Protection for Sale in Monopolistic Competition: Beyond the CES*

Technical Appendix

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*This Technical Appendix assumes that readers are familiar with the notation used in the main tex.

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1 Marginal Effects of Trade Policy on the Welfare Components

1.1 Reward to the Specific Factor

The aggregate reward to the specific factor used for the production of goods in sector j is $\Pi_j(t_j, s_j) = \lambda_j K_j \pi_{j,h}$, that can be re-written extensively as

$$\pi_{j,h} = (p_{j,h} - c_j)x_{j,h}N + (p^*_{j,h} + s_j - t^*_j - c_j)x^*_{j,h}N^*. \tag{T.1}$$

Clearly, the marginal effects of a change in the domestic trade policy instruments are the following:

$$\frac{\partial \Pi_j}{\partial t_j} = \lambda_j K_j N \left[\frac{\partial p_{j,h}}{\partial t_j} x_{j,h} + (p_{j,h} - c_j) \frac{\partial x_{j,h}}{\partial t_j} \right], \tag{T.2}$$

$$\frac{\partial \Pi_j}{\partial s_j} = \lambda_j K_j N^* \left[\left(\frac{\partial p^*_{j,h}}{\partial s_j} + 1 \right) x^*_{j,h} + \left(p^*_{j,h} - t^*_j + s_j - c_j \right) \frac{\partial x^*_{j,h}}{\partial s_j} \right]. \tag{T.3}$$

1.2 Trade Policy Revenues

Net per-capita trade policy revenues are defined as

$$R(\mathbf{t}, \mathbf{s}) = \sum_{j=1}^{n} (1 - \lambda_j) K_j t_j x_{j,f} - \frac{N^*}{N} \sum_{j=1}^{n} \lambda_j K_j s_j x^*_{j,h}, \tag{T.4}$$

therefore, the marginal effects of a change in the domestic trade policy instruments are

$$\frac{\partial R}{\partial t_j} = (1 - \lambda_j) K_j \left(x_{j,f} + t_j \frac{\partial x_{j,f}}{\partial t_j} \right), \tag{T.5}$$

$$\frac{\partial R}{\partial s_j} = -\frac{N^*}{N} K_j \lambda_j \left(x^*_{j,h} + s_j \frac{\partial x^*_{j,h}}{\partial s_j} \right). \tag{T.6}$$

1.3 Consumer Surplus

Consumer surplus is simply

$$\mathcal{S}(\mathbf{t}, \mathbf{s}) = \sum_{j=1}^{n} \left(U(X_j) - E_j \right). \tag{T.7}$$

The marginal effects of a change in the domestic trade policy instruments are

$$\frac{\partial S}{\partial t_j} = -\lambda_j K_j x_{j,h} \frac{\partial p_{j,h}}{\partial t_j} - (1 - \lambda_j) K_j x_{j,f} \frac{\partial p_{j,f}}{\partial t_j}, \tag{T.8}$$

$$\frac{\partial S}{\partial s_j} = 0. \tag{T.9}$$

1.4 Marginal Effects of Trade Policy on the Social Welfare

Using (T.2), (T.5), (T.8) the marginal effects of an increase in the import tariff, t_j, set by H is

$$\begin{aligned}\frac{\partial W}{\partial t_j} &= \frac{\partial \Pi_j}{\partial t_j} + N\left(\frac{\partial R}{\partial t_j} + \frac{\partial S}{\partial t_j}\right), \\ &= \lambda_j K_j N\left[\frac{\partial p_{j,h}}{\partial t_j} x_{j,h} + (p_{j,h} - c_j)\frac{\partial x_{j,h}}{\partial t_j}\right] + \\ & N\left[(1-\lambda_j) K_j \left(x_{j,f} + t_j \frac{\partial x_{j,f}}{\partial t_j}\right)\right] + \\ & - N\lambda_j K_j x_{j,h}\frac{\partial p_{j,h}}{\partial t_j} - N(1-\lambda_j) K_j x_{j,f}\frac{\partial p_{j,f}}{\partial t_j},\end{aligned} \tag{T.10}$$

that can be simplified to yield (22) of the main text.

At the optimum the above condition must be equalized to zero:

$$\begin{aligned}&\lambda_j (p_{j,h} - c_j)\frac{\partial x_{j,h}}{\partial t_j} + \\ &\left[(1-\lambda_j)\left(x_{j,f} + t_j^W \frac{\partial x_{j,f}}{\partial t_j}\right)\right] + \\ & - (1-\lambda_j) x_{j,f}\frac{\partial p_{j,f}}{\partial t_j} = 0,\end{aligned} \tag{T.11}$$

that can be re-written as

$$\frac{\lambda_j}{1-\lambda_j}(p_{j,h} - c_j)\frac{\partial x_{j,h}}{\partial t_j} + t_j^W \frac{\partial x_{j,f}}{\partial t_j} + x_{j,f}\left(1 - \frac{\partial p_{j,f}}{\partial t_j}\right) = 0, \tag{T.12}$$

or analogously

$$\frac{t_j^W}{p_{j,f}} = -\frac{\lambda_j}{1-\lambda_j}\frac{p_{j,h} - c_j}{p_{j,f}}\frac{\partial x_{j,h}}{\partial t_j} / \frac{\partial x_{j,f}}{\partial t_j} - \frac{x_{j,f}}{p_{j,f}} / \frac{\partial x_{j,f}}{\partial t_j}\left(1 - \frac{\partial p_{j,f}}{\partial t_j}\right).$$

The first term of the above equation can be manipulated as:

$$\begin{aligned}&-\frac{\lambda_j}{1-\lambda_j}\frac{p_{j,h} - c_j}{p_{j,f}}\frac{\partial x_{j,h}}{\partial t_j} / \frac{\partial x_{j,f}}{\partial t_j} = \\ &-\frac{\lambda_j}{1-\lambda_j}\frac{p_{j,h} - c_j}{p_{j,h}}\frac{p_{j,h}}{p_{j,f}}\frac{x_{j,h}}{x_{j,h}}\frac{x_{j,f}}{x_{j,f}}\frac{\partial x_{j,h}}{\partial t_j} / \frac{\partial x_{j,f}}{\partial t_j} = \\ & z_j \frac{\mu_{j,h} - 1}{\mu_{j,h}}\sigma_{x_{j,h}},\end{aligned}$$

while the second term can be written as:

$$
\begin{gathered}
-\left(\frac{x_{j,f}}{p_{j,f}}/\frac{\partial x_{j,f}}{\partial t_j}\right)\left(1-\frac{\partial p_{j,f}}{\partial t_j}\right)= \\
-\left(\frac{x_{j,f}}{p_{j,f}}/\frac{\partial x_{j,f}}{\partial t_j}\right)\left(\frac{\partial p_{j,f}}{\partial t_j}\right)\left(1-\frac{\partial p_{j,f}}{\partial t_j}\right)/\left(\frac{\partial p_{j,f}}{\partial t_j}\right)= \\
\frac{\theta_{j,f}}{\varepsilon_{x_{j,f}}},
\end{gathered}
$$

where $\theta_{j,f}=\left(1-\frac{\partial p_{j,f}}{\partial t_j}\right)/\left(\frac{\partial p_{j,f}}{\partial t_j}\right)>0$ *and* $\varepsilon_{x_{j,f}}=-\left(\frac{\partial x_{j,f}}{\partial t_j}/\frac{\partial p_{j,f}}{\partial t_j}\right)(p_{j,f}/x_{j,f})>0.$

The welfare-maximizing import tariff, t_j^W, then satisfies the following condition:

$$
\frac{t_j^W}{p_{j,f}}=\frac{\theta_{j,f}}{\varepsilon_{x_{j,f}}}+z_j\frac{\mu_{j,h}-1}{\mu_{j,h}}\frac{\sigma_{x_{j,h}}}{\varepsilon_{x_{j,f}}}, \tag{T.13}
$$

where $z_j=\lambda_j x_{j,h}p_{j,h}\left[(1-\lambda_j)\,x_{j,f}p_{j,f}\right]^{-1}$ and $\sigma_{x_{j,h}}=\left(\frac{\partial x_{j,h}}{\partial t_j}/\frac{\partial p_{j,f}}{\partial t_j}\right)\frac{p_{j,f}}{x_{j,h}}>0$. This shows Lemma 1.

Using (T.3), (T.6), (T.9) the marginal effects of an increase in the export subsidy, s_j, set by H is

$$
\begin{aligned}
\frac{\partial W}{\partial s_j}&=\frac{\partial \Pi_j}{\partial s_j}+N\left(\frac{\partial R}{\partial s_j}+\frac{\partial S}{\partial s_j}\right), \\
&\lambda_j K_j N^*\left[\left(\frac{\partial p^*_{j,h}}{\partial s_j}+1\right)x^*_{j,h}+\left(p^*_{j,h}-t^*_j+s_j-c_j\right)\frac{\partial x^*_{j,h}}{\partial s_j}\right]+ \\
&-N^*K_j\lambda_j\left(x^*_{j,h}+s_j\frac{\partial x^*_{j,h}}{\partial s_j}\right),
\end{aligned} \tag{T.14}
$$

that can be simplified to yield (30) of the main text for $t_j^*=0$.

At the optimum the above condition must be equalized to zero:

$$
\begin{aligned}
&\left[\left(\frac{\partial p^*_{j,h}}{\partial s_j}+1\right)x^*_{j,h}+\left(p^*_{j,h}-t^*_j+s^W_j-c_j\right)\frac{\partial x^*_{j,h}}{\partial s_j}\right]+ \\
&-\left(x^*_{j,h}+s^W_j\frac{\partial x^*_{j,h}}{\partial s_j}\right)=0,
\end{aligned} \tag{T.15}
$$

and can be manipulated as follows:

$$
s^W_j=\frac{\partial p^*_{j,h}}{\partial s_j}x^*_{j,h}/\frac{\partial x^*_{j,h}}{\partial s_j}+\left(p^*_{j,h}-t^*_j+s^W_j-c_j\right), \tag{T.16}
$$

$$
\frac{s^W_j}{p^*_{j,h}}=\frac{\partial p^*_{j,h}}{\partial s_j}x^*_{j,h}/\frac{\partial x^*_{j,h}}{\partial s_j}\frac{1}{p^*_{j,h}}+\frac{p^*_{j,h}-t^*_j+s^W_j-c_j}{p^*_{j,h}}, \tag{T.17}
$$

or better

$$
\frac{s_j}{p^*_{j,h}}=-\frac{1}{\varepsilon^*_{x_{j,h}}}+\frac{\mu^*_{j,h}-1}{\mu^*_{j,h}}, \tag{T.18}
$$

where $\varepsilon^*_{j,h}=-\left(\frac{\partial x^*_{j,h}}{\partial s_j}/\frac{\partial p^*_{j,h}}{\partial s_j}\right)\left(p^*_{j,h}/x^*_{j,h}\right)>0$. This shows Lemma 3.

1.5 Marginal Effects of Trade Policy on the Welfare of Lobby

Using (T.2), (T.5), (T.8) the marginal effects of an increase in the import tariff t_j on the welfare of a lobby i is

$$\begin{aligned}\frac{\partial W_i}{\partial t_j} &= \frac{\partial \Pi_i}{\partial t_j} + \alpha_i N \left(\frac{\partial R}{\partial t_j} + \frac{\partial \mathcal{S}}{\partial t_j} \right), \\ &= \delta_{ij} \lambda_j N K_j \left[\frac{\partial p_{j,h}}{\partial t_j} x_{j,h} + (p_{j,h} - c_j) \frac{\partial x_{j,h}}{\partial t_j} \right] + \\ &- \alpha_i N K_j \left[(1 - \lambda_j) \frac{\partial p_{j,f}}{\partial t_j} x_{j,f} + \lambda_j \frac{\partial p_{j,h}}{\partial t_j} x_{j,h} \right] + \\ &+ \alpha_i N K_j (1 - \lambda_j) \left(t_j \frac{\partial x_{j,f}}{\partial t_j} + x_{j,f} \right),\end{aligned} \tag{T.19}$$

that is (24) of the main text. Manipulations similar to those undertaken in the previous section lead to the results of LEMMA 2. At the optimum the above condition is equal to zero:

$$\begin{aligned}&\delta_{ij} \frac{\lambda_j}{1 - \lambda_j} \left[\frac{\partial p_{j,h}}{\partial t_j} x_{j,h} + (p_{j,h} - c_j) \frac{\partial x_{j,h}}{\partial t_j} \right] + \\ &- \alpha_i \left[\frac{\partial p_{j,f}}{\partial t_j} x_{j,f} + \frac{\lambda_j}{1 - \lambda_j} \frac{\partial p_{j,h}}{\partial t_j} x_{j,h} \right] + \\ &+ \alpha_i \left(t_j^L \frac{\partial x_{j,f}}{\partial t_j} + x_{j,f} \right) = 0.\end{aligned} \tag{T.20}$$

This can be easily re-written as

$$\begin{aligned}\frac{t_j^L}{p_{j,f}} &= -\frac{\delta_{ij}}{\alpha_i} \frac{\lambda_j}{1 - \lambda_j} \left[\frac{\partial p_{j,h}}{\partial t_j} x_{j,h} \frac{1}{p_{j,f}} + (p_{j,h} - c_j) \frac{\partial x_{j,h}}{\partial t_j} \frac{1}{p_{j,f}} \right] / \frac{\partial x_{j,f}}{\partial t_j} + \\ &+ \left[\frac{\partial p_{j,f}}{\partial t_j} x_{j,f} \frac{1}{p_{j,f}} + \frac{\lambda_j}{1 - \lambda_j} \frac{\partial p_{j,h}}{\partial t_j} \frac{1}{p_{j,f}} x_{j,h} \right] / \frac{\partial x_{j,f}}{\partial t_j} - x_{j,f} / \frac{\partial x_{j,f}}{\partial t_j} \frac{1}{p_{j,f}},\end{aligned} \tag{T.21}$$

that collecting common terms gives

$$\begin{aligned}\frac{t_j^L}{p_{j,f}} &= -\frac{1}{\alpha_i} \frac{\lambda_j}{1 - \lambda_j} \left[(\delta_{ij} - \alpha_i) \frac{\partial p_{j,h}}{\partial t_j} x_{j,h} \frac{1}{p_{j,f}} + \delta_{ij} (p_{j,h} - c_j) \frac{\partial x_{j,h}}{\partial t_j} \frac{1}{p_{j,f}} \right] / \frac{\partial x_{j,f}}{\partial t_j} + \\ &+ \left(\frac{\partial p_{j,f}}{\partial t_j} - 1 \right) x_{j,f} \frac{1}{p_{j,f}} / \frac{\partial x_{j,f}}{\partial t_j}.\end{aligned} \tag{T.22}$$

Further manipulations lead to

$$\begin{aligned}\frac{t_j^L}{p_{j,f}} &= -\frac{1}{\alpha_i} \frac{\lambda_j x_{j,h} p_{j,h}}{(1 - \lambda_j) p_{j,f} x_{j,f}} \left[(\delta_{ij} - \alpha_i) \frac{\partial p_{j,h}}{\partial t_j} \frac{x_{j,f}}{p_{j,h}} + \delta_{ij} \frac{p_{j,h} - c_j}{p_{j,h}} \frac{\partial x_{j,h}}{\partial t_j} \frac{x_{j,f}}{x_{j,h}} \right] / \frac{\partial x_{j,f}}{\partial t_j} + \\ &+ \frac{\frac{\partial p_{j,f}}{\partial t_j} - 1}{\frac{\partial p_{j,f}}{\partial t_j}} \frac{x_{j,f}}{p_{j,f}} \frac{\partial p_{j,f}}{\partial t_j} / \frac{\partial x_{j,f}}{\partial t_j},\end{aligned} \tag{T.23}$$

or equivalently

$$\frac{t_j^L}{p_{j,f}} = -\frac{1}{\alpha_i}\frac{\lambda_j x_{j,h} p_{j,h}}{(1-\lambda_j)\, p_{j,f} x_{j,f}} \left[\begin{array}{c} (\delta_{ij}-\alpha_i)\left(\frac{\partial p_{j,h}}{\partial t_j}/\frac{\partial p_{j,f}}{\partial t_j}\right)\frac{p_{j,f}}{p_{j,h}}\left(\frac{\partial p_{j,f}}{\partial t_j}/\frac{\partial x_{j,f}}{\partial t_j}\right)\frac{x_{j,f}}{p_{j,f}}+ \\ +\delta_{ij}\frac{p_{j,h}-c_j}{p_{j,h}}\left(\frac{\partial x_{j,h}}{\partial t_j}/\frac{\partial x_{j,f}}{\partial t_j}\right)\frac{x_{j,f}}{x_{j,h}} \end{array} \right] + \tag{T.24}$$
$$+\frac{\frac{\partial p_{j,f}}{\partial t_j}-1}{\frac{\partial p_{j,f}}{\partial t_j}}\left(\frac{\partial p_{j,f}}{\partial t_j}/\frac{\partial x_{j,f}}{\partial t_j}\right)\frac{x_{j,f}}{p_{j,f}},$$

that can, in turn, be written as

$$\frac{t_j^L}{p_{j,f}} = \frac{\theta_{j,f}}{\varepsilon_{x_{i,f}}} + \frac{z_j}{\alpha_i}\left(\frac{\delta_{ij}-\alpha_i}{\epsilon_{x_{j,f}}}\sigma_{p_{j,h}} + \delta_{ij}\frac{\mu_{j,h}-1}{\mu_{j,h}}\frac{\sigma_{x_{j,h}}}{\varepsilon_{x_{j,f}}}\right), \tag{T.25}$$

where $\sigma_{p_{j,h}} = \left(\frac{\partial p_{j,h}}{\partial t_j}/\frac{\partial p_{j,f}}{\partial t_j}\right)\frac{p_{j,f}}{p_{j,h}} > 0$. This shows Lemma 2.

Using (T.3), (T.6), (T.9) we obtain

$$\begin{aligned} \frac{\partial W_i}{\partial s_j} &= \frac{\partial \Pi_i}{\partial s_j} + \alpha_i N \frac{\partial R}{\partial s_j}, \\ &= \delta_{ij}\lambda_j K_j N^* \left[\left(\frac{\partial p_{j,h}^*}{\partial s_j}+1\right)x_{j,h}^* + \left(p_{j,h}^* - t_j^* + s_j - c_j\right)\frac{\partial x_{j,h}^*}{\partial s_j}\right] + \\ &\quad - \alpha_i \lambda_j K_j N^* \left(x_{j,h}^* + s_j \frac{\partial x_{j,h}^*}{\partial s_j}\right), \end{aligned} \tag{T.26}$$

that for $t_j^* = 0$ yields (32) of the main text. Manipulations similar to those undertaken in the previous section lead to the results of Lemma 4. Equalizing to zero the above condition and simplifying we obtain:

$$\frac{s_j^L}{p_{j,h}^*} = \frac{\delta_{ij}}{\alpha_i}\left[\left(\frac{\partial p_{j,h}^*}{\partial s_j}+1\right)\frac{x_{j,h}^*}{p_{j,h}^*}/\frac{\partial x_{j,h}^*}{\partial s_j} + \frac{p_{j,h}^* - t_j^* + s_j^L - c_j}{p_{j,h}^*}\right] - \frac{x_{j,h}^*}{p_{j,h}^*}/\frac{\partial x_{j,h}^*}{\partial s_j}. \tag{T.27}$$

Some simple manipulations give

$$\frac{s_j^L}{p_{j,h}^*} = \frac{\delta_{ij}}{\alpha_i}\left(\frac{\frac{\partial p_{j,h}^*}{\partial s_j}+1}{\frac{\partial p_{j,h}^*}{\partial s_j}}\frac{x_{j,h}^*}{p_{j,h}^*}\frac{\partial p_{j,h}^*}{\partial s_j}/\frac{\partial x_{j,h}^*}{\partial s_j} + \frac{p_{j,h}^* - t_j^* + s_j^L - c_j}{p_{j,h}^*}\right) - \frac{x_{j,h}^*}{p_{j,h}^*}/\frac{\partial x_{j,h}^*}{\partial s_j} \tag{T.28}$$

$$\frac{s_j^L}{p_{j,h}^*} = \frac{\delta_{ij}}{\alpha_i}\left(\frac{\theta_{j,h}^*}{\epsilon_{j,h}^*} + \frac{\mu_{j,h}^*-1}{\mu_{j,h}^*}\right) + \frac{1}{\frac{\partial p_{j,h}^*}{\partial s_j}}\frac{1}{\varepsilon_{j,h}^*} \tag{T.29}$$

which by using $\theta_{j,h}^* = -\left(1+\frac{1}{\frac{\partial p_{j,h}^*}{\partial s_j}}\right)$ becomes

$$\frac{s_j^L}{p_{j,h}^*} = \frac{\delta_{ij}}{\alpha_j}\left(\frac{\theta_{j,h}^*}{\varepsilon_{x_{j,h}}^*} + \frac{\mu_{j,h}^*-1}{\mu_{j,h}^*}\right) - \frac{\theta_{j,h}^*+1}{\varepsilon_{x_{j,h}}^*}. \tag{T.30}$$

This shows Lemma 4.

2 Political Equilibrium

Using the above results the marginal effects of a tariff and of a subsidy on the government objective function immediately follow:

$$
\begin{aligned}
\frac{\partial \widetilde{G}}{\partial t_j} &= \sum_{i \in L} \frac{\partial W_i}{\partial t_j} + a \frac{\partial W}{\partial t_j} \\
&= (I_j + a)\, N K_j \lambda_j \left[\frac{\partial p_{j,h}}{\partial t_j} x_{j,h} + (p_{j,h} - c_j) \frac{\partial x_{j,h}}{\partial t_j} \right] + \\
&+ (\alpha_L + a)\, N K_j \left\{ (1 - \lambda_j) \left[\frac{\partial x_{j,f}}{\partial t_j} t_j + \left(1 - \frac{\partial p_{j,f}}{\partial t_j} \right) x_{j,f} \right] - \lambda_j \frac{\partial p_{j,h}}{\partial t_j} x_{j,h} \right\},
\end{aligned}
\tag{T.31}
$$

$$
\begin{aligned}
\frac{\partial \widetilde{G}}{\partial s_j} &= \sum_{i \in L} \frac{\partial W_i}{\partial s_j} + a \frac{\partial W}{\partial s_j}, \\
&= (I_j + a)\, \lambda_j K_j N^* \left(\frac{\partial p^*_{j,h}}{\partial s_j} + 1 \right) x^*_{j,h} + \\
&+ (I_j + a)\, \lambda_j K_j N^* \left(p^*_{j,h} - t^*_j + s_j - c_j \right) \frac{\partial x^*_{j,h}}{\partial s_j} + \\
&- (a + \alpha_L)\, \lambda_j K_j N^* \left(x^*_{j,h} + s_j \frac{\partial x^*_{j,h}}{\partial s_j} \right).
\end{aligned}
\tag{T.32}
$$

These show results (27) and (35) of the paper. Manipulations similar to those undertaken in sections 1.4 and 1.5 lead to the results of PROPOSITIONS 1 and 2.