^{\phi_2}} + \lambda^M \varphi\exp(-\chi(M-\widetilde{M}))AL = 0, \tag{T-3}$$

$$-\chi + \lambda^M + \chi\lambda^M (1-U)\,\varphi\exp(-\chi(M-\widetilde{M}))AL - \beta(1-\delta_M)\lambda^M = 0, \tag{T-4}$$

while steady-state output, emissions and consumption are given by

$$Y = \exp(-\chi(M-\widetilde{M}))AL, \tag{T-5}$$

$$Z = (1-U)\,\varphi Y, \tag{T-6}$$

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$$C = Y\left(1 - \phi_1 U^{\phi_2}\right). \tag{T-7}$$

The equilibrium conditions listed above contain 7 equations in 7 variables, Y, Z, L, M, U, λ^M, C and 8 parameters $\beta, \delta_M, \varphi, \chi, A, \mu_L, \phi_1, \phi_2$. In order to obtain values for the steady-state levels of all variables and for the deep structural parameters, we also need to determine $\widetilde{M}$, $\tilde{Z}$.

We start by setting β, η, δ_M and ϕ_2

$$\beta = 0.99,$$

$$\eta = 1,$$

$$\delta_M = 1 - 0.9979.$$

$$\phi_2 = 2.8.$$

We normalize the steady-state level of output under the social planner solution to 1:

$$Y = 1,$$

and labor to 0.2

$$L = 0.2.$$

We now refer to the optimal and to the baseline runs of the RICE-2010 model (see Nordhaus 2008 and Nordhaus and Boyer 2000), using the simulation results for the year 2015. In the optimal scenario, world industrial emissions amount to 8.475 GTC, non-industrial emissions to 1.280 GTC, while output gross of abatement cost, but net of climate damage, is equal to 81.0561=81.3×(1-0.0030) trillion U.S. dollars. Having normalized output to one, these data deliver the steady-state values for Z, $\tilde{Z}$ and M in model units:

$$\begin{aligned}
Z &= 8.475/(81.3 \times (1 - 0.0030)), \\
\tilde{Z} &= 1.280/(81.3 \times (1 - 0.0030)), \\
M &= (Z + \tilde{Z})/\delta_M.
\end{aligned}$$

The pre-industrial atmospheric concentration of carbon amounts to about 600 GTC, therefore we have:

$$\tilde{M} = 600/(81.3 \times (1 - 0.0030)).$$

In the baseline scenario world industrial emissions amount to 10.004 GTC. We use this value to approximate our φ, measuring emissions intensity in the absence of abatement:[1]

$$\varphi = 10.004/(81.3 \times (1 - 0.0030)).$$

From (T-6) we obtain the abatement effort:

$$U = 1 - \frac{Z}{\varphi Y},$$

In the optimal scenario the abatement cost expressed as a fraction of output is 0.000255. Given U and ϕ_2, we then have the scale parameter ϕ_1

$$\phi_1 = 0.000255/U^{\phi_2}.$$

From (T-7) we compute C.

From (T-3), using (T-5), we now obtain the Lagrange multiplier λ^M

$$\lambda^M = \frac{\phi_1 \phi_2 U^{\phi_2 - 1}}{(1 - \phi_1 U^{\phi_2})\, \varphi Y}.$$

From (T-2) we easily compute the scale parameter μ_L

$$\mu_L = \frac{1 - \lambda^M (1 - U)\, \varphi Y}{L^{\eta + 1}}.$$

From (T-4), using (T-5), the implied value of χ immediately follows:

$$\chi = \frac{1 - \beta(1 - \delta_M)}{1 - \lambda^M (1 - U)\, \varphi Y} \lambda^M.$$

Finally, from (T-5) the scale parameter measuring technology is found to be

$$A = \frac{Y}{\exp(-\chi(M - \widetilde{M}))L}.$$

[1]This represents an approximation, since we interpret "the absence of abatement" as "the absence of optimal policy".

Steady State in the Ramsey Solution

We now describe the strategy adopted to compute the steady state under the assumption that a single authority has access to both monetary and environmental policy instruments (see section 4.1). We use the very same parametrization described above for the deep structural parameters $\beta, \delta_M, \varphi, \chi, A, \mu_L, \phi_1, \phi_2$ and consider the same levels for $\widetilde{M}$, $\tilde{Z}$. In this case two additional parameters must be calibrated, namely the price elasticity θ which is set equal to 6, and the parameter γ, measuring the degree of price rigidity set at 58.25. Given these structural parameters, we use the first-order conditions reported in the Appendix together with the constraints to the Ramsey problem and proceed as follows.

STEP 1: we use the fact that in Ramsey steady state $\Pi = 1$. By using this result, the relevant equations to compute the steady state are then the following:

$$\frac{1-\theta}{1-\phi_1 U^{\phi_2}} + \theta\mu_L L^{\eta+1} + \theta\frac{\phi_1 U^{\phi_2} + \phi_1\phi_2 U^{\phi_2 - 1}(1-U)}{1-\phi_1 U^{\phi_2}} = 0, \tag{T-8}$$

$$M = (1-\delta_M)M + (1-U)\,\varphi\exp(-\chi(M-\widetilde{M}))AL + \tilde{Z}, \tag{T-9}$$

$$\frac{1}{L} - \mu_L L^{\eta} - \lambda^{\Pi}(\eta+1)\,\theta\mu_L L^{\eta} - \lambda^{M}(1-U)\,\varphi\exp(-\chi(M-\widetilde{M}))A = 0, \tag{T-10}$$

$$\begin{aligned} -\frac{\phi_1\phi_2 U^{\phi_2-1}}{1-\phi_1 U^{\phi_2}} - \theta\lambda^{\Pi}\frac{(\phi_2-1)(1-U)\,\phi_1\phi_2 U^{\phi_2-2}}{1-\phi_1 U^{\phi_2}} + \\ +\frac{\phi_1\phi_2 U^{\phi_2-1}}{(1-\phi_1 U^{\phi_2})^2}\lambda^{\Pi}\left\{\theta - 1 - \theta\left[\phi_1 U^{\phi_2} + \phi_1\phi_2 U^{\phi_2-1}(1-U)\right]\right\} \\ +\varphi AL\exp(-\chi(M_t-\widetilde{M})) = 0, \end{aligned} \tag{T-11}$$

$$-\chi + \lambda^M - \beta(1-\delta_M)\lambda^M + \chi(1-U)\,\varphi\exp(-\chi(M-\widetilde{M}))AL. \tag{T-12}$$

STEP 2: we solve (T-8) for L

$$L = \left[\frac{1}{(1-\phi_1 U^{\phi_2})\,\mu_L}\left(\frac{\theta-1}{\theta} - \phi_1 U^{\phi_2} - \phi_1\phi_2 U^{\phi_2-1}(1-U)\right)\right]^{\frac{1}{\eta+1}}, \tag{T-13}$$

expressing it as a function of U.

STEP 3: we guess a value for the optimal abatement effort U and find a solution for L. By using (T-9) we are also able to find a numerical solution for M.

STEP 4: with the numerical values for L and M, and our guess for U in hand, we can use equations (T-10)-(T-12) to compute the Lagrange multipliers λ^M and λ^{Π}. Since we have more equations than unknowns, following Schmitt-Grohé and Uribe (2012), we adopt a projection-based approach to determine λ^M and λ^Π. In particular, let

$$\Omega = \begin{pmatrix} -(\eta+1)\,\theta\mu_L L^{\eta+1} & -(1-U)\,\varphi AL\exp\left(-\chi\left(M_t-\widetilde{M}\right)\right) \\ \Omega_{21} & \varphi AL\exp(-\chi(M_t-\widetilde{M})) \\ & 1-\beta(1-\delta_M)+\chi\,(1-U)\,\varphi AL\exp(-\chi(M_t-\widetilde{M})) \end{pmatrix}$$

with

$$\Omega_{21} = -\theta\frac{(\phi_2-1)(1-U)\phi_1\phi_2 U^{\phi_2-2}}{1-\phi_1 U^{\phi_2}} + \frac{\phi_1\phi_2 U^{\phi_2-1}}{\left(1-\phi_1 U^{\phi_2}\right)^2}\left\{\theta-1-\theta\left[\phi_1 U^{\phi_2}+\phi_1\phi_2 U^{\phi_2-1}(1-U)\right]\right\},$$

$$\Theta = \begin{pmatrix} 1-\mu_L L^{\eta+1} \\ -\frac{\phi_1\phi_2 U^{\phi_2-1}}{1-\phi_1 U^{\phi_2}} \\ -\chi \end{pmatrix} \text{ and } \Delta = \begin{pmatrix} \lambda_\Pi \\ \lambda^M \end{pmatrix}.$$

The system of three equations in two unknowns can then be written as

$$\Omega\Delta + \Theta = 0,$$

We then construct the OLS projection

$$\widehat{\Delta} = -\Omega'\Theta\left(\Omega'\Omega\right)^{-1},$$

compute the regression residual $\widehat{\epsilon}$

$$\widehat{\epsilon} = \Omega\widehat{\Delta} + \Theta,$$

and find the residual sum of square $\widehat{\epsilon}'\widehat{\epsilon}$.

Finally, we repeat STEPS 3 and 4 until we find the value of U that minimizes the residual sum of square. The value so found is the optimal level of abatement under the Ramsey planner solution. The steady-state solution for all the other variables immediately follows.

References

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