Innovation, Growth and Optimal Monetary Policy*

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Abstract

This paper examines how innovation-led growth affects optimal monetary policy. We consider the Ramsey policy in a New Keynesian model where R&D leads to an expanding variety of intermediate goods and compare the results with those obtained when the expansion occurs exogenously. Positive trend inflation is found to be optimal under both assumptions, but much higher with profit-seeking innovation. Optimal monetary policy must be counter-cyclical in response to both technology and public spending shocks, yet the intensity of the reaction crucially depends on the presence of an R&D sector. However, the small amount of short-run deviations of prices from the non-zero trend inflation observed in response to shocks suggests inflation targeting as a robust policy recommendation.

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1 Introduction

Macroeconomics traditionally studies the role of monetary policy in influencing business cycle fluctuations in models featuring exogenous growth or no growth at all. However, there is ample evidence that short-term fluctuations affect growth-enhancing activities (i.e. savings, investments, R&D activities) and modify the growth trend of the entire economy.[1] Over the postwar period significant oscillations between periods of robust growth versus relative stagnation have been observed in many industrialized countries. Comin and Gertler (2006) and Comin et al. (2009) suggest that these medium-frequency oscillations may, to a significant degree, be triggered by business cycle disturbances propagated by R&D activities.[2] What are then the implications for monetary policy? An early attempt to study optimal monetary policy, while considering the interaction between short-run dynamics and long-run growth, can be found in Blackburn and Pelloni (2005), and more recently in Annicchiarico and Rossi (2013), who both base their analysis on a stochastic version of the AK model with knowledge spillovers *à la* Romer (1986). In Romer (1986) there is no profit-seeking innovation activity by firms: however, this is a feature which is crucial, as we will show, to shape the optimal design of monetary policy in response to shocks.

In this paper we extend a prototypical New Keynesian (NK) model, with monopolistically competitive final and intermediate good sectors, by incorporating in it an R&D sector leading to an expansion in the variety of the intermediates, as in Romer (1990). In particular, we use a simplified version of Comin and Gertler (2006), augmented to allow for nominal price rigidities *à la* Rotemberg in the two imperfectly competitive sectors.

In the model R&D activity is stronger during expansions because its rewards are higher too. In fact, innovation creates monopoly power which will be exploited on a larger scale when aggregate demand is higher. The mechanism underlying the co-movement between R&D activity and output is, therefore, close to that described by Fatás (2000), where positive shocks may also induce long-run effects by permanently shifting out the production possibilities frontier of the economy.

In this framework we consider the problem of a Ramsey planner which sets monetary policy so as to maximize the expected utility of households, given the constraints represented by the general equilibrium conditions of the market economy. We consider two sources of uncertainty: the level of total factor productivity and of the real government purchases which are assumed to be fully financed by lump-sum taxes. Finally, to single out the role played by the innovation activities in determining monetary policy, we compare the results obtained in an economy where technical progress is driven by R&D expenses with those arising when the expansion happens exogenously.

The paper then asks the following fundamental questions: what is the optimal trend inflation in an economy with innovation-led growth? How does monetary policy optimally respond to business

[1] In the words of Lawrence H. Summers, "reversion back to trend is actually less common than evidence that the recession not only reduces the level of GDP, but reduces the trend rate of growth of GDP, what Larry Ball has referred to as super hysteresis" (Summers 2015, p. 8).

[2] In this respect, following the seminal work of Ramey and Ramey (1995) the question of precisely how cyclical fluctuations may affect long-run growth has been the subject of a broad body of research. See e.g. Martin and Rogers (1997), Aghion and Saint-Paul (1998), De Hek (1999), Martin and Rogers (2000), Jones et al. (2005), Aghion et al. (2010) and Oikawa (2010). However, there are very few investigations that analyze the role of monetary factors (e.g. Dotsey and Sarte 2000 and Varvarigos 2008), while an even smaller subset introduce nominal rigidities to study the interplay between uncertainty and growth (e.g. Pelloni 1997, Blackburn and Pelloni 2004, 2005, Annicchiarico et al. 2011, Annicchiarico and Pelloni 2014, Cozzi et al. 2017 and Pinchetti 2017).

cycles in this framework? How does the nature of the shocks hitting the economy shape this optimal response?

Our key result is that significant deviations from zero trend inflation are optimal regardless of the assumptions on long-run growth.[3] This finding is striking because the optimality of long-run zero inflation is a core result of NK models and one which is, in fact, robust across most modeling variations (see e.g. King and Wolman 1999, Schmitt-Grohé and Uribe 2008, Schmitt-Grohé and Uribe 2010 and Woodford 2011). We find that an important quantitative determinant of the optimal long-run rate of inflation is the presence of innovation activities in the economy, along with the degree of nominal rigidities.

To understand our finding of positive trend inflation we have to focus on the market failures in the model, which the Ramsey planner can affect by controlling monetary policy. The presence of the two non-competitive sectors gives rise to the static inefficiency familiar from the standard analysis of monopoly due to which the level of production is too low. When technical progress is driven by R&D, a second, dynamic, market failure stems from the incomplete appropriability of the social surplus from a new invention: the pace of innovation is then too low.

Higher markups in the final and intermediate goods sectors lower the level of production in each sector and have effects on each of these market failures. Because of the presence of price adjustment costs the monetary authority has some control over markups that can be eroded or magnified by changes in inflation. The desirability of this control does not arise in a standard NK model, where the time discount factor of the Ramsey planner and of firms is the same. In our model the two are different because of firms' entry in the intermediate sector, as we will show.[4] In this context the Ramsey planner must find a compromise between the welfare gains of inflation, via the decrease in markups and the increase in the level of economic activity, and the welfare losses, via the price adjustment costs inflation entails. Consistently with our explanation, we find that when technical progress is driven by R&D, the optimal trend inflation is higher as the Ramsey planner factors in not only the static welfare benefit from a higher scale of production, but also the dynamic one accruing from higher growth.

Our analysis also sheds some light on the optimal stabilization role of monetary policy in an economy hit by two canonical shocks in the NK literature i.e. a technology and a government spending shock.

In the short run, optimal monetary policy requires moderate deviations from full inflation targeting in response to both shocks, no matter whether long-run growth is endogenous or exogenous. However, the Ramsey planner would allow for larger deviations from price stability in response to technological shocks when innovation is endogenous, because the distortions due to imperfect competition reduce the market size for innovations.[5] On the contrary, when the economy is hit by government spending shocks the response of the Ramsey planner is more attenuated in an endogenous growth setting and tends to be accommodative at the earlier stages of the dynamic adjustment

[3]This result is shown to hold qualitatively also under a pricing scheme *à la* Calvo. See Online Appendix C.

[4]There is strong empirical evidence in favor of nominal rigidities with varying patterns of price adjustment across sectors and industries (see e.g. Nakamura and Steinsson 2013). The presence of two layers of rigidities is important for our results. The way the assumption of nominal rigidities in both sectors affects the determination of optimal trend inflation and the relationship between inflation and growth in the long-run is described in subsection 3.1.

[5]This result is qualitatively consistent with the previous findings of Annicchiarico and Rossi (2013), but the response of inflation is higher in this setting with endogenous R&D.

to the shock.[6] Overall, the small amount of short-run deviations of inflation from its long-run level, observed in response to shocks, suggests inflation targeting as a robust policy prescription.

We complete our analysis by studying a class of optimal operational monetary rules that make the current interest rate adjust in response to the past interest rates and to the current rates of inflation and output growth. We consider three different rules: 'flexible inflation targeting', 'strict inflation targeting' and 'nominal output growth targeting' and we calculate the optimal one, i.e. the one, whose adoption instead of a benchmark Taylor rule with no smoothing, maximizes a second-order-accurate measure of welfare gains.

We find that in both models the interest rate has to respond strongly to inflation. These findings go in the same direction as the ones obtained under the Ramsey policy, namely that it is optimal to stabilize inflation around a non-zero inflation target. However, as is the case in most laboratories, these short-run results are acutely conditional on the nature of the shocks hitting the economy.[7]

This study contributes to the NK literature on optimal monetary policy which is quite vast, but as its positive counterpart usually abstracts from the endogeneity of growth, e.g. Woodford (2002), Khan et al. (2003), Schmitt-Grohé and Uribe (2004), Benigno and Woodford (2005), Schmitt-Grohé and Uribe (2007b), Faia (2008). On the other hand, studies on optimal monetary policy, inflation and economic growth often adopt a deterministic framework of analysis. Recent contributions include Arato (2009), Vaona (2012), Chu and Cozzi (2014), Chen (2015), Arawatari et al. (2016), Chu and Ji (2016), Chu et al. (2017), Zheng et al. (2017) and Oikawa and Ueda (2018a).[8] An exception is the work of Ikeda and Kurozumi (2019) who like us study monetary policy in an economy with nominal rigidities and R&D leading to an expanding variety of goods. Ikeda and Kurozumi (2019), however, do not study the Ramsey policy, but focus on optimal operational interest rules. This of course implies taking the trend inflation rate as a given. Their model features financial frictions in the form of credit constraints on the borrowing activity by firms: they find that in the face of an adverse financial shock, i.e. of a tightening of credit constraints, monetary policy should strongly react to output growth.[9]

The organization of the paper is as follows. In Section 2 we present the NK endogenous growth model with innovation and those of the exogenous growth counterpart. Section 3 characterizes optimal monetary policy in the long- and in the short-run for both growth settings. Section 4 derives the optimal operational monetary policy rule. Section 5 concludes.

[6] In NK models featuring growth *à la* Romer (1986) the optimal monetary policy in response to a positive public consumption shock is such that the real rate is always positive along the adjustment path. See Annicchiarico and Rossi (2013).

[7] In fact the optimal operational monetary policy rule requires a substantial reaction to output growth when we consider a capital quality shock (i.e. a disturbance in the value of capital) as in Gertler and Karadi (2011), see Online Appendix G. The capital quality shock is often used in the business cycle literature as an exogenous trigger of asset price dynamics to mimic a financial shock.

[8] For a positive analysis of the trade-off between the short-run positive and long-run negative effects of monetary easing, see Oikawa and Ueda (2018b) who conduct their study in a Schumpeterian model of creative destruction.

[9] This result is in line to what we find when we study the optimal operational rule in the face of a capital quality shock in Online Appendix G.

2 A NK Model with R&D

There are three sectors in the economy, namely, a perfectly competitive R&D sector, a monopolistically competitive intermediate good sector and a monopolistically competitive final good sector. R&D activity leads to the creation of patents on new intermediate goods used along with existing ones in the production of final goods. An expansion in the variety of intermediate goods is the ultimate source of technological progress and, therefore, of sustained growth.

2.1 Final Good-Producing Firms

In the final goods sector each firm has monopoly power over its particular good. As is common practice, we further assume that at the top of this sector there is an output aggregator who assembles final (differentiated) goods $Y_{i,t}$, $i \in [0,1]$, into a composite product, Y_t, which we refer to as final output, by relying on a constant-return-to-scale technology of the type $Y_t = \left(\int_0^1 Y_{i,t}^{1-\frac{1}{\theta_Y}} di \right)^{\frac{\theta_Y}{\theta_Y - 1}}$, with $\theta_Y > 1$ denoting the elasticity of substitution between the various goods. If the price of each variety, $P_{i,t}$, is taken as given, by profit maximization we have the usual set of demand schedules $Y_{i,t} = (P_{i,t}/P_t)^{-\theta_Y} Y_t$ for all $i \in [0,1]$, where $P_t = \left(\int_0^1 P_{i,t}^{1-\theta_Y} di \right)^{\frac{1}{1-\theta_Y}}$ is the Dixit-Stiglitz aggregate price index. The aggregator will, in turn, sell units of the final output index at their unit cost P_t.

All final good firms have access to the same technology, represented by the following production function:

$$Y_{i,t} = A_t \left(K_{i,t}^{1-\alpha} N_{i,t}^{\alpha} \right)^v G_{i,t}^{1-v}, \tag{1}$$

where $\alpha \in (0,1)$, $v \in (0,1)$ and A_t measures aggregate productivity and is subject to shocks. The production of the generic final good $Y_{i,t}$, requires the use of capital $K_{i,t}$, labor inputs $N_{i,t}$ and of a CES composite of intermediate inputs $G_{i,t} = \left(\int_0^{Z_t} M_{i,j,t}^{1-\frac{1}{\theta_M}} dj \right)^{\frac{\theta_M}{\theta_M - 1}}$, where $M_{i,j,t}$ is intermediate good $j \in [0, Z_t]$, Z_t is a measure of product variety and $\theta_M > 1$ denotes the elasticity of substitution between the intermediate goods. We attach a time subscript to Z_t since product variety will be growing over time.

The optimal choice of factor inputs is the solution to a static cost minimization problem, where the nominal wage W_t, the rental cost of capital $P_t R_t^K$ and the price of each intermediate good $P_{j,t}^M$ are taken as given. In a symmetric equilibrium the first-order conditions are then found to be:

$$\frac{W_t}{P_t} = \alpha v MC_t \frac{Y_t}{N_t}, \tag{2}$$

$$R_t^K = (1-\alpha) v MC_t \frac{Y_t}{K_t}, \tag{3}$$

$$\frac{P_{j,t}^M}{P_t} = (1-v) MC_t Y_t \frac{M_{j,t}^{-\frac{1}{\theta_M}}}{G_t^{1-\frac{1}{\theta_M}}}, \text{ for } j \in [0, Z_t], \tag{4}$$

where MC_t denotes the real marginal cost.

The typical firm i sets the price $P_{i,t}$ by maximizing the present discounted value of expected profits, subject to the demand constraint $Y_{i,t} = (P_{i,t}/P_t)^{-\theta_Y} Y_t$, the available technology for production (1) and the adjustment cost of the Rotemberg type $\frac{\gamma_Y}{2}\left(\frac{P_{i,t}}{P_{i,t-1}}-1\right)^2 Y_t$ with γ_Y measuring the degree of price rigidity. This price adjustment cost increases in magnitude with the size of the price change and with the size of economic activity. At the optimum, and after having imposed symmetry across firms, we have the following optimal pricing condition:

$$(\theta_Y - 1)\, Y_t - \theta_Y MC_t Y_t + \gamma_Y (\Pi_{Y,t} - 1)\Pi_{Y,t} Y_t - \gamma_Y E_t \Lambda^R_{t,t+1} \left(\Pi_{Y,t+1} - 1\right) \Pi_{Y,t+1} Y_{t+1} = 0, \qquad (5)$$

where E_t is the rational expectation operator, $\Pi_{Y,t} = P_t/P_{t-1}$ and $\Lambda^R_{t,t+1}$ is the real stochastic discount factor used at time t by shareholders to value date $t+1$ real profits and is related to the household discount factor β and to their marginal utility of wealth λ_t (i.e. $\Lambda^R_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t}$), as we will see below. Equation (5) is often referred to as the New Keynesian Phillips curve and describes the equilibrium relationship between inflation and the marginal cost.

2.2 Intermediate Goods Producing Firms

The intermediate goods sector is populated by a continuum of firms belonging to an interval of length Z_t acting as monopolistic competitors, given the demand schedules set by the final good firms. Intermediate goods producers transform one unit of the final good (i.e. the CES composite of final differentiated goods) into one unit of their respective intermediate good. In other words, the production is roundabout. As in the final good sector, in this sector firms are assumed to face nominal rigidity in the form of a quadratic adjustment cost function *à la* Rotemberg. At time t each intermediate firm j sets the price $P^M_{j,t}$ so as to maximize the present discounted value of expected profits, given the demand schedule (4) and the adjustment cost $\frac{\gamma_M}{2}\left(\frac{P^M_{j,t}}{P^M_{j,t-1}}-1\right)^2 M_t$, with γ_M measuring the degree of price rigidity. Given the above assumptions in period t real profits of firm j can be written as:

$$D_{j,t} = \frac{P^M_{j,t} - P_t}{P_t} M_t - \frac{\gamma_M}{2}\left(\frac{P^M_{j,t}}{P^M_{j,t-1}} - 1\right)^2 M_t. \qquad (6)$$

In addition, we assume that firms operating in this sector face a positive probability of being hit by a negative shock forcing them to exit from the market. Let $\phi \in (0,1)$ denote the survival rate of firms operating in this sector. At the optimum, anticipating that the equilibrium is symmetric, the following optimal pricing condition holds:

$$(\theta_M - 1)\, M_t p^M_t - \theta_M M_t + \gamma_M \left(\Pi_{M,t} - 1\right) \Pi_{M,t} M_t - \gamma_M \phi E_t \Lambda^R_{t,t+1} \left(\Pi_{M,t+1} - 1\right) \Pi_{M,t+1} M_{t+1} = 0, \quad (7)$$

where $\Pi_{M,t} = P^M_t / P^M_{t-1}$ and $p^M_t = P^M_t/P_t$, while $\phi E_t \Lambda^R_{t,t+1}$ is the discount factor adjusted for the survival rate. Given the roundabout nature of th e available technology, p^M_t measures the markup capturing the degree of market power prevailing in this sector.

After having imposed symmetry, (4) can be expressed as follows:

$$M_t = \left[\frac{1}{p^M_t} MC_t (1-v) A_t \left({K_t}^{1-\alpha} N^\alpha_t\right)^v Z_t^{\theta_M (1-v)/(\theta_M - 1) - 1}\right]^{\frac{1}{v}}. \qquad (8)$$

From the above expression we observe that the equilibrium quantity of the intermediate good is negatively affected by the degree of market power in both sectors.

The value of owning exclusive rights to produce intermediate goods, denoted by V_t, is equal to the present discounted value of the current and future profits this allows:

$$V_t = D_t + \phi E_t \Lambda^R_{t,t+1} V_{t+1}. \tag{9}$$

where, from (6), $D_t = \left(p_t^M - 1 - \gamma_M \left(\Pi_{M,t} - 1\right)^2 /2\right) M_t$. The effect of imperfect competition in this sector on the value of patents is then twofold. On the one hand, less competition has a direct positive effect on profits, through the effects on the markup. On the other hand, less competition has a negative effect on profits through the negative impact it has on M_t. We will see that the latter effect tends to dominate the former, i.e. profits are pro-cyclical. The pro-cyclical behavior of profits implies that the value of patents is also pro-cyclical. Since the value of patents is the payoff to innovation, as described below, this implies that the payoff to innovation is pro-cyclical as well.

2.3 R&D Sector

In the R&D sector researchers develop blueprints for new intermediate goods. The patents are then sold to firms that produce the new goods. For simplicity we assume that innovators finance their activity by borrowing from households. Assuming free entry in the R&D sector, the price of a new patent will be equal to its value for a new firm i.e. V_t. The R&D sector is characterized by a linear technology. Let S_t be the total amount of R&D expenditure in terms of the final output and ξ_t be its productivity level. Given the intermediate product survival rate ϕ, the law of motion for the measure of intermediate goods Z_t is then

$$Z_{t+1} = \xi_t S_t + \phi Z_t, \tag{10}$$

where, as in Comin and Gertler (2006), the technology coefficient ξ_t involves a congestion externality effect capturing decreasing returns to scale in the innovation sector:

$$\xi_t = \hat{\xi} \left(Z_t / S_t\right)^{1-\varepsilon}, \ \varepsilon \in (0,1), \tag{11}$$

with ε measuring the elasticity in the creation of new intermediate goods with respect to R&D and $\hat{\xi}$ being a scale parameter. Perfect competition in the R&D sector implies that the following break-even condition must hold:

$$E_t \Lambda^R_{t,t+1} V_{t+1} (Z_{t+1} - \phi Z_t) = S_t, \tag{12}$$

where V_{t+1} is the price of an innovation at time $t+1$. The above condition simply says that the expected sales revenues, $E_t \Lambda^R_{t,t+1} V_{t+1}(Z_{t+1} - \phi Z_t)$, must be equal to the cost S_t. This condition can be equivalently formulated using (10) as

$$1/\xi_t = E_t \left(\Lambda^R_{t,t+1} V_{t+1}\right), \tag{13}$$

which simply implies that the marginal cost $1/\xi_t$ equals the expected marginal revenue $E_t \left(\Lambda^R_{t,t+1} V_{t+1}\right)$.

2.4 Households

The infinitely lived representative household faces the following time-separable expected utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \mu_n \frac{N_t^{1+\varphi}}{1+\varphi} \right), \tag{14}$$

where β is the subjective discount factor, μ_n is a positive scale parameter measuring the disutility of labor, $\varphi > 0$ measures the inverse of the Frisch elasticity of labor supply and C_t is consumption of the final composite good. Households make one-period loans to innovators, own monopoly rights on firms and also own the capital stock and let this capital to firms in a perfectly competitive rental market at the real rental rate R_t^K. The period budget constraint takes the form

$$P_t C_t + \Lambda_t B_t = B_{t-1} + W_t N_t + P_t R_t^K K_t - P_t I_t + T_t, \tag{15}$$

for $t = 0, 1, 2...$, where K_t is physical capital carried over from period $t-1$, I_t denotes investments, T_t represents the lump-sum component of income, which includes dividends from the ownership of the firms and non-distortionary taxation. B_t denotes the quantity of one-period nominal riskless bonds purchased in t at price Λ_t that will pay one unit of the numérarire in period $t+1$.

Investment increases the stock of capital according to a standard law of motion:

$$K_{t+1} = (1-\delta) K_t + I_t, \tag{16}$$

where $\delta \in (0,1)$ is the depreciation rate of capital. The typical household will choose the sequences $\{C_t, B_{t+1}, K_{t+1}, I_t, N_t\}_{t=0}^{\infty}$ so as to maximize (14), subject to (15) and (16). At the optimum the following conditions hold:

$$C_t^{-1} = \lambda_t, \tag{17}$$

$$\Lambda_t = \beta E_t \frac{\lambda_{t+1}/P_{t+1}}{\lambda_t/P_t} = \frac{1}{R_t}, \tag{18}$$

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(R_{t+1}^k + 1 - \delta \right), \tag{19}$$

$$\mu_n \frac{N_t^{\varphi}}{\lambda_t} = \frac{W_t}{P_t}, \tag{20}$$

where λ_t denotes the Lagrange multiplier associated to the flow budget constraint (15), expressed in real terms, and measures the marginal utility of consumption (17), condition (18) reflects the optimal choice between current and future consumption and expresses the relationship between Λ_t and the risk-free (gross) nominal interest rate R_t, (19) refers to the optimality condition with respect to capital, whereas (20) reflects the optimal choice for labor supply.

2.5 Market Clearing

Final output (i.e. the final composite good) is used for consumption, investment in physical capital, factor input used in the production of intermediate goods, R&D, public expenditure and nominal adjustment costs on prices. In equilibrium the following aggregate resource constraint must hold:

$$Y_t = C_t + I_t + Z_t M_t + S_t + c_t^G Y_t + \frac{\gamma_Y}{2} \left(\Pi_{Y,t} - 1 \right)^2 Y_t + \frac{\gamma_M}{2} \left(\Pi_{M,t} - 1 \right)^2 Z_t M_t, \tag{21}$$

where c_t^G denotes the public consumption to output ratio, therefore $c_t^G Y_t$ is public consumption, fully financed by lump-sum taxation. This assumption is made to capture the fact that government expenses grow with the economy. The ratio c_t^G is subject to shocks.

Using (8) into the production function (1) final output can be expressed as

$$Y_t = A_t^{\frac{1}{v}} \left[\frac{1}{p_t^M} MC_t(1-v) \right]^{\frac{1-v}{v}} \left(K_t{}^{1-\alpha} N_t^{\alpha} \right) Z_t^{\frac{1-v}{v(\theta_M - 1)}}, \tag{22}$$

For the existence of a balanced growth path the aggregate production function must be homogeneous of degree one in the accumulating factors K_t and Z_t. Hence we need the following parameter restriction:

$$\frac{1-v}{v\,(\theta_M - 1)} = \alpha, \tag{23}$$

which also ensures stationarity of M_t.[10]

In this economy a number of variables, such as output, consumption etc. will not be stationary along the balanced-growth path. We note that non-stationary variables at time t are cointegrated with Z_t, while the same variables at time $t+1$ are cointegrated with Z_{t+1}. We divide variables by the appropriate cointegrating factor and denote the corresponding stationary variables with lowercase letters. In particular, for any variable, X_t, we have $x_t = X_t/Z_t$. In addition we denote $w_t = \frac{W_t}{Z_t P_t}$ and $g_{Z,t+1} = Z_{t+1}/Z_t$. The two sources of uncertainty A_t and c_t^G are assumed to evolve as $\log A_t = (1-\rho_A)\log A + \rho_A \log A_{t-1} + \varepsilon_t^A$, with $0 < \rho_A < 1$, $\varepsilon_t^A \sim i.i.d.N(0, \sigma_A^2)$, and $\log c_t^G = (1-\rho_G)\log c_t^G + \rho_G \log c_{t-1}^G + \varepsilon_t^G$, with $0 < \rho_G < 1$, $\varepsilon_t^G \sim i.i.d.N(0, \sigma_G^2)$. The equilibrium conditions of the model expressed in efficiency units are summarized in Online Appendix A.

2.6 A NK Model with Exogenous Growth

To isolate the role of innovation we also consider a version of the model incorporating an exogenous growth mechanism. The structure of the economy is the same, so to obtain easily comparable results, but we now assume that the intermediate good sector expands at an exogenously set growth rate:

$$Z_{t+1} = g_Z Z_t, \tag{24}$$

where g_Z denotes a deterministic growth factor, so that there is no more a role for R&D activity. This way of introducing exogenous technical progress was chosen to focus sharply on the consequences for optimal monetary policy of the assumption of innovation-led growth.[11] The assumption of Z_t growing exogenously could also be taken to represent the limit case of a follower economy which copies some prototypes of goods invented abroad. Therefore, the above equation replaces (10)-(13), while the resource constraint of the economy becomes

$$Y_t = C_t + I_t + Z_t M_t + c_t^G Y_t + \frac{\gamma_Y}{2} \left(\Pi_{Y,t} - 1 \right)^2 Y_t + \frac{\gamma_M}{2} \left(\Pi_{M,t} - 1 \right)^2 Z_t M_t, \tag{25}$$

[10] From (8) M_t is stationary provided that $(1-\alpha)\,v + \frac{\theta_M(1-v)}{\theta_M - 1} = 1$. It is straightforward to show that this restriction holds under (23), implying that (8) can be written as $M_t = \left[\frac{1}{p_t^M} MC_t(1-v) A_t \right]^{\frac{1}{v}} \left(\frac{K_t}{Z_t} \right)^{1-\alpha} N_t^{\alpha}$.

[11] Comparing results obtained in the same model but with no growth or with A_t growing exogenously would have made the interpretation of the differences less transparent.

which replaces (21). Observe that equation (9) is still valid, but there is no feedback from the equation and the rest of the equilibrium conditions (i.e. when solved recursively the equation gives the time sequence for the value of the firm).[12]

Using the same notation adopted in the previous section, the exogenous growth model in efficiency units is summarized in Online Appendix A.

2.7 Market Failures

Before turning to the implications of the model for monetary policy, we describe in more detail the distortions which the monetary authorities try with their action to minimize.

A first source of inefficiency is due to costly price adjustment. This pricing assumption gives rise to a wedge between aggregate demand and aggregate output, since a part of output is used for adjusting prices. Clearly, this wedge is eliminated at zero inflation.

A second source of inefficiency stems from the existence of monopolistically competitive producers: the quantity produced of each intermediate and final good is inefficiently low as the price is higher than the marginal cost. This is a static distortion familiar from the standard analysis of monopoly.

A third source of inefficiency arises in the model with R&D where, as in Romer (1990), the invention of a new intermediate has a cost to be paid upfront. However, only the present discounted value of profits, as opposed to all of social surplus originating from the design for the new good, is taken into account by an entrepreneur when deciding whether to pay for it. The rate of innovation is, therefore, inefficiently low.

Finally, there is a disparity in the price setting behavior of firms in the two monopolistically competitive sectors, no matter whether long-run growth is exogenous or endogenous. This disparity arises because the firm size and the cost of price adjustment per firm do not grow asymptotically in the intermediate goods sector.[13]

3 Ramsey Monetary Policy

We now consider the problem of a monetary authority (Ramsey planner) which maximizes the expected utility of households, given the constraints represented by the general equilibrium conditions of the market economies outlined in Section 2.[14] We assume that the Ramsey planner is able to commit to the contingent policy rule it announces at time 0 (i.e. there is ex-ante commitment to a feedback policy so as to have the ability to dynamically adapt the policy to the changed economic conditions). The discount factor of the planner is β. The Ramsey planner then maximizes (14), which can be easily expressed in efficiency units, subject to the constraints represented by the equilibrium conditions of the market economy. See Online Appendix B for details.

[12]This is because in the exogenous case the value of the firm will not be equal to the cost of innovation, as the firm has not to pay for a patent on a new good.

[13]This disparity becomes evident when the equations are stationarized, as in Online Appendix A.

[14]The Ramsey approach allows to study the optimal policy around a distorted steady state, as the one in our model. See Khan et al. (2003), Benigno and Woodford (2005) and Faia (2009) for a discussion on welfare analysis with a distorted steady state.

In what follows we will make two uses of the first-order conditions from the Ramsey policy problem.[15] First, we will use these conditions to characterize the optimal trend inflation by finding a fixed point of the deterministic version of these conditions (i.e assuming no shocks). Second, we will analyze the dynamic response of the economy to exogenous shocks.

3.1 Trend Inflation

The optimal trend inflation rate is computed by solving the first-order conditions of the Ramsey plan in steady state. The inflation rate so computed reflects the so-called *modified golden rule steady state*.[16] Notably, in a simple cashless NK model with Rotemberg pricing the optimal trend inflation is zero.[17] We will see that this result does not necessarily hold in the two growth models of this paper.

For comparison purposes and to make the analysis more transparent our benchmark parametrization is based on the market equilibrium for both models. For each of the two, we compute the deterministic steady state and then proceed with the calibration. The time frequency is quarterly. We start with the conventional parameters. The subjective discount factor β is set at 0.99. The labor share α is set equal to 2/3. The physical capital depreciation rate δ is 0.025. We set the inverse of the Frisch elasticity of labor supply φ at 1 which represents an intermediate value in the range of macro and micro data estimates. The elasticity of substitution between differentiated final goods θ_Y is set at 6. The scale parameter μ_n is set to deliver a steady-state fraction of time spent working $N = 0.17$. Steady-state inflation in the market equilibrium is set at zero, therefore $\Pi_P = \Pi_M = 1$. Finally, c_t^G i set at 0.1 in steady state. The coefficients γ_Y and γ_M governing price stickiness are set to be consistent with a Calvo pricing setting with a probability that price will stay unchanged of 0.75.[18]. Similarly to Schmitt-Grohé and Uribe (2007b) the persistence of the technology shock is $\rho_a = 0.86$, while that of the government spending shock is $\rho_g = 0.87$.

Now we turn our attention to the parameters related to R&D and innovation. Our calibration mainly follows Comin and Gertler (2006). We consider an annual trend growth rate of output of 2% in the market equilibrium, i.e. $g_Z = 1.02^{1/4}$, and an annual obsolescence rate for intermediate goods equal to 3%, yielding $\phi = (1 - 0.03)^{1/4}$. The productivity parameters $\hat{\xi}$ in the R&D technology function and the technology parameter in the final good production function, A, are chosen consistently. The gross markup in the intermediate good sector is set at 1.6, i.e. $\theta_M = 2.67$. We set the elasticity of new intermediate goods with respect to R&D spending at $\varepsilon = 0.5$, so as to ensure

[15] The first-order conditions stemming from these problems are optimal from a "timeless perspective", rather than from the perspective of the particular date at which the policy is actually adopted. This is to rule out the possibility that the Ramsey planner could renege on previous announcements. Technically speaking, given the above Ramsey problems, this "timeless perspective" implies that we can focus on the first-order conditions at time $t \geq 1$.

[16] This inflation rate differs from the *golden rule steady state*, that is the inflation rate that maximizes total utility of households along the deterministic balanced growth path. The latter notion of optimality overlooks uncertainty and the transitional dynamics implied by the Ramsey solution. For details, see King and Wolman (1999).

[17] See e.g. Schmitt-Grohé and Uribe (2008) for an analysis of optimal monetary policy in a prototype NK model with Rotemberg pricing.

[18] The coefficients γ_Y and γ_M are in fact set so that the slope of the log-linearized version of both NK Phillips curves are equal to the one that would arise under Calvo pricing in a baseline NK model for a frequency of price adjustment of 1/4. For details, see Keen and Wang (2007). Given the calibration set for θ_Y and θ_M, we have $\gamma_Y = 58.25$ and $\gamma_M = 19.22$.

real determinacy of the Ramsey equilibrium.[19]

Given this baseline parametrization, we are able to numerically compute the steady state solution to the Ramsey problem and quantify the optimal long-run inflation rate for both models.[20] In the benchmark case we observe that the Ramsey optimal inflation rate is 0.663% per year in the endogenous growth model and 0.175% in the exogenous growth model. This result stands in sharp contrast to the common finding emerging in many cashless NK models, where the Ramsey planner cannot use inflation as a device to reduce market inefficiencies and, therefore, opts to neutralize the adjustment costs by setting trend inflation at zero.

However, when we remove the hypothesis of price rigidities in the intermediate goods sector, both growth models replicate the standard result of zero optimal trend inflation. We deduce that what drives the optimal trend inflation above zero is the possibility of controlling the markup in the intermediate goods sector. As in the standard NK model, in this case, the optimality of zero trend inflation derives from the fact that the planner will find it optimal to avoid the costly price adjustments in the final goods sector which reduce the overall resources available and create a wedge between aggregate demand and output.[21]

The above result can be explained as follows. In both growth settings, intermediate goods producers when pricing goods attach a lower weight to the costs of future expected inflation. On a balanced growth path, in fact, the intermediate sector expands at the extensive margin, while production at the intensive margin is constant. For this reason the relevant discount factor for pricing decisions is lower in this sector than in the final goods sector. This implies that even when considering the dynamic trade-offs implied by the Ramsey solution, the benefits of price changes smoothing do not vanish in steady state, leaving room for positive trend inflation. In Online Appendix C we show that this result also holds under a Calvo pricing scheme where, however, the trade-off faced by the Ramsey planner is different.

Under the benchmark calibration, in the endogenous growth case, the optimal trend inflation is higher than in the exogenous growth one. In fact the higher level of economic activity made possible by lower markups reduces not only the static externality due to monopolistic competition, but also the dynamic externality i.e. the appropriability problem and the inefficiently low growth it entails. In other words, a higher scale of production favors R&D spending and a higher innovation rate. In the baseline calibration, in fact, the model generates a monotonically positive relationship between inflation and growth.[22]

[19] It can be shown that for $\varepsilon > 0.7$ the Ramsey equilibrium is unstable, while under a monetary rule of the Taylor type the model displays stability and uniqueness of the rational expectations solution also for $\varepsilon = 1$.

[20] In other words to compute the optimal long-run inflation we opt for a two-step procedure. First, we calibrate both versions of the model so as to characterize the decentralized long-run equilibrium (such that inflation is zero and annual growth is equal to 2%). Then, we use the first-order conditions of the Ramsey problem to compute an initial vector for the Lagrange multipliers. Using this equilibrium as an initial candidate for the optimal solution we rely on a non-linear solver to numerically find the Ramsey steady state.

[21] In a previous version of this paper the analysis was carried out under this hypothesis. See Annicchiarico and Pelloni (2016). Of course, this result also arises since the model does not embody any money demand distortions. See Khan et al. (2003).

[22] In Online Appendix B we show that, in the baseline calibration and in the absence of price adjustment costs in the intermediate goods sector, the model is able to generate a monotonically positive relationship between these two variables. In this case inflation by reducing markups and increasing the market size for new products is found to foster growth. The relationship is instead found to be hump-shaped when price adjustment costs arise only in the intermediate goods sector. In this case the ability of inflation of increasing demand is moderate and is soon

Figure 1 plots optimal trend inflation for different parametrizations. The higher the degree of nominal rigidities in the intermediate good sector, the higher the level of optimal trend inflation in both growth models, while the opposite is true for high price adjustment costs in the final goods sector. Intuitively, higher nominal adjustment costs in the intermediate goods sector improve the ability of the Ramsey planner to use inflation as a device to reduce the welfare losses from market failures. By contrast, higher nominal adjustment costs in the final goods sector imply a decrease of the benefits of positive trend inflation. We note that for low adjustment costs in the final goods sector the optimal trend inflation in the exogenous growth model is higher than in the endogenous growth one.[23] From Figure 1 we also observe that a larger elasticity in the creation of blueprints for intermediate goods with respect to R&D spending implies a higher optimal trend inflation. Intuitively, a higher ε implies a higher marginal return to R&D spending, making it more convenient for the Ramsey planner to decrease markups as a way to free up resources to be channelled toward R&D activity.[24] Similarly, a higher obsolescence rate will push the Ramsey planner to take advantage of positive inflation as a means to engineer a reduction of markups. When the rate of substitution of the old intermediates by the new intermediates is high, the expected market size for the innovation becomes crucial for innovators.

3.2 Dynamics

In this section we characterize the dynamic properties of Ramsey allocations in response to shocks on technology and public consumption. The models are calibrated according to the parameterization outlined in the previous section and then solved by using a 'pure' perturbation method based on a second-order approximation around the non-stochastic Ramsey steady state.[25]

Figure 2 shows the Ramsey optimal impulse response functions to a one percent jump in technology for output, consumption, investment, hours, TFP growth, inflation, real interest rate, markups, R&D spending and profits. All results are reported as percentage deviations from the steady state, except for inflation, nominal and real interest rates, which are expressed as percentage-point deviations and for TFP growth which is expressed in annualized rates.[26] Continuous lines show the impulse response functions of the Ramsey plan in the endogenous growth model, while dotted lines refer to the Ramsey plan in the exogenous growth model.

As expected, output, consumption, investment, hours and R&D spending positively react to the technology shock and then gradually reverse back to the steady-state level. However, inflation initially increases, while the nominal interest rate increases by more yielding a higher real rate. Later the economy experiences a decline of inflation and lower real interest rates. During all the adjustment path markups are below their steady state level. Clearly, the Ramsey planner will find

counterbalanced by the negative effects of higher price adjustment costs and by the diminished incentives to innovation induced by lower markups. In Appendix C we show how the relationship between inflation and growth changes under Calvo pricing.

[23] In Online Appendix C we show, instead, that under Calvo pricing optimal trend inflation is always higher in the endogenous growth model.

[24] The opposite result holds under Calvo pricing, where for a higher return of R&D spending the Ramsey planner tries to minimize the distortions related to price dispersion.

[25] In Online Appendix D we show the dynamic properties of the two models under an interest rate rule of the Taylor type.

[26] From (22) and (23), $TFP_t = A_t^{\frac{1}{v}} Z_t^{\alpha}$.

it optimal that the economy initially expands, with inflation acting as a tax on monopolistic profits, via a temporary negative effect on the price markups of final and intermediate goods producers.[27] Quantitatively, however, we observe a small amount of inflation variation during all the adjustment process.[28]

Turning to the differences between the two growth settings, we notice that with endogenous innovation all these effects tend to be slightly stronger and/or more persistent. In the model with R&D inflation initially increases by more, while the real interest rate stays above its steady state level for longer than in the exogenous growth model. Hours increase by more when growth is endogenous, while the expansion of consumption is slightly lower. The difference arises because a fraction of the increased output goes to R&D to sustain higher growth rates of output. We observe, in fact, a sharp increase in R&D spending which sustains aggregate demand, so that the effects on output are slightly higher with endogenous innovation. These results can be explained by noting that a higher technology level increases the marginal product of intermediate goods as well, so boosting their demand and the real profits received by their producers. In other words, the Ramsey planner finds it optimal to decrease the markups in order to induce a positive market size effect so boosting innovation incentives. Overall, despite these differences, the behavior of the economy is very similar under the two growth settings.

Figure 3 displays the response of the economy to a one percent positive government spending shock. We observe that in both settings the Ramsey planner tries to stabilize consumption in the face of government purchase shocks. Moreover, the optimizing monetary authority tightens monetary policy to raise markups in both sectors when government demand is high. The negative effects of this policy reaction on aggregate demand are such to induce a very slight decrease of output in the endogenous growth model and a sharper decline of inflation. This counterintuitive result can be explained by noting the sharp decrease of investments and the drop of R&D spending. The response of these variables is able to absorb a part of the expansionary shock on aggregate demand. In addition, the lower level of output, and the smaller market size for innovation exacerbates the negative response of the R&D expenditure to the shock. The idea is that in the face of an increase in public spending, that in this economy is modeled as pure waste, the Ramsey planner will find it optimal to (very) slightly shrink output and reduce the response of hours, so decreasing the disutility of labor.[29]

[27] Qualitatively similar results stem out in a simple NK model with growth *à la* Romer (1986), however inflation is found to react less than in our setting. See Annicchiarico and Rossi (2013). Our results are also consistent with those obtained by Faia (2008) in a NK model with capital accumulation and Rotemberg price adjustment, but differ substantially with those obtained by Khan et al. (2003) who develop their analysis in a simple NK model with labor as the only production input.

[28] In Online Appendix D we show that under a Taylor rule a positive shock on technology determines an initial decline of inflation and an increase in the markups of both sectors. In Online Appendix E we show that the Ramsey monetary authority will use inflation as a way to lower the markups, so inducing an expansion of varieties of the intermediate good sector also in response to positive shocks to R&D productivity.

[29] Of course, this dynamics is the outcome of a number of simplifying assumptions and, therefore, not what we observe in reality following a government shock, for a variety of reasons. A first one is that in the real world government consumption is not a stochastic process the consequences of whose variability the monetary authorities will try to offset. An other obvious one is that monetary policy is not conducted according to the Ramsey scheme. In Online Appendix D we show that when monetary policy is conducted according to a Taylor rule, the model with endogenous innovation behaves as expected: following a positive shock on public spending output slightly increases and inflation goes up.

We further observe that in the economy with endogenous innovation, at least initially, the resulting real interest rate is slightly below its long-run level, suggesting that the Ramsey planner will undertake a slightly accommodative monetary policy. On the other hand, in the model with exogenous innovation, the inflation and the nominal interest rate responses are such that the real rate is always positive along the adjustment path. In this context the optimal monetary policy calls for a higher real rate, so as to moderate the temporary expansionary effects of aggregate demand on output. It turns out that with endogenous innovation the optimizing monetary authority will find it optimal to tighten monetary policy when government demand is high to a lesser extent than in a model with exogenous growth.[30] We note that in both cases the Ramsey planner manages to stabilize the economy, being the deviations of the variables from their steady state quite modest.

Overall, the moderate short-run variation of prices around the non-zero trend inflation observed in response to both shocks would suggest inflation targeting as a robust policy recommendation.

4 Optimal Operational Interest Rate Rules

This Section looks at optimal monetary rules in the two growth settings. Optimal rules are obtained by searching for the policy parameters of an interest rate rule so as to maximize consumer welfare, given the competitive equilibrium conditions in the economy. In our search for the optimal policy, we limit our attention to operational rules. To be operational a monetary rule must be based on observable variables, must ensure the uniqueness of the rational expectations equilibrium and must respect a non-negativity constraint on the nominal interest rate. To this purpose we consider the following class of interest rate rules:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\iota_R} \left[\left(\frac{\Pi_{Y,t}}{\Pi_Y}\right)^{\iota_{\Pi_Y}} \left(\frac{Y_t}{g_Z Y_{t-1}}\right)^{\iota_{g_Y}}\right]^{1-\iota_R}, \tag{26}$$

where variables without the time subscript refer to the deterministic balanced growth path under the Ramsey policy, while ι_R, ι_{Π_Y} and ι_{g_Y} are policy parameters.[31] In our search for optimal rules we restrict the policy parameters as follows: ι_{Π_Y} is restricted to lie in the interval [1, 10], ι_{g_Y} in [0, 10], both with a step of size 0.0625, and ι_R is restricted to lie in [0, 0.9] with a step of size 0.1. We approximate the non-negativity constraint on the nominal interest rate by searching for the rule generating low volatility of the nominal interest rate around its steady state.[32] Our measure of welfare is the conditional expected value of lifetime utility at time 0.

Following Ikeda and Kurozumi (2019) and using their terminology, we consider three different rules, 'flexible inflation targeting' where no further restrictions are imposed in conducting our grid search, 'strict inflation targeting' where $\iota_{g_Y} = 0$ and 'nominal output growth targeting' where we

[30]In a NK model with endogenous growth as in Romer (1986), the optimal monetary policy in response to a positive public consumption shock prescribes a positive reaction of the real rate along all the adjustment path, similarly to what observed in the model with exogenous innovation. See Annicchiarico and Rossi (2013).

[31]As in Ikeda and Kurozumi (2019) we focus on a Taylor rule in which monetary authorities react to output growth, that is observable, rather than to the output gap. Similarly to Schmitt-Grohé and Uribe (2007b) the standard deviation of the technology shock is $\sigma_a = 0.0064$, while that of the government spending shock is $\sigma_g = 0.016$.

[32]Following Schmitt-Grohé and Uribe (2007a) we impose the condition: $2\sigma_R < R - 1$, where σ_R denotes the standard deviation of the nominal interest rate.

impose the restriction $\iota_{\Pi_Y} = \iota_{g_Y}$. Table 1 summarizes the results and reports a second-order-accurate measure of the welfare gains from adopting each optimal operational rule instead of a benchmark Taylor rule with $\iota_{\Pi_Y} = 1.5$, $\iota_{g_Y} = 0.25$ and $\iota_R = 0$. These gains are measured in consumption-equivalent terms. See Online Appendix G for details.

Table 1 also reports the results obtained under two non-optimized rules, i.e. a Taylor rule with smoothing and a simple Taylor rule. We note that a simple Taylor rule induces real indeterminacy. This confirms that in NK models determinacy is made harder to achieve by the addition of capital and investment spending, as shown by Carlstrom and Fuerst (2005) and by an endogenous growth mechanism, as shown by Micheli (2018).[33]

We observe that with endogenous innovation the optimal operational monetary rule prescribes a strong response to inflation, high persistence, but no reaction to output growth. With exogenous innovation the optimal operational rule features an even stronger response to inflation than that observed for the endogenous growth model, but a positive, although small, reactivity to output. This result is mainly driven by the different optimal response to public spending shocks in the two models, consistently with the findings obtained under the Ramsey monetary policy. In the exogenous growth model a more vigorous reaction to public spending shock is in fact optimal. Finally, we observe that the welfare gains deriving from adopting a 'flexible inflation targeting' rule are higher in the model with endogenous innovation.

These findings go in the same direction as the results obtained under the Ramsey policy, namely that is optimal to stabilize inflation around a non-zero inflation target.[34]

5 Conclusion

In this paper we have studied optimal monetary policy in a standard NK model with two monopolistic sectors, one producing final goods, one producing intermediate goods. Growth takes the form of an expansion in the variety of intermediates as in Romer (1990). In the benchmark model this expansion is the outcome of profit-motivated R&D, but to make results more transparent we also consider the case where the expansion occurs at a fixed rate i.e. growth is exogenous. We have shown that, no matter whether growth is endogenous or exogenous, optimal trend inflation can be significantly above zero because the Ramsey planner uses inflation as a device to affect the markups and reduce the deadweight losses arising in the imperfectly competitive sectors of the

[33] More specifically, Carlstrom and Fuerst (2005) show that when monetary policy is conducted by a current-looking Taylor rule and prices are extremely sticky a NK model with capital and investment spending will display indeterminacy for a range of values of parameter ι_{Π_Y} larger than 1. In the two models we consider in this paper by decreasing the degree of stickiness in the final good sector by 10%, the equilibrium is determined for $\iota_{\Pi_Y} > 1$ in the exogenous growth model, and for $\iota_{\Pi_Y} > 1.05$ in the endogenous growth model. This last result is consistent with Micheli (2018) who shows that under endogenous growth the region of determinacy is restricted.

[34] In this sense our findings are consistent with those of Schmitt-Grohé and Uribe (2007b), among others, but also with those of Annicchiarico and Rossi (2013) who show that in a NK model characterized by endogenous growth with serendipitous learning *à la* Romer the optimal operational interest rate rule responding to current variables features a positive response to inflation and a small response to output. Ikeda and Kurozumi (2019) show, instead, that when uncertainty of the model comes through financial shocks, then it is optimal to respond vigorously to output growth. In Online Appendix G we show that in the presence of capital quality shocks the optimal operational monetary policy rule requires a substantial reaction to output growth, consistently with the findings obtained by Ikeda and Kurozumi (2019) in their model with financial frictions.

economy. This use of inflation is made possible by the entry of firms in steady state in the intermediate sector. An important determinant of the optimal long-run rate of inflation is the endogeneity of growth, along with the degree of nominal rigidities. In the economy with endogenous growth optimal trend inflation is always higher. In fact, the decrease of the markups and increase in the level of economic activity so engineered not only reduce the static distortions from monopoly, but also the dynamic one from the spillover to innovation in the model.

In the short run the Ramsey policy requires small deviations from full inflation targeting in response to both technology and government spending shocks. However, the intensity of the reaction to expansionary supply or demand shocks crucially depends on the underlying growth mechanism. In response to positive shocks on productivity, with endogenous growth, in fact, the Ramsey planner will tolerate larger deviations of the inflation rate above its optimal steady state in the attempt to induce a stronger reduction of the markups and sustain a higher expansion, so as to create the conditions for a stronger positive market size effect for the new products. On the other hand, in response to a positive government shock, where optimality calls for a decline in the price level, an increase in the real interest rate, a fall in consumption and higher markup in the final good sector, we observe that in the endogenous growth setting the optimizing monetary authority will tend to tighten monetary policy to a lesser extent than in the model with exogenous growth.

Overall, we find further reasons why optimal monetary policy might depart from price stability, by showing the non-trivial role played by the underlying growth mechanism in shaping the optimal policy.

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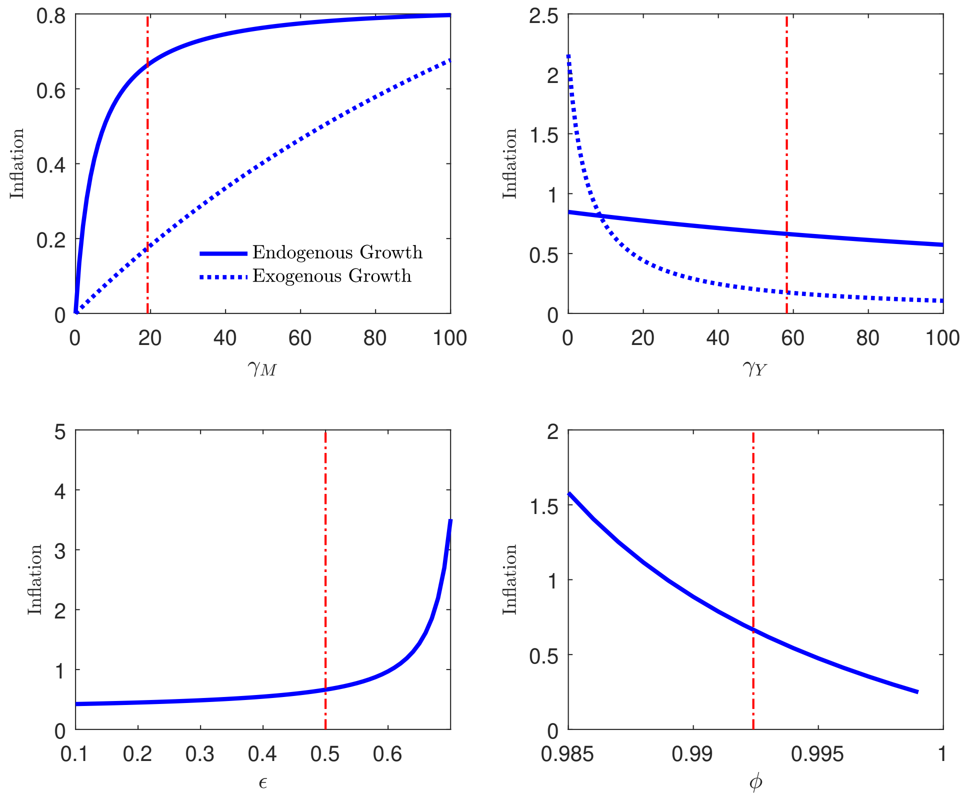
Table 1: Optimal Operational Monetary Policy Rules and Welfare Gain

	Endogenous Growth Model				Exogenous Growth Model			
	ι_{Π_Y}	ι_{g_Y}	ι_R	Welfare Gain (%)	ι_{Π_Y}	ι_{g_Y}	ι_R	Welfare Gain (%)
Optimized Rules								
- Flexible Inflation Targeting	3	0	0.5	0.0822	7.625	0.125	0.8	0.0338
- Strict Inflation Targeting - $\iota_Y = 0$	3	0	0.5	0.0822	10	0	0.3	0.0337
- Nominal GDP Growth Targeting - $\iota_{\Pi_Y} = \iota_Y$	10	10	0	-0.0669	10	10	0	-0.0421
Non-Optimized Rules								
Taylor Rule with Smoothing	1.5	0.25	0.7	0.0261	1.5	0.25	0.7	0.0128
Simple Taylor Rule	1.5	-	-	Indeterminacy	1.5	-	-	Indeterminacy

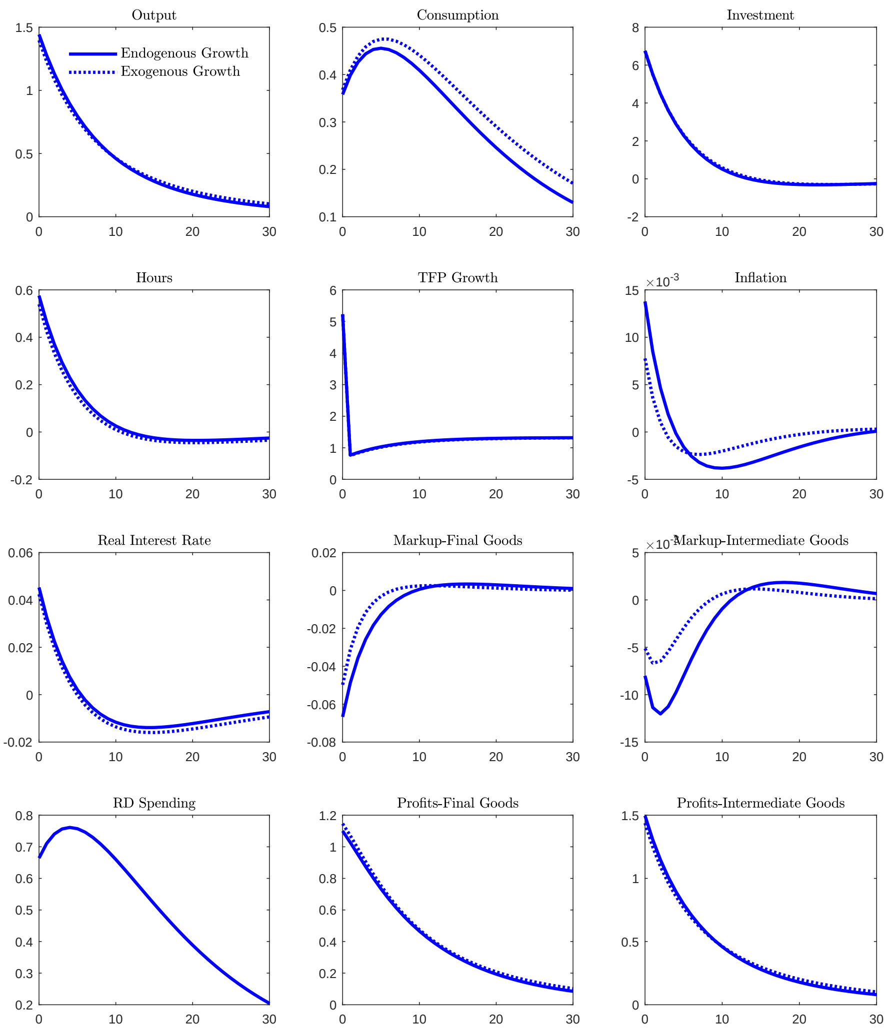
Note: For each monetary policy rule the welfare gain is measured relatively to a benchmark Taylor rule with $\iota_{\Pi_Y} = 1.5$, $\iota_{g_Y} = 0.25$, $\iota_R = 0$.

Figure 1: Annual Optimal Trend Inflation for Different Model Parametrizations (%)



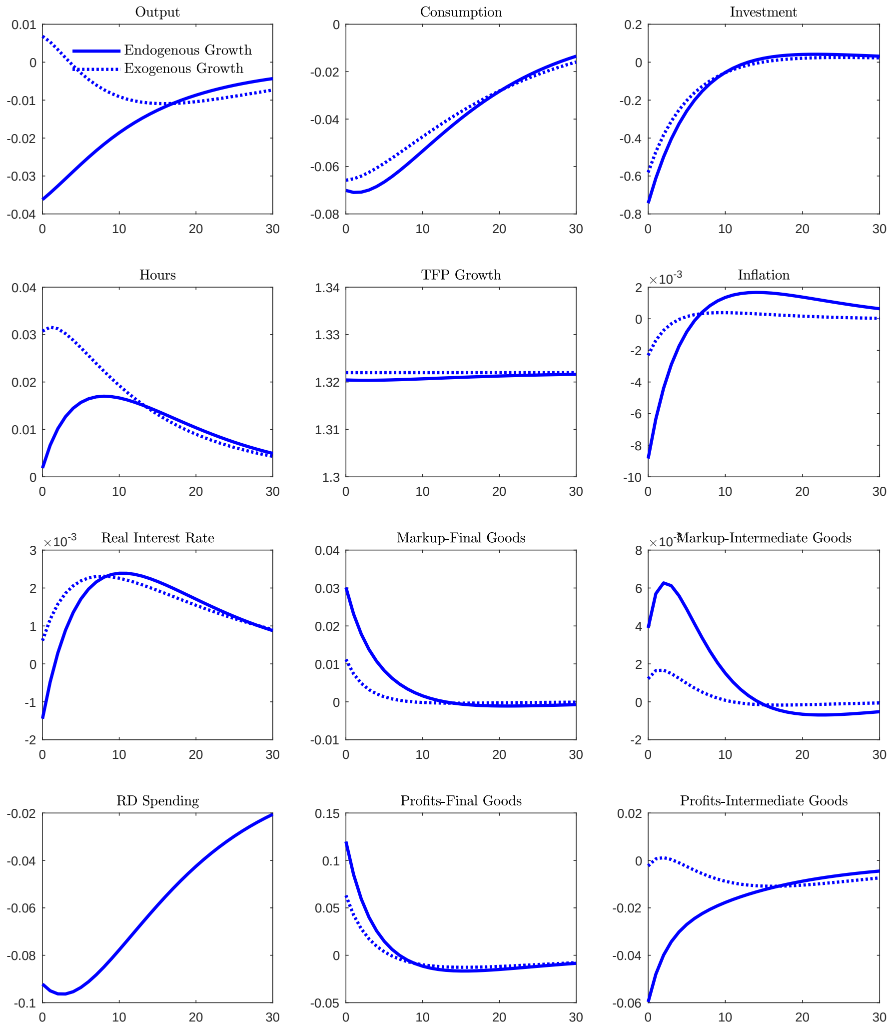
Note: The figure shows optimal trend inflation (the annual inflation rate in %) in the two growth models for different parametrizations, where γ_M is the degree of nominal rigidities in the intermediate good sector, γ_Y is the degree of nominal rigidities in the final good sector, ϵ measures the elasticity of new intermediate goods with respect to R&D and ϕ is the survival rate of intermediate good producers. Vertical lines refer to the baseline calibration.

Figure 2: Impulse Responses to a 1% Technology Shock



Note: The figure shows the impulse response to a shock on A_t. All results are reported as percentage deviations from the steady state, except for inflation, nominal and real interest rates, which are expressed as percentage-point deviations and for the TFP growth which is expressed in annualized rates. Continuous lines show the impulse response functions of the Ramsey plan in the endogenous growth model, while dotted lines refer to the Ramsey plan in the exogenous growth model.

Figure 3: Impulse Responses to a 1% Public Spending Shock



Note: The figure shows the impulse response to a shock on c_t^G. All results are reported as percentage deviations from the steady state, except for inflation, nominal and real interest rates, which are expressed as percentage-point deviations and for the TFP growth which is expressed in annualized rates. Continuous lines show the impulse response functions of the Ramsey plan in the endogenous growth model, while dotted lines refer to the Ramsey plan in the exogenous growth model.