Innovation, Growth and Optimal Monetary Policy - Online Appendixes

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Online Appendix A

This Appendix reports the equilibrium conditions of the two growth models, where non-stationary variables are expressed in efficiency units. Table A-1 summarizes the equilibrium conditions of the endogenous growth model, while Table A-2 reports those referring to the exogenous growth model.

Table A-1: Endogenous Growth Model in Efficiency Units

$$y_t = c_t + i_t + M_t + s_t + c_t^G y_t + \frac{\gamma_Y}{2}\left(\Pi_{Y,t}-1\right)^2 y_t + \frac{\gamma_M}{2}\left(\Pi_{M,t}-1\right)^2 M_t$$

$$y_t = A_t^{\frac{1}{v}}\left[\frac{1}{p_t^M}MC_t(1-v)\right]^{\frac{1-v}{v}} k_t^{1-\alpha}N_t^{\alpha}$$

$$M_t = \left[\frac{1}{p_t^M}MC_t(1-v)A_t\right]^{\frac{1}{v}} k_t^{1-\alpha}N_t^{\alpha}$$

$$w_t = \alpha v MC_t \frac{y_t}{N_t}$$

$$R_t^K = (1-\alpha)\, v MC_t \frac{y_t}{k_t}$$

$$k_{t+1}g_{Z,t+1} = (1-\delta)\, k_t + i_t$$

$$E_t \frac{\Lambda_{t,t+1}^R}{\Pi_{Y,t+1}} = \frac{1}{R_t}$$

$$1 = E_t \Lambda_{t,t+1}^R \left(R_{t+1}^k + 1 - \delta\right)$$

$$\mu_n N_t^{\varphi} c_t = w_t$$

$$\theta_Y - 1 - \theta_Y MC_t + \gamma_Y(\Pi_{Y,t}-1)\Pi_{Y,t} - \gamma_Y \beta E_t \frac{c_t}{c_{t+1}}\left(\Pi_{Y,t+1}-1\right)\Pi_{Y,t+1}\frac{y_{t+1}}{y_t} = 0$$

$$g_{Z,t+1} = \xi_t s_t + \phi$$

$$\xi_t = \hat{\xi}\,(1/s_t)^{1-\varepsilon}$$

$$V_t = (p_t^M - 1)M_t - \frac{\gamma_M}{2}\left(\Pi_{M,t}-1\right)^2 M_t + \phi E_t \Lambda_{t,t+1}^R V_{t+1}$$

$$p_t^M = p_{t-1}^M \frac{\Pi_{M,t}}{\Pi_{P,t}}$$

$$(\theta_M - 1)\, p_t^M - \theta_M + \gamma_M\left(\Pi_{M,t}-1\right)\Pi_{M,t} - \gamma_M \phi E_t \Lambda_{t,t+1}^R \left(\Pi_{M,t+1}-1\right)\Pi_{M,t+1}\frac{M_{t+1}}{M_t} = 0$$

$$1/\xi_t = E_t\left(\Lambda_{t,t+1}^R V_{t+1}\right)$$

$$\Lambda_{t,t+1}^R = \beta \frac{c_t}{g_{Z,t+1}c_{t+1}}$$

Table A-2: Exogenous Growth Model in Efficiency Units

$$y_t = c_t + i_t + M_t + s_t + c_t^G y_t + \frac{\gamma_Y}{2}\left(\Pi_{Y,t} - 1\right)^2 y_t + \frac{\gamma_M}{2}\left(\Pi_{M,t} - 1\right)^2 M_t$$

$$y_t = A_t^{\frac{1}{v}}\left[\frac{1}{p_t^M} MC_t(1-v)\right]^{\frac{1-v}{v}} k_t^{1-\alpha} N_t^{\alpha}$$

$$M_t = \left[\frac{1}{p_t^M} MC_t(1-v)A_t\right]^{\frac{1}{v}} k_t^{1-\alpha} N_t^{\alpha}$$

$$w_t = \alpha v MC_t \frac{y_t}{N_t}$$

$$R_t^K = (1-\alpha)\, v MC_t \frac{y_t}{k_t}$$

$$k_{t+1} g_Z = (1-\delta)\, k_t + i_t$$

$$E_t \frac{\Lambda_{t,t+1}^R}{\Pi_{Y,t+1}} = \frac{1}{R_t}$$

$$1 = E_t \Lambda_{t,t+1}^R \left(R_{t+1}^k + 1 - \delta\right)$$

$$\mu_n N_t^{\varphi} c_t = w_t$$

$$\theta_Y - 1 - \theta_Y MC_t + \gamma_Y(\Pi_{Y,t} - 1)\Pi_{Y,t} - \gamma_Y \beta E_t \frac{c_t}{c_{t+1}}\left(\Pi_{Y,t+1} - 1\right)\Pi_{Y,t+1}\frac{y_{t+1}}{y_t} = 0$$

$$p_t^M = p_{t-1}^M \frac{\Pi_{M,t}}{\Pi_{P,t}}$$

$$(\theta_M - 1)\, p_t^M - \theta_M + \gamma_M\left(\Pi_{M,t} - 1\right)\Pi_{M,t} - \gamma_M E_t \Lambda_{t,t+1}^R\left(\Pi_{M,t+1} - 1\right)\Pi_{M,t+1}\frac{M_{t+1}}{M_t} = 0$$

$$\Lambda_{t,t+1}^R = \beta \frac{c_t}{g_Z c_{t+1}}$$

Online Appendix B

Welfare Measure in Stationary Variables

The lifetime utility function of the typical individual (14) can be written in recursive form as

$$V_t = \log C_t - \mu_n \frac{{N_t}^{1+\varphi}}{1+\varphi} + \beta E_t V_{t+1}. \tag{A-1}$$

By adding and subtracting $\frac{1}{1-\beta} \log Z_t$ and $\frac{\beta}{1-\beta} \log Z_{t+1}$ we get

$$\begin{aligned} V_t = {} & \log C_t - \mu_n \frac{{N_t}^{1+\varphi}}{1+\varphi} + \\ & - \log Z_t + \frac{1}{1-\beta} \log Z_t - \frac{\beta}{1-\beta} \log Z_t + \\ & + \frac{\beta}{1-\beta} \log Z_{t+1} - \frac{\beta}{1-\beta} \log Z_{t+1} + \beta E_t V_{t+1}. \end{aligned} \tag{A-2}$$

where we have used the fact that $\frac{1}{1-\beta} \log Z_t = \log Z_t + \frac{\beta}{1-\beta} \log Z_t$. Collecting terms and defining $\upsilon_t = V_t - \frac{1}{1-\beta} \ln Z_t$ yield:

$$\upsilon_t = E_t \sum_{j=0}^{\infty} \beta^j \left(\log c_{t+j} - \mu_n \frac{N_{t+j}^{1+\varphi}}{1+\varphi} + \frac{\beta}{1-\beta} \log g_{Z,t+1+j} \right). \tag{A-3}$$

Ramsey Monetary Policy in the Endogenous Growth Model

We start by considering the Ramsey problem in the endogenous growth model. For the sake of simplicity we solve the Ramsey problem starting from the constraints already expressed in efficiency units. Having reduced the number of constraints of Table A-1, the Lagrangian representation of the Ramsey problem is found to be:

$$\begin{aligned} & \underset{\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}}{Min} \underset{\{\mathbf{d}_t\}_{t=0}^{\infty}}{Max} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\left(\log c_t - \mu_n \tfrac{N_t^{1+\varphi}}{1+\varphi} + \tfrac{\beta}{1-\beta} \log g_{Z,t+1} \right) + \right.\right. \\ & + \lambda_{1,t} \left(y_t - c_t - k_{t+1} g_{Z,t+1} + (1-\delta) k_t - s_t - M_t - c_t^G y_t - \tfrac{\gamma_M}{2} \left(\Pi_{M,t} - 1 \right)^2 M_t - \tfrac{\gamma_Y}{2} \left(\Pi_{Y,t} - 1 \right)^2 y_t \right) + \\ & + \lambda_{2,t} \left(A_t^{\frac{1}{v}} \left(\tfrac{1}{p_t^M} MC_t (1-v) \right)^{\frac{1-v}{v}} {k_t}^{1-\alpha} N_t^{\alpha} - y_t \right) + \\ & + \lambda_{3,t} \left(\beta \left((1-\alpha) \tfrac{\mu_n {N_{t+1}}^{\varphi+1}}{\alpha k_{t+1}} + \tfrac{1-\delta}{c_{t+1}} \right) - \tfrac{g_{Z,t+1}}{c_t} \right) + \\ & + \lambda_{4,t} \left[\left((\theta_Y - 1) \tfrac{y_t}{c_t} - \theta_Y MC_t \tfrac{y_t}{c_t} + \gamma_Y (\Pi_{Y,t} - 1) \Pi_{Y,t} \tfrac{y_t}{c_t} - \beta \gamma_Y E_t \left(\Pi_{Y,t+1} - 1 \right) \Pi_{Y,t+1} \tfrac{y_{t+1}}{c_{t+1}} \right) \right] + \\ & + \lambda_{5,t} \left(\hat{\xi} s_t^{\varepsilon} + \phi - g_{Z,t+1} \right) + \\ & + \lambda_{6,t} \left(-V_t \tfrac{g_{Z,t+1}}{c_t} + (p_t^M - 1) M_t \tfrac{g_{Z,t+1}}{c_t} - \tfrac{\gamma_M}{2} \left(\Pi_{M,t} - 1 \right)^2 M_t \tfrac{g_{Z,t+1}}{c_t} + \phi \beta E_t \tfrac{V_{t+1}}{c_{t+1}} \right) + \\ & + \lambda_{7,t} \left(-\tfrac{1}{\hat{\xi}} s_t^{1-\varepsilon} \tfrac{g_{Z,t+1}}{c_t} + \beta E_t \tfrac{V_{t+1}}{c_{t+1}} \right) + \\ & + \lambda_{8,t} \left(\left(\tfrac{1}{p_t^M} MC_t (1-v) A_t \right)^{\frac{1}{v}} {k_t}^{1-\alpha} N_t^{\alpha} - M_t \right) + \end{aligned}$$

$$+\lambda_{9,t}\left(\frac{c_t\mu_n N_t{}^{\varphi+1}}{\alpha v y_t}-MC_t\right)+$$
$$+\lambda_{10,t}\left(p_t^M\frac{\Pi_{P,t}}{\Pi_{M,t}}-p_{t-1}^M\right)+$$
$$+\lambda_{11,t}\left(\begin{array}{c}(\theta_M-1)\,M_t\frac{g_{Z,t+1}}{c_t}p_t^M-\theta_M M_t\frac{g_{Z,t+1}}{c_t}+\gamma_M\frac{g_{Z,t+1}}{c_t}\left(\Pi_{M,t}-1\right)M_t\Pi_{M,t}+\\-\beta\phi E_t\frac{1}{c_{t+1}}\gamma_M\left(\Pi_{M,t+1}-1\right)M_{t+1}\Pi_{M,t+1}\end{array}\right)\Bigg]\Bigg\},$$

where $\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}=\{\lambda_{1,t},\,\lambda_{2,t},\,\lambda_{3,t},\,\lambda_{4,t},\,\lambda_{5,t},\,\lambda_{6,t},\,\lambda_{7,t},\,\lambda_{8,t},\,\lambda_{9,t},\,\lambda_{10,t},\,\lambda_{11,t}\}_{t=0}^{\infty}$ denote the Lagrange multipliers of the constraints and $\{\mathbf{d}_t\}_{t=0}^{\infty}=\{c_t,\,g_{Z,t+1},\,k_{t+1},\,M_t,\,MC_t,\,N_t,\,p_t^M,\,s_t,\,V_t,\,y_t,\,\Pi_{M,t},\,\Pi_{Y,t}\}_{t=0}^{\infty}$. Notice that in the absence of monetary frictions, the nominal interest rate only enters the consumption Euler equation, $\beta E_t\frac{c_t}{\Pi_{Y,t+1}g_{Z,t+1}c_{t+1}}=\frac{1}{R_t}$, that is why this last condition can be omitted from the set of constraints. It is the intertemporal Euler equation that determines the nominal rate of interest R_t. In what follows we will only focus on the first order conditions with respect to inflation.

We start by considering the special case in which $\gamma_M=0$. In the case of flexible prices in the intermediate good sector p_t^M is constant and equal to $\frac{\theta_M}{\theta_M-1}$, while $\Pi_{M,t}$ is always equal to $\Pi_{Y,t}$. At the optimum, the following first-order condition with respect to $\Pi_{Y,t}$ must hold:

$$-\lambda_{1,t}\gamma_Y\left(\Pi_{Y,t}-1\right)y_t+\left(\lambda_{4,t}-\lambda_{4,t-1}\right)\gamma_Y\frac{y_t}{c_t}(2\Pi_{Y,t}-1)=0, \tag{A-4}$$

where the first term reflects the marginal effects of inflation on welfare deriving from the negative effects of nominal adjustment costs on the resource constraint, while the second term measures the marginal benefits of smoothing out price changes. Clearly, in steady state, the above condition boils down to $\lambda_1\gamma_Y\left(\Pi_Y-1\right)y=0$. Since $\lambda_1>0$, the optimal steady state inflation rate is then found to be equal to zero, i.e. $\Pi_Y=1$. In steady state the effects of intertemporal cost smoothing vanish, thus the Ramsey planner cannot use inflation as a device to reduce the markup. As discussed in the main text, in this case the endogenous growth model replicates the standard prediction of the baseline NK model, namely that the optimal long-run inflation rate is zero.

In the general case, instead, the relevant first-order conditions describing the optimal time paths of inflation for both sectors are found to be:

$$-\lambda_{1,t}\gamma_Y\left(\Pi_{Y,t}-1\right)y_t+\lambda_{4,t}\gamma_Y\frac{y_t}{c_t}(2\Pi_{Y,t}-1)-\lambda_{4,t-1}\gamma_Y\frac{y_t}{c_t}\left(2\Pi_{Y,t}-1\right)+\lambda_{10,t}\frac{p_t^M}{\Pi_{M,t}}=0, \tag{A-5}$$

$$\begin{array}{c}-\lambda_{1,t}\gamma_M\left(\Pi_{M,t}-1\right)M_t-\lambda_{6,t}\gamma_M\left(\Pi_{M,t}-1\right)M_t\frac{g_{Z,t+1}}{c_t}-\lambda_{10,t}p_t^M\frac{\Pi_{P,t}}{\Pi_{M,t}^2}+\\+\lambda_{11,t}\gamma_M\frac{g_{Z,t+1}}{c_t}\left(2\Pi_{M,t}-1\right)M_t-\lambda_{11,t-1}\frac{\phi}{c_t}\gamma_M\left(2\Pi_{M,t}-1\right)M_t=0.\end{array} \tag{A-6}$$

Imposing the steady state and combining the two above conditions we obtain

$$\begin{array}{c}-\lambda_1\left(\gamma_M M+\gamma_Y y\right)\left(\Pi_Y-1\right)-\lambda_6\gamma_M\left(\Pi_Y-1\right)M\frac{g_Z}{c}+\\+\lambda_{11}\gamma_M\frac{g_Z}{c}\left(1-\frac{\phi}{g_Z}\right)\left(2\Pi_Y-1\right)M=0,\end{array} \tag{A-7}$$

where we have used the fact $\Pi_Y = \Pi_M$. The first term measures the marginal negative effect of inflation on welfare through the resource constraint, the second term reflects the negative effects of inflation deriving from a lower firm value, while the last term refers to marginal benefits of smoothing out price changes between two consecutive periods. Clearly, this last term does not vanish at zero inflation.

Ramsey Monetary Policy in the Exogenous Growth Model

Starting from the exogenous growth model expressed in efficiency units of Table A-2 and reducing the number of constraints by substitution, the Ramsey problem can be written as:

$$
\begin{aligned}
&\underset{\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}}{Min}\ \underset{\{\mathbf{d}_t\}_{t=0}^{\infty}}{Max}\ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\left(\log c_t - \mu_n \tfrac{N_t^{1+\varphi}}{1+\varphi} + \tfrac{\beta}{1-\beta} \log g_Z \right) + \right.\right. \\
&+\lambda_{1,t} \left[y_t - c_t - k_{t+1} g_Z + (1-\delta) k_t - M_t - \tfrac{\gamma_Y}{2} \left(\Pi_{Y,t} - 1\right)^2 y_t - c_t^G y_t - \tfrac{\gamma_M}{2} \left(\Pi_{M,t} - 1\right)^2 M_t \right] + \\
&+\lambda_{2,t} \left(A_t^{\frac{1}{v}} \left[\tfrac{1}{p_t^M} MC_t (1-v) \right]^{\frac{1-v}{v}} k_t{}^{1-\alpha} N_t^{\alpha} - y_t \right) + \\
&+\lambda_{3,t} \left[\beta \left((1-\alpha) \tfrac{\mu_n N_{t+1}{}^{\varphi+1}}{\alpha k_{t+1}} + \tfrac{1-\delta}{c_{t+1}} \right) - \tfrac{g_Z}{c_t} \right] + \\
&+\lambda_{4,t} \left[(\theta_Y - 1) \tfrac{y_t}{c_t} - \theta_Y MC_t \tfrac{y_t}{c_t} + \gamma_Y (\Pi_{Y,t} - 1) \Pi_{Y,t} \tfrac{y_t}{c_t} - \beta \gamma_Y E_t \left(\Pi_{Y,t+1} - 1\right) \Pi_{Y,t+1} \tfrac{y_{t+1}}{c_{t+1}} \right] + \\
&+\lambda_{5,t} \left(\left[\tfrac{1}{p_t^M} MC_t (1-v) A_t \right]^{\frac{1}{v}} k_t{}^{1-\alpha} N_t^{\alpha} - M_t \right) + \\
&+\lambda_{6,t} \left(\tfrac{c_t \mu_n N_t{}^{\varphi+1}}{\alpha v y_t} - MC_t \right) + \\
&+\lambda_{7,t} \left(p_t^M \tfrac{\Pi_{P,t}}{\Pi_{M,t}} - p_{t-1}^M \right) + \\
&\left.\left. +\ \lambda_{8,t} \left(\begin{array}{c} (\theta_M - 1) M_t \frac{p_t^M}{c_t} g_Z - \theta_M M_t \frac{g_Z}{c_t} + \gamma_M \frac{g_Z}{c_t} \left(\Pi_{M,t} - 1\right) M_t \Pi_{M,t} + \\ -\beta E_t \frac{1}{c_{t+1}} \gamma_M \left(\Pi_{M,t+1} - 1\right) M_{t+1} \Pi_{M,t+1} \end{array} \right) \right] \right\},
\end{aligned}
$$

where $\{\mathbf{\Lambda}_t\}_{t=0}^{\infty} = \{\lambda_{1,t},\ \lambda_{2,t},\ \lambda_{3,t},\ \lambda_{4,t},\ \lambda_{5,t},\ \lambda_{6,t},\ \lambda_{7,t},\ \lambda_{8,t}\}_{t=0}^{\infty}$ denote the Lagrange multipliers of the constraints and $\{\mathbf{d}_t\}_{t=0}^{\infty} = \{c_t,\ k_{t+1}, M_t, MC_t,\ N_t,\ p_t^M,\ y_t,\ \Pi_{M,t},\ \Pi_{Y,t}\}_{t=0}^{\infty}$.

Proceeding as done in the previous section, we observe that, in the special case of flexible prices in the intermediate good sector, in steady state the first order condition with respect to Π_Y becomes $\lambda_1 \frac{\gamma_Y}{2} (\Pi_Y - 1)\, y = 0$, implying the optimality of zero inflation. In the general case, the first order conditions with respect to $\Pi_{Y,t}$ and $\Pi_{M,t}$, computed in steady state, can be combined to obtain:

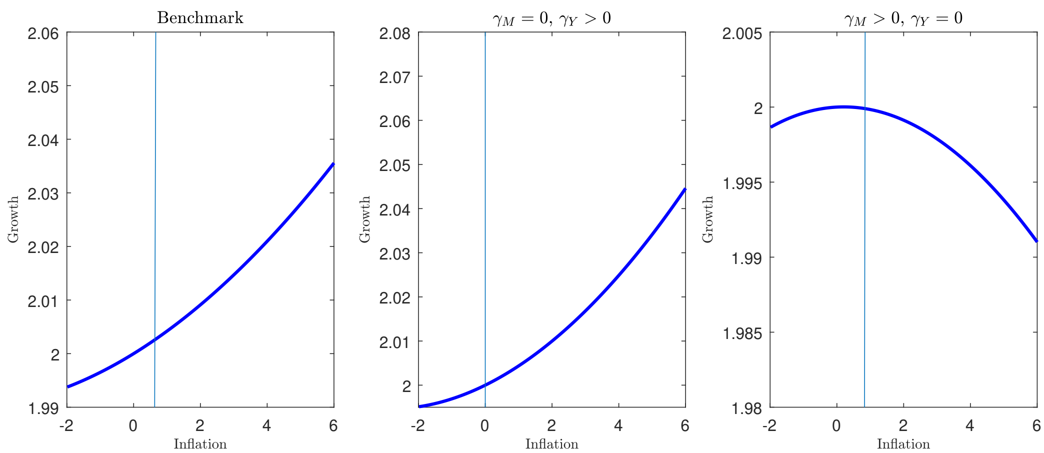
$$
-\lambda_1 \left(\gamma_M M + \gamma_Y Y\right) \left(\Pi_Y - 1\right) - \lambda_6 \gamma_M \left(\Pi_Y - 1\right) M \frac{g_Z}{c} + \lambda_{11} \gamma_M \frac{g_Z}{c} \left(1 - \frac{1}{g_Z}\right) \left(2\Pi_Y - 1\right) M = 0, \tag{A-8}
$$

where the last term representing the marginal benefits of intertemporal smoothing price changes does not vanish at zero inflation, opening up to the possibility of exploiting inflation as device to reduce markups.

Inflation and Growth

In this section we show the relationship between inflation and growth in the model of Table A-1. Using the baseline calibration illustrated in Section 3, we compute the steady state different inflation rates and for three different parametrizations of the nominal adjustment cost on price. Figure B-1 presents the results. The vertical continuous lines refer to the optimal trend inflation stemming from the Ramsey policy.

Figure B-1: Growth and Inflation in Steady State - Decentralized Equilibrium (Annual Rates %)



Note: The figure shows the relationship between long-run growth and trend inflation in the benchmark case (γ_M, $\gamma_Y > 0$), in the case of nominal rigidities only in the final good sector ($\gamma_M = 0$, $\gamma_Y > 0$) and in the case of nominal rigidities only in the intermediate good sector ($\gamma_M > 0$, $\gamma_Y = 0$). Growth and inflation are both expressed as annual rates in %. Vertical lines refer to the optimal trend inflation under Ramsey policy.

Online Appendix C

The Endogenous Growth Model under Calvo Pricing

In this Appendix we modify the NK model with endogenous growth model presented in the main text by modelling price rigidities *à la* Calvo instead that *à la* Rotemberg.

We assume that each firm i of the final good sector may reset its price with probability $1-\kappa_Y$ in any given period, independently of the time elapsed since the last adjustment. Let $\hat{P}_t$ denote the price set in period t by firms resetting their price in that period, then the optimal pricing condition can be written as:

$$\frac{\hat{P}_t}{P_t} = \frac{\theta_Y}{\theta_Y - 1} \frac{E_t \sum_{k=0}^{\infty} \kappa_Y^k \Lambda_{t,t+k}^R MC_{t+k} \left(\frac{P_{t+k}}{P_t}\right)^{\theta_Y} Y_{t+k}}{E_t \sum_{k=0}^{\infty} \kappa_Y^k \Lambda_{t,t+k}^R \left(\frac{P_{t+k}}{P_t}\right)^{\theta_Y - 1} Y_{t+k}}, \tag{A-9}$$

where $\Lambda_{t,t+k}^R = \beta^k \frac{\lambda_{t+k}}{\lambda_t}$. The optimal condition (A-9) can be simplified and then re-written as

$$\widehat{p}_t^Y = \frac{\theta_Y}{\theta_Y - 1} \frac{\Upsilon_t}{\Xi_t}, \tag{A-10}$$

where $\widehat{p}_t^Y = \frac{\hat{P}_t}{P_t}$ and

$$\Upsilon_t = \frac{y_t}{c_t} MC_t + \beta \kappa_Y E_t \Pi_{Y,t+1}^{\theta_Y} \Upsilon_{t+1}, \tag{A-11}$$

$$\Xi_t = \frac{y_t}{c_t} + \beta \kappa_Y E_t \Pi_{Y,t+1}^{\theta_Y - 1} \Xi_{+1}, \tag{A-12}$$

where we have used the fact that $\lambda_t = \frac{1}{C_t}$ and then expressed Y_t and C_t in efficiency units as $y_t = Y_t/Z_t$ and $c_t = C_t/Z_t$.

The aggregate price dynamics are described by

$$1 = \kappa_Y \Pi_{Y,t}^{\theta_Y - 1} + (1-\kappa_Y) \left(\widehat{p}_t^Y\right)^{1-\theta_Y}, \tag{A-13}$$

while price dispersion $\Delta_t^Y = \int_0^1 \left(\frac{P_{i,t}^Y}{P_t^Y}\right)^{-\theta_Y} di$ evolves as follows

$$\Delta_t^Y = (1-\kappa_Y) \left(\widehat{p}_t^Y\right)^{-\theta_Y} + \kappa_Y \Pi_{Y,t}^{\theta_Y} \Delta_{t-1}^Y. \tag{A-14}$$

Notably price dispersion is generated by Calvo price staggering and generates a wedge between aggregate output and factor inputs making aggregate production less efficient. In equilibrium, in fact, the aggregate production function expressed in efficiency units is

$$y_t = \frac{1}{\Delta_t^Y} A_t^{\frac{1}{v}} \left[\frac{1}{p_t^M} MC_t (1-v)\right]^{\frac{1-v}{v}} {k_t}^{1-\alpha} N_t^{\alpha}, \tag{A-15}$$

where $\Delta_t^Y \geq 1$. Clearly, the higher is price dispersion the more inputs are needed to produce a given level of output. Under Calvo pricing then trend inflation generates an efficiency loss.

However, trend inflation is able to reduce the average markup, decreasing the inefficiency loss due to monopolistic competition.

A similar problem is faced by the typical firm belonging to the intermediate good sector, where each firm j may reset its price with probability $1-\kappa_M$ in any given period, independently of the time elapsed since the last adjustment. Let $\hat{P}_t^M$ denote the price set in period t by firms able to revise their price in that period. The optimal pricing condition of the generic firm able to reset its price at time t can then be written as

$$\frac{\hat{P}_t^M}{P_t^M} = \frac{\theta_M}{\theta_M - 1} \frac{E_t \sum_{k=0}^{\infty} (\kappa_M \phi)^k \Lambda_{t,t+k}^R \left(\frac{P_{t+k}^M}{P_t^M} \right)^{\theta_M} M_{t+k}}{E_t \sum_{k=0}^{\infty} (\kappa_M \phi)^k \Lambda_{t,t+k}^R p_{t+k}^M \left(\frac{P_{t+k}^M}{P_t^M} \right)^{\theta_M - 1} M_{t+k}}, \tag{A-16}$$

where $P_t^M = \left[\frac{1}{Z_t} \int_0^{Z_t} \left(P_{j,t}^M \right)^{1-\theta_M} dj \right]^{\frac{1}{1-\theta_M}}$, while $M_t = G_t / Z_t^{\frac{\theta_M}{\theta_M - 1}}$ is the average level of production of intermediate good producers.

The above condition can be simplified and then re-written as follows

$$\widehat{p}_t^M = \frac{\theta_M}{\theta_M - 1} \frac{\Theta_t}{\Psi_t}, \tag{A-17}$$

where $\widehat{p}_t^M = \frac{\hat{P}_t^M}{P_t^M}$ and

$$\Theta_t = \frac{M_t}{c_t} + \beta \frac{\kappa_M \phi}{g_{Z,t+1}} E_t \Pi_{M,t+1}^{\theta_M} \Theta_{t+1}, \tag{A-18}$$

$$\Psi_t = \frac{M_t}{c_t} p_t^M + \beta \frac{\kappa_M \phi}{g_{Z,t+1}} E_t \Pi_{M,t+1}^{\theta_M - 1} \Psi_{t+1}. \tag{A-19}$$

Note that p_t^M denotes the average markup prevailing in the intermediate good sector.

The aggregate price dynamics are described by

$$1 = \kappa_M \Pi_{M,t}^{\theta_M - 1} + (1 - \kappa_M) \left(\widehat{p}_t^M \right)^{1-\theta_M}, \tag{A-20}$$

while price dispersion $\Delta_t^M = \frac{1}{Z_t} \int_0^{Z_t} \left(\frac{P_{j,t}^M}{P_t^M} \right)^{-\theta_M} dj$ evolves as follows

$$\Delta_t^M = (1 - \kappa_M) \left(\widehat{p}_t^M \right)^{-\theta_M} + \kappa_M \Pi_{M,t}^{\theta_M} \Delta_{t-1}^M, \tag{A-21}$$

where we have implicitly assumed that in each period the price dispersion on the fraction of obsolete varieties, new varieties and surviving varieties are equal.

Given the roundabout technology of the intermediate good sector, in equilibrium the quantity of final output used to produce intermediate goods amounts to

$$\int_0^{Z_t} Y_{j,t} dj = \int_0^{Z_t} M_{j,t} dj = M_t Z_t \Delta_t^M. \tag{A-22}$$

It follows that price dispersion in the intermediate good sector implies a waste of the final output.

The equilibrium conditions of the model under Calvo pricing are summarized in Table C-1, where all non-stationary variables are expressed in efficiency units.

The Calvo pricing gives rise to price dispersion in both sectors, creating an inefficiency loss on the supply side of the economy (i.e. in both sectors more inputs are needed to produce one unit of output). Price dispersion is eliminated at zero inflation.

The Exogenous Growth Model Growth Model under Calvo Pricing

The NK model with exogenous growth presented in the main text is modified by introducing price rigidities *à la* Calvo. By making the same assumptions formulated in the previous section with regard to the endogenous growth model, the equilibrium conditions of the exogenous growth model under Calvo pricing can be summarized as in Table C-2.

Optimal Trend Inflation under Calvo Pricing

Endogenous Growth

For the endogenous growth model under the Calvo pricing assumption the Lagrangian representation of the Ramsey problem can be written as

$$
\begin{aligned}
&\underset{\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}}{Min}\ \underset{\{\mathbf{d}_t\}_{t=0}^{\infty}}{Max}\ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\left(\log c_t - \mu_n \frac{N_t^{1+\varphi}}{1+\varphi} + \frac{\beta}{1-\beta} \log g_{Z,t+1} \right) + \right.\right. \\
&+\lambda_{1,t} \left[y_t - c_t - k_{t+1} g_{Z,t+1} + (1-\delta) k_t - s_t - c_t^G y_t - \Delta_t^M M_t \right] + \\
&+\lambda_{2,t} \left(A_t^{\frac{1}{v}} \left[\frac{1}{p_t^M} MC_t (1-v) \right]^{\frac{1-v}{v}} {k_t}^{1-\alpha} N_t^{\alpha} - y_t \Delta_t^Y \right) + \\
&+\lambda_{3,t} \left[\beta \left((1-\alpha) \frac{\mu_n {N_{t+1}}^{\varphi+1}}{\alpha k_{t+1}} + \frac{1-\delta}{c_{t+1}} \right) - \frac{g_{Z,t+1}}{c_t} \right] + \\
&+\lambda_{4,t} \left(\hat{\xi} s_t^{\varepsilon} + \phi - g_{Z,t+1} \right) + \\
&+\lambda_{5,t} \left(-\frac{1}{\hat{\xi}} s_t^{1-\varepsilon} \frac{g_{Z,t+1}}{c_t} + \beta E_t \frac{V_{t+1}}{c_{t+1}} \right) + \\
&+\lambda_{6,t} \left(\left(\frac{1}{p_t^M} MC_t (1-v) A_t \right)^{\frac{1}{v}} {k_t}^{1-\alpha} N_t^{\alpha} - M_t \right) + \\
&+\lambda_{7,t} \left(-V_t \frac{g_{Z,t+1}}{c_t} + (p_t^M - 1) M_t \frac{g_{Z,t+1}}{c_t} + \phi \beta E_t \frac{V_{t+1}}{c_{t+1}} \right) + \\
&+\lambda_{8,t} \left(\frac{c_t \mu_n {N_t}^{\varphi+1}}{\alpha v y_t} - MC_t \right) + \\
&+\lambda_{9,t} \left(p_t^M \frac{\Pi_{Y,t}}{\Pi_{M,t}} - p_{t-1}^M \right) + \\
&+\lambda_{10,t} \left(g_{Z,t+1} \Theta_t - \frac{g_{Z,t+1}}{c_t} M_t - \beta \kappa_M \phi E_t \Pi_{M,t+1}^{\theta_M} \Theta_{t+1} \right) + \\
&+\lambda_{11,t} \left(g_{Z,t+1} \Psi_t - \frac{g_{Z,t+1}}{c_t} M_t p_t^M - \beta \kappa_M \phi E_t \Pi_{M,t+1}^{\theta_M - 1} \Psi_{t+1} \right) + \\
&+\lambda_{12,t} \left(1 - \kappa_M \left(\Pi_{M,t} \right)^{\theta_M - 1} - (1-\kappa_M) \left(\widehat{p}_t^M \right)^{1-\theta_M} \right) + \\
&+\lambda_{13,t} \left(\Delta_t^M - (1-\kappa_M) \left(\widehat{p}_t^M \right)^{-\theta_M} - \kappa_M \Pi_{M,t}^{\theta_M} \Delta_{t-1}^M \right) + \\
&+\lambda_{14,t} \left(\widehat{p}_t^M - \frac{\theta_M}{\theta_M - 1} \frac{\Theta_t}{\Psi_t} \right) +
\end{aligned}
$$

$$+\lambda_{15,t}\left(\Upsilon_t-\tfrac{y_t}{c_t}MC_t-\beta\kappa_Y E_t\Pi_{Y,t+1}^{\theta_Y}\Upsilon_{t+1}\right)+$$
$$+\lambda_{16,t}\left(\Xi_t-\tfrac{y_t}{c_t}-\beta\kappa_Y E_t\Pi_{Y,t+1}^{\theta_Y-1}\Xi_{t+1}\right)+$$
$$+\lambda_{17,t}\left(1-\kappa_Y\left(\Pi_{Y,t}\right)^{\theta_Y-1}-(1-\kappa_Y)\left(\widehat{p}_t^Y\right)^{1-\theta_Y}\right)+$$
$$+\lambda_{18,t}\left(\Delta_t^Y-(1-\kappa_Y)\left(\widehat{p}_t^Y\right)^{-\theta_Y}-\kappa_Y\Pi_{Y,t}^{\theta_Y}\Delta_{t-1}^Y\right)+$$
$$+\lambda_{19,t}\left(\widehat{p}_t^Y-\tfrac{\theta_Y}{\theta_Y-1}\tfrac{\Upsilon_t}{\Xi_t}\right)\Big]\Big\},$$

where $\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}=\{\lambda_{1,t},\ \lambda_{2,t},\ \lambda_{3,t},\ \lambda_{4,t},\ \lambda_{5,t},\ \lambda_{6,t},\ \lambda_{7,t},\ \lambda_{8,t},\ \lambda_{9,t},\ \lambda_{10,t},\ \lambda_{11,t},\ \lambda_{12,t},\ \lambda_{13,t},\ \lambda_{14,t},\ \lambda_{15,t},\ \lambda_{16,t},\ \lambda_{17,t},\ \lambda_{18,t},\ \lambda_{19,t}\}_{t=0}^{\infty}$ denote the Lagrange multipliers of the constraints and $\{\mathbf{d}_t\}_{t=0}^{\infty}=\{c_t,\ g_{Z,t+1},\ k_{t+1}, M_t, MC_t,\ N_t,\ p_t^M, s_t, V_t, y_t,\ \Pi_{M,t}, \Pi_{Y,t}, \Delta_t^Y, \widehat{p}_t^Y, \Xi_t, \Upsilon_t, \Delta_t^M, \widehat{p}_t^M, \Theta_t, \Psi_t\}_{t=0}^{\infty}$. Observe that also in this case we can omit the consumption Euler equation from the set of constraints.

We now show that in the special case in which $\kappa_M=0$, that is under flexible prices in the intermediate good sector, the optimal level of trend inflation is zero. We proceed as in Schmitt-Grohé and Uribe (2010) and show that in steady state the first-order conditions with respect to $\Pi_{Y,t}$ and $\widehat{p}_t^Y$ are not independent when $\Pi_{Y,t}=1$.

Consider the first-order conditions with respect to $\Pi_{Y,t}$, $\widehat{p}_t^Y$, Υ_t and Ξ_t assuming that $\kappa_M=0$, so that $\Pi_{M,t}=\Pi_{Y,t}$ and $p_t^M=\frac{\theta_M}{\theta_M-1}$:

$$\begin{gathered}-\lambda_{15,t-1}\theta_Y\Pi_{Y,t}^{\theta_Y-1}\Upsilon_t-\lambda_{16,t-1}\left(\theta_Y-1\right)\Pi_{Y,t}^{\theta_Y-2}\Xi_t+\\-\lambda_{17,t}\left(\theta_Y-1\right)\left(\Pi_{Y,t}\right)^{\theta_Y-2}-\lambda_{18,t}\theta_Y\left(\Pi_{Y,t}\right)^{\theta_Y-1}\Delta_{t-1}^Y=0,\end{gathered}\tag{A-23}$$

$$\lambda_{17,t}\left(\theta_Y-1\right)\left(1-\kappa_Y\right)\left(\widehat{p}_t^Y\right)^{-\theta_Y}+\lambda_{18,t}\theta_Y(1-\kappa_Y)\left(\widehat{p}_t^Y\right)^{-\theta_Y-1}+\lambda_{19,t}\widehat{p}_t^Y=0,\tag{A-24}$$

$$\lambda_{15,t}-\lambda_{15,t-1}\kappa_Y\Pi_{Y,t}^{\theta_Y}-\lambda_{19,t}\frac{\theta_Y}{\theta_Y-1}\frac{1}{\Xi_t}=0,\tag{A-25}$$

$$\lambda_{16,t}-\lambda_{16,t-1}\kappa_Y\Pi_{Y,t}^{\theta_Y-1}+\lambda_{19,t}\frac{\theta_Y}{\theta_Y-1}\frac{\Upsilon_t}{\Xi_t^2}=0.\tag{A-26}$$

Assume now that in steady state trend inflation is zero, that is $\Pi_Y=1$, thus $\widehat{p}^Y=1=\frac{\theta_Y}{\theta_Y-1}\frac{\Upsilon}{\Xi}$ and $\Delta^Y=1$. The above conditions boil down to

$$\lambda_{15}\theta_Y\Upsilon+\lambda_{16}\left(\theta_Y-1\right)\Xi+\lambda_{17}\left(\theta_Y-1\right)+\lambda_{18}\theta_Y=0,\tag{A-27}$$

$$\lambda_{17}\left(\theta_Y-1\right)\left(1-\kappa_Y\right)+\lambda_{18}\theta_Y(1-\kappa_Y)+\lambda_{19}=0,\tag{A-28}$$

$$\lambda_{15}-\lambda_{15}\kappa_Y-\lambda_{19}\frac{\theta_Y}{\theta_Y-1}\frac{1}{\Xi}=0,\tag{A-29}$$

$$\lambda_{16}-\lambda_{16}\kappa_Y+\lambda_{19}\frac{\theta_Y}{\theta_Y-1}\frac{\Upsilon}{\Xi^2}=0.\tag{A-30}$$

Conditions (A-29) and (A-30) can then be combined to obtain:

$$\begin{aligned}\lambda_{16}&=-\lambda_{15}\frac{\theta_Y-1}{\theta_Y},\\\lambda_{19}&=\lambda_{15}\left(1-\kappa_Y\right)\Upsilon.\end{aligned}$$

Using the above results into (A-27) and (A-28) it can be easily shown that both conditions boil down to $\Upsilon\lambda_{15}+\lambda_{17}(\theta_Y-1)+\lambda_{18}\theta_Y=0$, therefore when in the intermediate good sector prices are flexible zero trend inflation is optimal.

Consider now the case of flexible prices in the final good sector (i.e. $\kappa_Y=0$), but of sticky prices in the intermediate good sector. The first-order conditions with respect to $\Pi_{M,t}$, $\widehat{p}_t^M$, Θ_t and Ψ_t are, respectively:

$$
\begin{gathered}
-\lambda_{10,t-1}\phi\theta_M\Pi_{M,t}^{\theta_M-1}\Theta_t-\lambda_{11,t-1}\phi\left(\theta_M-1\right)\Pi_{M.t}^{\theta_M-2}\Psi_t+ \\
-\lambda_{12,t}\left(\theta_M-1\right)\Pi_{M,t}^{\theta_M-2}-\lambda_{13,t}\theta_M\Pi_{M,t}^{\theta_M-1}\Delta_{t-1}^M=0,
\end{gathered}
\tag{A-31}
$$

$$
\lambda_{12,t}(1-\kappa_M)(\theta_M-1)\left(\widehat{p}_t^M\right)^{-\theta_M}+\lambda_{13,t}\theta_M(1-\kappa_M)\left(\widehat{p}_t^M\right)^{-\theta_M-1}+\lambda_{14,t}=0, \tag{A-32}
$$

$$
\lambda_{10,t}g_{Z,t+1}-\lambda_{10,t-1}\kappa_M\phi\Pi_{M,t}^{\theta_M}-\lambda_{14,t}\frac{\theta_M}{\theta_M-1}\frac{1}{\Psi_t}=0, \tag{A-33}
$$

$$
\lambda_{11,t}g_{Z,t+1}-\lambda_{11,t-1}\kappa_M\phi\Pi_{M,t}^{\theta_M-1}+\lambda_{14,t}\frac{\theta_M}{\theta_M-1}\frac{\Theta_t}{\Psi_t^2}=0. \tag{A-34}
$$

Assume that in steady state trend inflation is zero, that is $\Pi_M=1$, the $\widehat{p}^M=1=\frac{\theta_M}{\theta_M-1}\frac{\Theta}{\Psi}$ and $\Delta^M=1$. The above conditions become:

$$
-\lambda_{10}\phi\theta_M\Theta-\lambda_{11}\phi\left(\theta_M-1\right)\Psi-\lambda_{12}\left(\theta_M-1\right)-\lambda_{13}\theta_M=0, \tag{A-35}
$$

$$
\lambda_{12}(1-\kappa_M)(\theta_M-1)+\lambda_{13}\theta_M(1-\kappa_M)+\lambda_{14}=0, \tag{A-36}
$$

$$
\lambda_{10}g_Z-\lambda_{10}\kappa_M\phi-\lambda_{14}\frac{\theta_M}{\theta_M-1}\frac{1}{\Psi}=0, \tag{A-37}
$$

$$
\lambda_{11}g_Z-\lambda_{11}\kappa_M\phi+\lambda_{14}\frac{\theta_M}{\theta_M-1}\frac{\Theta}{\Psi^2}=0. \tag{A-38}
$$

The last two conditions can be combined to obtain:

$$
\begin{aligned}
\lambda_{11}&=-\lambda_{10}\frac{\theta_M-1}{\theta_M}, \\
\lambda_{14}&=\lambda_{10}\left(g_Z-\kappa_M\phi\right)\Theta
\end{aligned}
$$

Substituting these results into (A-35) and (A-36) it can be easily verified that these conditions are independent, implying that zero trend inflation cannot be the optimal solution when prices are sticky in the intermediate good sector.

Exogenous Growth

For the exogenous growth model under the Calvo pricing assumption the Lagrangian representation of the Ramsey problem can be expressed as

$$
\begin{aligned}
&\underset{\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}}{Min}\ \underset{\{\mathbf{d}_t\}_{t=0}^{\infty}}{Max}\ E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\left(\log c_t-\mu_n\frac{N_t^{1+\varphi}}{1+\varphi}+\frac{\beta}{1-\beta}\log g_Z\right)+\right.\right. \\
&+\lambda_{1,t}\left[y_t-c_t-k_{t+1}g_Z+(1-\delta)k_t-c_t^Gy_t-\Delta_t^MM_t\right]+
\end{aligned}
$$

$$+\lambda_{2,t}\left(A_t^{\frac{1}{v}}\left[\frac{1}{p_t^M}MC_t(1-v)\right]^{\frac{1-v}{v}}{k_t}^{1-\alpha}N_t^{\alpha}-y_t\Delta_t^Y\right)+$$
$$+\lambda_{3,t}\left[\beta\left((1-\alpha)\frac{\mu_n {N_{t+1}}^{\varphi+1}}{\alpha k_{t+1}}+\frac{1-\delta}{c_{t+1}}\right)-\frac{g_{Z,t+1}}{c_t}\right]+$$
$$+\lambda_{4,t}\left(\left(\frac{1}{p_t^M}MC_t(1-v)A_t\right)^{\frac{1}{v}}{k_t}^{1-\alpha}N_t^{\alpha}-M_t\right)+$$
$$+\lambda_{5,t}\left(\frac{c_t\mu_n {N_t}^{\varphi+1}}{\alpha v y_t}-MC_t\right)+$$
$$+\lambda_{6,t}\left(p_t^M\frac{\Pi_{Y,t}}{\Pi_{M,t}}-p_{t-1}^M\right)+$$
$$+\lambda_{7,t}\left(g_Z\Theta_t-\frac{g_Z}{c_t}M_t-\beta\kappa_M\phi E_t\Pi_{M,t+1}^{\theta_M}\Theta_{t+1}\right)+$$
$$+\lambda_{8,t}\left(g_Z\Psi_t-\frac{g_Z}{c_t}p_t^M M_t-\beta\kappa_M\phi E_t\Pi_{M,t+1}^{\theta_M-1}\Psi_{t+1}\right)+$$
$$+\lambda_{9,t}\left(1-\kappa_M\left(\Pi_{M,t}\right)^{\theta_M-1}-(1-\kappa_M)\left(\widehat{p}_t^M\right)^{1-\theta_M}\right)+$$
$$+\lambda_{10,t}\left(\Delta_t^M-(1-\kappa_M)\left(\widehat{p}_t^M\right)^{-\theta_M}-\kappa_M\Pi_{M,t}^{\theta_M}\Delta_{t-1}^M\right)+$$
$$+\lambda_{11,t}\left(\widehat{p}_t^M-\frac{\theta_M}{\theta_M-1}\frac{\Theta_t}{\Psi_t}\right)+$$
$$+\lambda_{12,t}\left(\Upsilon_t-\frac{1}{c_t}MC_t y_t-\beta\kappa_Y E_t\Pi_{Y,t+1}^{\theta_Y}\Upsilon_{t+1}\right)+$$
$$+\lambda_{13,t}\left(\Xi_t-\frac{1}{c_t}y_t-\beta\kappa_Y E_t\Pi_{Y,t+1}^{\theta_Y-1}\Xi_{t+1}\right)+$$
$$+\lambda_{15,t}\left(1-\kappa_Y\Pi_{Y,t}^{\theta_Y-1}-(1-\kappa_Y)\left(\widehat{p}_t^Y\right)^{1-\theta_Y}\right)+$$
$$+\lambda_{15,t}\left(\Delta_t^Y-(1-\kappa_Y)\left(\widehat{p}_t^Y\right)^{-\theta_Y}-\kappa_Y\Pi_{Y,t}^{\theta_Y}\Delta_{t-1}^Y\right)+$$
$$+\,\lambda_{16,t}\left(\widehat{p}_t^Y-\frac{\theta_Y}{\theta_Y-1}\frac{\Upsilon_t}{\Xi_t}\right)\Big]\Big\}$$

where $\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}=\{\lambda_{1,t},\ \lambda_{2,t},\ \lambda_{3,t},\ \lambda_{4,t},\ \lambda_{5,t},\ \lambda_{6,t},\ \lambda_{7,t},\ \lambda_{8,t},\ \lambda_{9,t},\ \lambda_{10,t},\ \lambda_{11,t},\ \lambda_{12,t},\ \lambda_{13,t},\ \lambda_{14,t},\ \lambda_{15,t},\ \lambda_{16,t}\}_{t=0}^{\infty}$ denote the Lagrange multipliers of the constraints and $\{\mathbf{d}_t\}_{t=0}^{\infty}=\{c_t,\ k_{t+1},\ M_t,\ MC_t,\ N_t,\ p_t^M,\ y_t,\ \Pi_{M,t},\Pi_{Y,t},\ \Delta_t^Y,\ \widehat{p}_t^Y,\ \Xi_t,\ \Upsilon_t,\ \Delta_t^M,\ \widehat{p}_t^M,\ \Theta_t,\ \Psi_t\}_{t=0}^{\infty}$.

Proceeding as done in the previous section, it is straightforward to show that when prices are sticky only in the final good sector, optimal trend inflation will be equal to zero. On the other hand, when prices are sticky in the intermediate good sector zero inflation cannot be the optimal solution in steady state.

Optimal Trend Inflation: Numerical Results

Given the baseline parametrization of Section 3, we compute the steady state solution of the Ramsey problem and quantify the optimal long-run inflation rate for both growth models under the Calvo pricing hypothesis. For a degree of price stickiness consistent with the one assumed under Rotemberg pricing, that is $\kappa_Y=\kappa_M=0.75$, we find that the Ramsey optimal inflation rate is 0.144% per year in the endogenous growth model and 0.082% in the exogenous growth model.

Figure C-1 plots optimal trend inflation for different parametrizations of both models. We observe that, consistently with the findings obtained under Rotemberg pricing, a higher degree of price rigidities in the intermediate good sector implies a higher trend inflation, while a higher degree of nominal rigidities in the final good sector implies a lower optimal inflation rate.

Consider now the role played by elasticity ε that determines the marginal return of R&D spending. We observe that under Calvo pricing the optimal level of trend inflation declines with it, contrary to the result of obtained under costly price adjustment. In this case the Ramsey planner will use inflation as a device to reduce average markups and increase the scale of production especially when ε is low. By contrast, when ε is high it will be optimal to reduce price dispersion and keep markups high. Finally, we observe that with staggered prices the effects of the obsolescence rate on the optimal inflation rate are negligible.

On the one hand our robustness check confirms our main result regarding the optimality of positive trend inflation, on the other hand the kind of inefficiency induced by Calvo pricing, through price dispersion, seems to interact differently with the underlying market structure and growth engine. The difference between the two pricing schemes is worth to be investigated, but we leave this aspect for future research.

Inflation and Growth under Calvo Pricing

In this appendix we show the steady state relationship between inflation and growth in the model of Table C-1 for three different parametrizations of nominal rigidities. Figure C-2 presents the results. The vertical continuous lines refer to the optimal trend inflation stemming from the Ramsey policy under Calvo pricing. We observe that contrary to the results obtained under costly price adjustment, the relationship between growth and inflation is hump shaped in the presence of nominal rigidities in the final good sector.

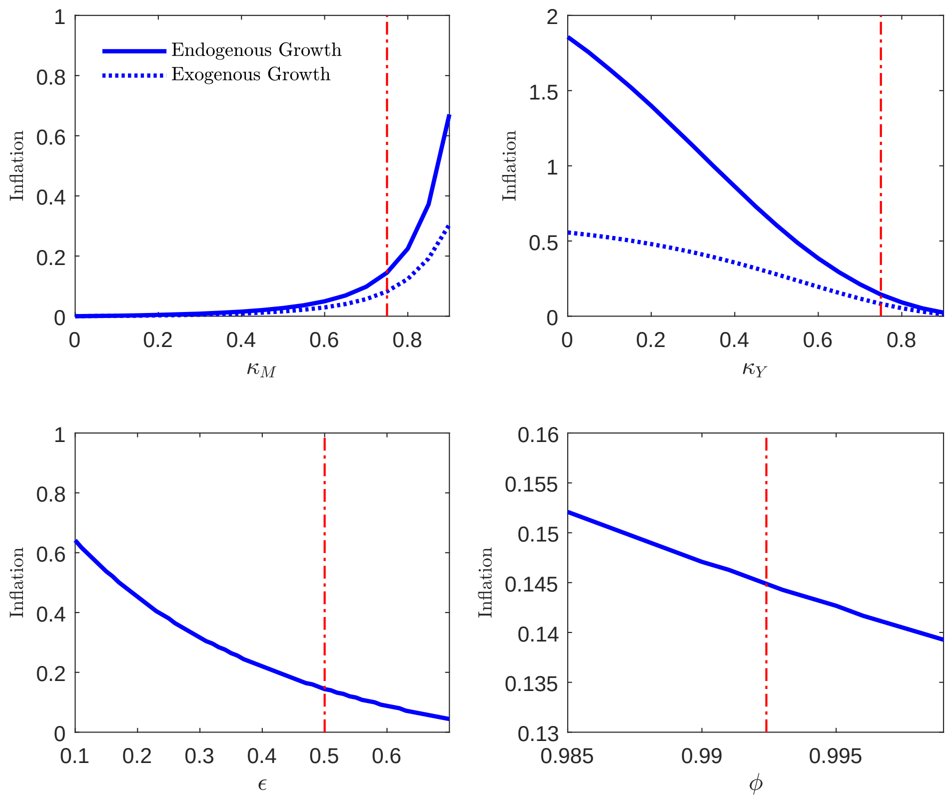
Table C-1: Endogenous Growth Model under Calvo Pricing

$$y_t = c_t + i_t + s_t + c_t^G y_t + \Delta_t^M M_t$$
$$y_t = \frac{1}{\Delta_t^Y} A_t^{\frac{1}{v}} \left[\frac{1}{p_t^M} MC_t (1-v)\right]^{\frac{1-v}{v}} k_t^{1-\alpha} N_t^{\alpha}$$
$$M_t = \left[\frac{1}{p_t^M} MC_t (1-v) A_t\right]^{\frac{1}{v}} k_t^{1-\alpha} N_t^{\alpha}$$
$$w_t = \alpha v MC_t \frac{y_t}{N_t}$$
$$R_t^K = (1-\alpha)\, v MC_t \frac{y_t}{k_t}$$
$$k_{t+1} g_{Z,t+1} = (1-\delta)\, k_t + i_t$$
$$E_t \frac{\Lambda_{t,t+1}^R}{\Pi_{Y,t+1}} = \frac{1}{R_t}$$
$$1 = E_t \Lambda_{t,t+1}^R \left(R_{t+1}^k + 1 - \delta\right)$$
$$\mu_n N_t^{\varphi} c_t = w_t$$
$$\widehat{p}_t^Y = \frac{\theta_Y}{\theta_Y - 1} \frac{\Upsilon_t}{\Xi_t}$$
$$\Upsilon_t = \frac{y_t}{c_t} MC_t + \beta \kappa_Y E_t \Pi_{Y,t+1}^{\theta_Y} \Upsilon_{t+1}$$
$$\Xi_t = \frac{y_t}{c_t} + \beta \kappa_Y E_t \Pi_{Y,t+1}^{\theta_Y - 1} \Xi_{+1}$$
$$1 = \kappa_Y \Pi_{Y,t}^{\theta_Y - 1} + (1-\kappa_Y) \left(\widehat{p}_t^Y\right)^{1-\theta_Y}$$
$$\Delta_t^Y = (1-\kappa_Y) \left(\widehat{p}_t^Y\right)^{-\theta_Y} + \kappa_Y \Pi_{Y,t}^{\theta_Y} \Delta_{t-1}^Y$$
$$g_{Z,t+1} = \xi_t s_t + \phi$$
$$\xi_t = \hat{\xi}\, (1/s_t)^{1-\varepsilon}$$
$$V_t = (p_t^M - 1) M_t - \frac{\gamma_M}{2} \left(\Pi_{M,t} - 1\right)^2 M_t + \phi E_t \Lambda_{t,t+1}^R V_{t+1}$$
$$p_t^M = p_{t-1}^M \frac{\Pi_{M,t}}{\Pi_{P,t}}$$
$$\widehat{p}_t^M = \frac{\theta_M}{\theta_M - 1} \frac{\Theta_t}{\Psi_t}$$
$$\Theta_t = \frac{M_t}{c_t} + \beta \frac{\kappa_M \phi}{g_{Z,t+1}} E_t \Pi_{M,t+1}^{\theta_M} \Theta_{t+1}$$
$$\Psi_t = \frac{M_t}{c_t} p_t^M + \beta \frac{\kappa_M \phi}{g_{Z,t+1}} E_t \Pi_{M,t+1}^{\theta_M - 1} \Psi_{t+1}$$
$$1 = \kappa_M \Pi_{M,t}^{\theta_M - 1} + (1-\kappa_M) \left(\widehat{p}_t^M\right)^{1-\theta_M}$$
$$\Delta_t^M = (1-\kappa_M) \left(\widehat{p}_t^M\right)^{-\theta_M} + \kappa_M \Pi_{M,t}^{\theta_M} \Delta_{t-1}^M$$
$$1/\xi_t = E_t \left(\Lambda_{t,t+1}^R V_{t+1}\right)$$
$$\Lambda_{t,t+1}^R = \beta \frac{c_t}{g_{Z,t+1} c_{t+1}}$$

Table C-2: Exogenous Growth Model under Calvo Pricing

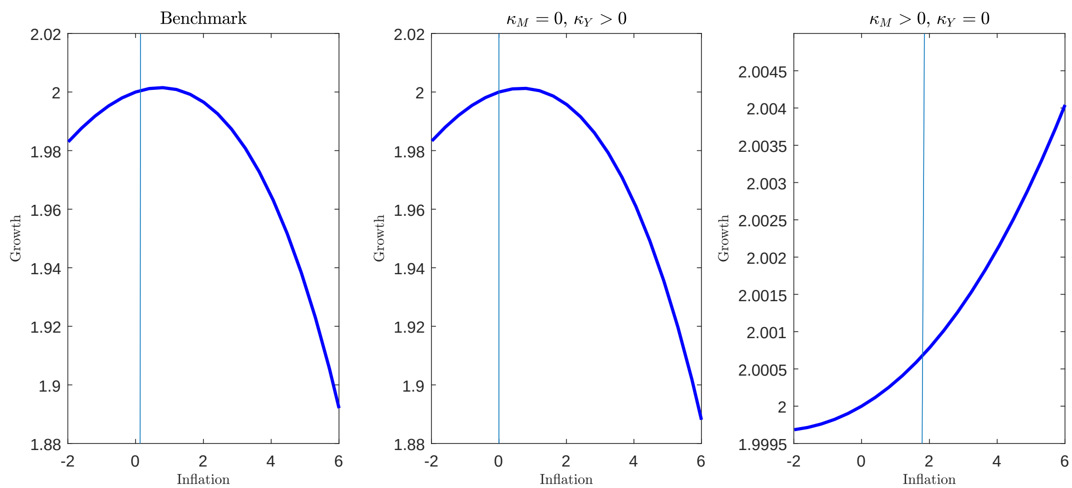
$$y_t = c_t + i_t + M_t + s_t + c_t^G y_t$$
$$y_t = A_t^{\frac{1}{v}} \left[\frac{1}{p_t^M} MC_t(1-v)\right]^{\frac{1-v}{v}} k_t^{1-\alpha} N_t^{\alpha}$$
$$M_t = \left[\frac{1}{p_t^M} MC_t(1-v) A_t\right]^{\frac{1}{v}} k_t^{1-\alpha} N_t^{\alpha}$$
$$w_t = \alpha v MC_t \frac{y_t}{N_t}$$
$$R_t^K = (1-\alpha)\, v MC_t \frac{y_t}{k_t}$$
$$k_{t+1} g_Z = (1-\delta)\, k_t + i_t$$
$$E_t \frac{\Lambda_{t,t+1}^R}{\Pi_{Y,t+1}} = \frac{1}{R_t}$$
$$1 = E_t \Lambda_{t,t+1}^R \left(R_{t+1}^k + 1 - \delta\right)$$
$$\mu_n N_t^{\varphi} c_t = w_t$$
$$\widehat{p}_t^Y = \frac{\theta_Y}{\theta_Y - 1} \frac{\Upsilon_t}{\Xi_t}$$
$$\Upsilon_t = \frac{y_t}{c_t} MC_t + \beta \kappa_Y E_t \Pi_{Y,t+1}^{\theta_Y} \Upsilon_{t+1}$$
$$\Xi_t = \frac{y_t}{c_t} + \beta \kappa_Y E_t \Pi_{Y,t+1}^{\theta_Y - 1} \Xi_{+1}$$
$$1 = \kappa_Y \Pi_{Y,t}^{\theta_Y - 1} + (1-\kappa_Y) \left(\widehat{p}_t^Y\right)^{1-\theta_Y}$$
$$\Delta_t^Y = (1-\kappa_Y) \left(\widehat{p}_t^Y\right)^{-\theta_Y} + \kappa_Y \Pi_{Y,t}^{\theta_Y} \Delta_{t-1}^Y$$
$$p_t^M = p_{t-1}^M \frac{\Pi_{M,t}}{\Pi_{P,t}}$$
$$\widehat{p}_t^M = \frac{\theta_M}{\theta_M - 1} \frac{\Theta_t}{\Psi_t}$$
$$\Theta_t = \psi_t M_t + \beta \frac{\kappa_M}{g_Z} E_t \Pi_{M,t+1}^{\theta_M} \Theta_{t+1}$$
$$\Psi_t = \psi_t p_t^M M_t + \frac{\beta \kappa_M}{g_Z} E_t \Pi_{M,t+1}^{\theta_M - 1} \Psi_{t+1}$$
$$1 = \kappa_M \Pi_{M,t}^{\theta_M - 1} + (1-\kappa_M) \left(\widehat{p}_t^M\right)^{1-\theta_M}$$
$$\Delta_t^M = (1-\kappa_M) \left(\widehat{p}_t^M\right)^{-\theta_M} + \kappa_M \Pi_{M,t}^{\theta_M} \Delta_{t-1}^M$$
$$\Lambda_{t,t+1}^R = \beta \frac{c_t}{g_Z c_{t+1}}$$

Figure C-1: Annual Optimal Trend Inflation for Different Model Parametrizations under Calvo Pricing (%)



Note: The figure shows optimal trend inflation (the annual inflation rate in %) in the two growth models for different parametrizations under the assumption of Calvo pricing, where κ_M is the degree of nominal rigidities in the intermediate good sector, κ_Y is the degree of nominal rigidities in the final good sector, ϵ measures the elasticity of new intermediate goods with respect to R&D and ϕ is the survival rate of intermediate good producers. Vertical lines refer to the baseline calibration.

Figure C-2: Growth and Inflation in Steady State under Calvo Pricing- Decentralized Equilibrium (Annual Rates %)



Note: The figure shows the relationship between long-run growth and trend inflation in the benchmark case (κ_M, $\kappa_Y > 0$), in the case of nominal rigidities only in the final good sector ($\kappa_M = 0$, $\kappa_Y > 0$) and in the case of nominal rigidities only in the intermediate good sector ($\kappa_M > 0$, $kappa_Y = 0$). Growth and inflation are both expressed as annual rates in %. Vertical lines refer to the optimal trend inflation under Ramsey policy.

Online Appendix D

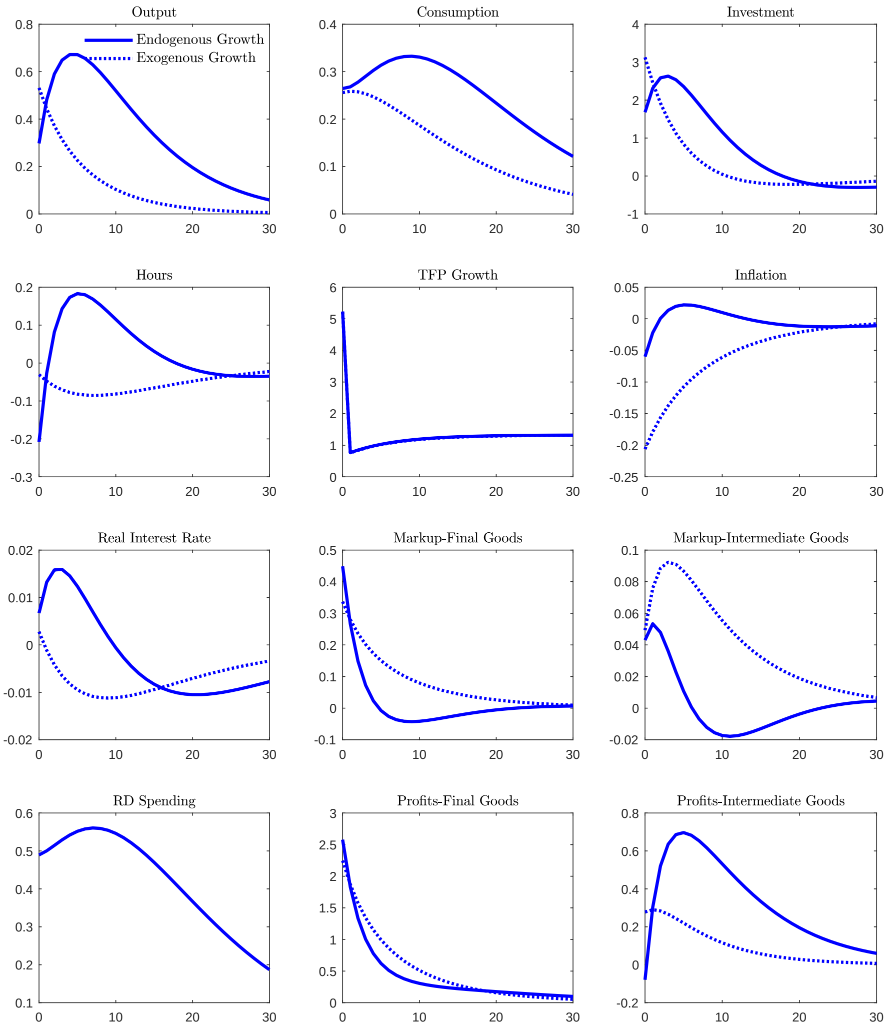
Dynamics under Taylor Rule

In this Appendix we show the impulse response functions to technology and public spending shocks under the assumption that monetary policy is conducted according to a standard Taylor rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\iota_R} \left[\left(\frac{\Pi_{Y,t}}{\Pi_Y}\right)^{\iota_{\Pi_Y}} \left(\frac{Y_t}{g_Z Y_{t-1}}\right)^{\iota_{g_Y}}\right]^{1-\iota_R}, \tag{A-39}$$

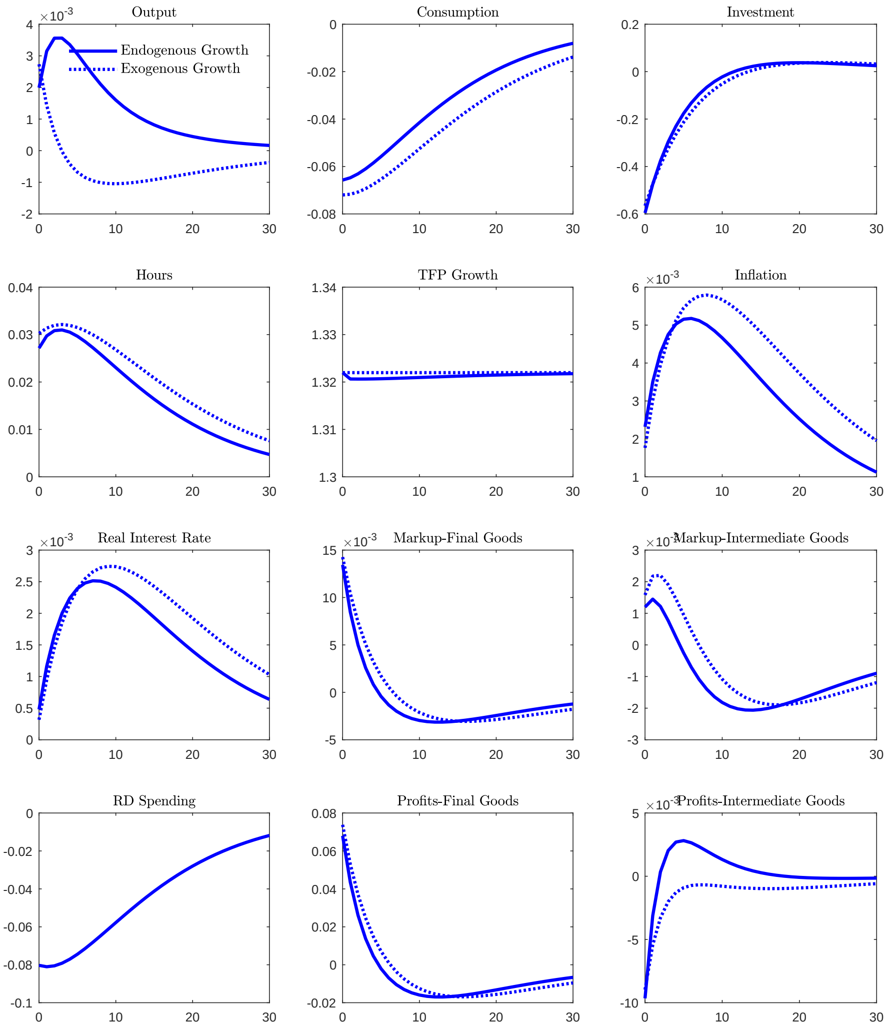
where variables without the time subscript refer to the deterministic balanced growth path under the Ramsey policy, while ι_R, ι_{Π_Y} and ι_{g_Y} are policy parameters. We set $\iota_{\Pi_Y} = 1.5$, $\iota_{g_Y} = 0.25$ and $\iota_R = 0$. See Figures D-1 and D-2.

Figure D-1: Impulse Responses to a 1% Technology Shock under a Benchmark Taylor Rule



Note: The figure shows the impulse response to a shock on A_t under a benchmark Taylor rule. All results are reported as percentage deviations from the steady state, except for inflation, nominal and real interest rates, which are expressed as percentage-point deviations and for the TFP growth which is expressed in annualized rates. Continuous lines show the impulse response functions in the endogenous growth model, while dotted lines refer to the exogenous growth model.

Figure D-2: Impulse Responses to a 1% Public Spending Shock under a Benchmark Taylor Rule



The figure shows the impulse response to a shock on c_t^G under a benchmark Taylor rule. All results are reported as percentage deviations from the steady state, except for inflation, nominal and real interest rates, which are expressed as percentage-point deviations and for the TFP growth which is expressed in annualized rates. Continuous lines show the impulse response functions of the Ramsey plan in the endogenous growth model, while dotted lines refer to the Ramsey plan in the exogenous growth model.

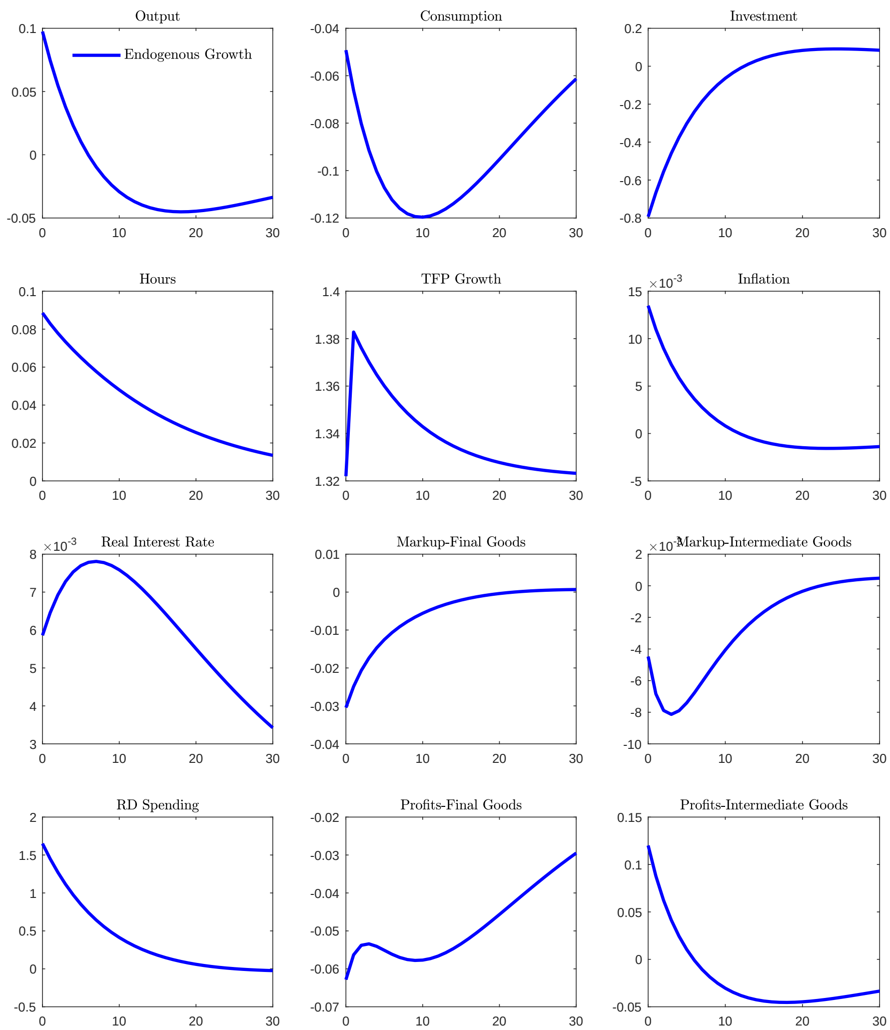
Online Appendix E

In this Appendix we explore the optimal dynamic response to R&D productivity shocks in the endogenous growth model. In particular, we assume that the coefficient $\hat{\xi}$ in (11) is time varying and follows a process of the form $\log \hat{\xi}_t = (1 - \rho_{\hat{\xi}}) \log \hat{\xi} + \rho_{\hat{\xi}} \log \hat{\xi}_{t-1} + \varepsilon_t^{\hat{\xi}}$, with $0 < \rho_{\hat{\xi}} < 1$, $\varepsilon_t^{\hat{\xi}} \sim i.i.d.N(0, \sigma_{\hat{\xi}}^2)$. Figure E-1 plots the dynamic responses to a one percent positive shock on R&D productivity under Ramsey monetary policy assuming a high autocorrelation of the shock, namely $\rho_{\hat{\xi}} = 0.9$. Also in this case the Ramsey planner tolerates temporary deviations of the price level from its optimal long-run trend. In the final good sector markups and profits decline sharply, while in the intermediate good sector profits increase. By using monetary policy the Ramsey planner is able to sustain the positive effects on output and therefore to increase the market size for innovation and innovation incentives during the periods of higher R&D productivity. However, the response of consumption and output to this shock deserves an explanation.

We observe that consumption declines, despite this shock generates an expansion of output. Recalling that we represent consumption in efficiency units (i.e. $c_t = C_t/Z_t$), like all non-stationary variables of the model, consider Figure E-2, where we plot the impulse response of the growth rates of intermediate goods, TFP, output and consumption, in response to the positive R&D productivity shock. On impact the model generates a decline of the growth rate of consumption, despite the sharp expansion in the growth rate of new varieties in the intermediate good sector that drives the TFP growth rate upward and so output growth. In the absence of any adjustment costs in the R&D sector, the TFP reaches a peak right after the positive shock. In these circumstances the market conditions push households to slightly reduce consumption and finance R&D spending on impact. A sort of substitution effect then prevails over the income effect.[1] However, already in the following period consumption growth rate is above its balanced growth path and only slowly converges to it. This explain the dynamics of consumption in efficiency units of Figure E-1. Output growth, instead, increases on impact, driven upward by the immediate increase in the TFP and by the expansion of hours. However, already in the second period, we note a slow down of the output growth rate. During the adjustment process we observe that output growth slowly converges toward its balanced growth path from above.

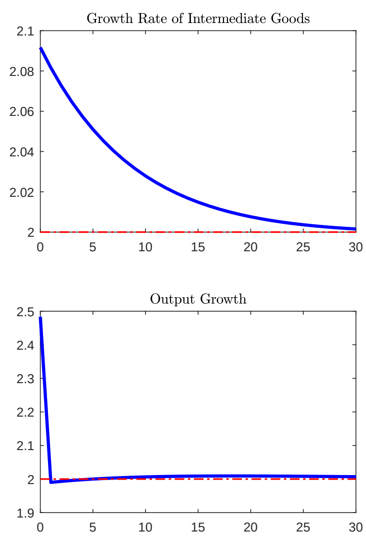
[1] In the presence of adjustment costs in the R&D sector, instead, the increase in the R&D spending would be more gradual and so the TFP growth reaction to this shock. Thus, on impact, the response of consumption would be driven by a positive income effect, rather then by a negative substitution effect. This is in fact the kind of dynamics we observe in response to similar shocks hitting the R&D sector in medium scale endogenous growth models featuring Schumpeterian mechanisms and adjustment costs in R&D investments. See e.g. Cozzi et al. (2017) and Pinchetti (2017).

Figure E-1: Impulse Responses to a 1% R&D Productivity Shock



Note: The figure shows the impulse response under Ramsey monetary policy to a shock on $\hat{\xi}_t$. All results are reported as percentage deviations from the steady state, except for inflation, nominal and real interest rates, which are expressed as percentage-point deviations and for the TFP growth which is expressed in annualized rates.

Figure E-2: Impulse Responses to a 1% R&D Productivity Shock - Growth Rates (Annual Rates %)



Note: The figure shows the impulse response under Ramsey monetary policy to a shock on $\hat{\xi}_t$. All growth rates are annualized. Horizontal lines refer to the balanced-growth-path growth rates.

Online Appendix F

In Table 2 the welfare gains of a particular operational monetary rule relatively to a standard Taylor rule are computed as follows. Let TR denote the Taylor rule regime (with $\iota_{\Pi_Y} = 1.5$, $\iota_{g_Y} = 0.25$ and $\iota_R = 0$), that is our reference policy regime, and AR denote an alternative monetary regime (any of the other rules considered in Table 1).

Following Schmitt-Grohé and Uribe (2007), our measure of welfare is the *conditional expectation* of lifetime utility at time zero, that from (A-3) is

$$\upsilon_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \mu_n \frac{N_t^{1+\varphi}}{1+\varphi} + \frac{\beta}{1-\beta} \log g_{Z,t+1} \right). \tag{F-1}$$

where $U(c_t, N_t, g_{Z,t+1}) = \log c_t - \mu_n \frac{N_t^{1+\varphi}}{1+\varphi} + \frac{\beta}{1-\beta} \log g_{Z,t+1}$. We also assume that at $t = 0$ the economy is on its balanced growth path, that is all stationary variables of the economy are equal to their respective steady-state values. We define υ_0^{TR} the welfare associated to reference regime TR and υ_0^{AR} the welfare associated to the alternative regime TR. Then we have:

$$\upsilon_0^{TR} = E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t^{TR}, N_t^{TR}, g_{Z,t+1}^{TR}\right), \tag{F-2}$$

$$\upsilon_0^{AR} = E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t^{AR}, N_t^{AR}, g_{Z,t+1}^{AR}\right), \tag{F-3}$$

Let χ denote the welfare gain of adopting the policy regime AR instead of the policy regime TR. In particular, χ is measured as the additional fraction of consumption that a household would be willing to obtain to be as well off under regime TR as under the alternative monetary policy regime AR. Then, by definition, it must be

$$\upsilon_0^{AR} = E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t^{TR}(1+\chi), N_t^{TR}, g_{Z,t+1}^{TR}\right). \tag{A-40}$$

Therefore

$$\upsilon_0^{AR} - \upsilon_0^{TR} = \frac{1}{1-\beta} \log(1+\chi). \tag{F-5}$$

The welfare gains reported in Table 1 are then computed from (F-5), using a second-order approximation of welfare around the deterministic steady state. See Schmitt-Grohé and Uribe (2007) for further details.

Online Appendix G

In this Appendix we show the results regarding the optimal operational monetary policy rule under the assumption that in both growth models uncertainty only comes through capital quality shocks rather than from technological and public spending shocks. In particular, we assume that the accumulated capital stock K_t is subject to a quality shock determining the level of effective capital for use in production. Notably, this type of shock captures any exogenous variation in the value of installed capital able to trigger sudden variations in its market value and changes in investment expenditure. In DSGE models shocks on the quality of capital are used to reproduce the effects of a recession originating from an adverse shock on asset prices. A negative shock on the quality of capital reduces directly and simultaneously both the supply and the demand schedules of the economy (see e.g. Gertler and Karadi 2011).

Therefore, the stock of capital held by households evolves as

$$K_{t+1} = I_t + (1-\delta)e^{u_{K,t}}K_t, \tag{G-1}$$

where u_K is the exogenous processes capturing the capital-quality shocks such that $u_{K,t} = \rho_K u_{K,t-1} + \varepsilon_t^K$, with $0 < \rho_K < 1$, $\varepsilon_t^K \sim i.i.d.N(0, \sigma_K^2)$, while the production function becomes

$$Y_t = A_t^{\frac{1}{v}} \left[\frac{1}{p_t^M} MC_t(1-v)\right]^{\frac{1-v}{v}} \left(e^{u_{K,t}}K_t\right)^{1-\alpha} \left(Z_t N_t\right)^{\alpha}. \tag{G-2}$$

Table (G-1) reports the optimal operational monetary policy rule under the assumptions that the only source of uncertainty of the economy is given by shocks on the quality of capital.

We observe that in this case, even in the absence of financial frictions, the optimal operational flexible inflation targeting rule features a high reactivity to output in both growth models, contrary to what observed when the economy is hit by technological and public spending shocks.

These results are in line with those of Ikeda and Kurozumi (2019) who find that, when the economy is hit by financial shocks, the optimal monetary policy rule features a very vigourous response to output growth. In their model financial frictions magnify the recessionary effects of adverse shocks, that is why they also find sizeable welfare gains from output stabilization.

Table G-1: Optimal Operational Monetary Policy Rules and Welfare Costs - Capital Quality Shocks

	Endogenous Growth Model				Exogenous Growth Model			
	ι_{Π_Y}	ι_{g_Y}	ι_R	Welfare Gain (%)	ι_{Π_Y}	ι_{g_Y}	ι_R	Welfare Gain (%)
Optimized Rules								
- Flexible Inflation Targeting	10	3.1875	0	0.4386	10	2.1875	0.6	0.3754
- Strict Inflation Targeting - $\iota_Y = 0$	10	-	0	0.4228	10	-	0	0.3714
- Nominal GDP Growth targeting - $\iota_{\Pi_Y} = \iota_Y$	2.3475	2.3475	0	0.4370	2.3750	2.3750	0.7	0.3742
Non-Optimized Rules								
Taylor Rule with Smoothing	1.5	0.25	0.7	-0.6848	1.5	0.25	0.7	-0.3669
Simple Taylor Rule	1.5	-	-	Indeterminacy	1.5	-	-	Indeterminacy

Note: For each monetary policy rule the welfare gain is measured relatively to a benchmark Taylor rule with $\iota_{\Pi_Y} = 1.5$, $\iota_{g_Y} = 0.25$, $\iota_R = 0$. The results are obtained assuming $\rho_K = 0.8$ and $\sigma_K = 0.005$.

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