Innovation, Growth and Optimal Monetary Policy*

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February 2018

Abstract

This paper examines how innovation-led growth is likely to affect optimal monetary policy. We consider the Ramsey policy in a New Keynesian model where R&D leads to an expanding variety of intermediate goods and compare the results with those obtained when the expansion occurs exogenously. A first striking result is that positive trend inflation is found to be optimal under both assumptions, but much higher with profit-seeking innovation. A second result is that optimal monetary policy must be counter-cyclical in response to both technology and public spending shocks, but again the intensity of the reaction crucially depends on the presence of an R&D sector. However, the small amount of short-run deviations of prices from the non-zero trend inflation observed in response to shocks suggests inflation targeting as a robust policy recommendation.

Keywords: Endogenous Growth, R&D, Optimal Monetary Policy, Ramsey Problem.
JEL codes: E32, E52, O42.

*We have benefited from presentations at the DEGIT XXI Conference - University of Nottingham, the 4th Macro Banking and Finance Workshop - Sapienza University of Rome, the 2nd FGN International Conference - University of Saint Gallen, the Workshop on Economic Growth, Innovation, and Corporate Governance - SKEMA Business School, the 13th Biennial Athenian Policy Forum Conference Program and the conference on Finance and Economic Growth in the Aftermath of the Crisis - University of Milan.

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1 Introduction

Macroeconomics traditionally studies the role of monetary policy in influencing business cycle fluctuations in models featuring exogenous growth or no growth at all. However, in the words of Lawrence H. Summers, “reversion back to trend is actually less common than evidence that the recession not only reduces the level of GDP, but reduces the trend rate of growth of GDP, what Larry Ball has referred to as super hysteresis” (Summers 2015, p. 8). For instance, there is ample evidence that short-term fluctuations are likely to affect growth-enhancing activities (i.e. savings, investments, R&D activities) and modify the growth trend of the entire economy.[1] Over the postwar period significant oscillations between periods of robust growth versus relative stagnation have been observed in many industrialized countries. Comin and Gertler (2006) and Comin et al. (2009) suggest that these medium-frequency oscillations may, to a significant degree, be triggered by business cycle disturbances at the high frequency propagated by R&D activities. What are then the implications for monetary policy? An early attempt to study optimal monetary policy while considering the interaction between short-run dynamics and long-run growth can be found in Blackburn and Pelloni (2005) and more recently in Annicchiarico and Rossi (2013), who both base their analysis on a stochastic version of the AK model with knowledge spillovers *à la* Romer (1986). In this model there is no profit-seeking innovation activity by firms: however this is a feature which is crucial, as we will show, to shape the optimal design of monetary policy in response to shocks.

In this paper we extend a prototypical New Keynesian (NK) model, with monopolistically competitive final and intermediate goods sectors, by incorporating in it an R&D sector leading to an expansion in the variety of the intermediates, as in Romer (1990). Looking at what we do from a different angle, we use a somewhat simplified version of Comin and Gertler (2006) augmented by allowing for price rigidities *à la* Rotemberg in the two imperfectly competitive sectors.[2] In the model R&D activity is stronger during expansions because its rewards are higher too. In fact, innovation creates monopoly power which will be exploited on a larger scale when aggregate demand is higher. The mechanism underlying the co-movement between R&D activity and output is therefore close to that described by Fatás (2000), where also positive shocks may induce long-run effects by permanently shifting out the production possibilities frontier of the economy.

The paper then asks the following fundamental questions: what is the optimal trend inflation in an economy with innovation-led growth? How does monetary policy optimally respond to business cycles in this framework? How does the nature of the shocks hitting the economy shape this optimal response? To address these questions we study the Ramsey monetary policy in a calibrated model-economy where technical progress represented by an expansion in the variety of inputs

[1] In this respect, following the seminal work of Ramey and Ramey (1995) the question of precisely how cyclical fluctuations might affect long-run growth has been the subject of a broad body of research. See e.g. Martin and Rogers (1997), Aghion and Saint-Paul (1998), De Hek (1999), Martin and Rogers (2000), Jones et al. (2005), Aghion et al. (2010) and Oikawa (2010). However, there are very few investigations that analyze the role of monetary factors (e.g. Dotsey and Sarte 2000 and Varvarigos 2008), while an even smaller subset introduce nominal rigidities to study the interplay between uncertainty and growth under various monetary regimes (e.g. Pelloni 1997, Blackburn and Pelloni 2004, 2005, Annicchiarico et al. 2011 and Annicchiarico and Pelloni 2014).

[2] Similar models have been used by Comin et al. (2009) and Kung and Schmid (2015) to study the link between innovation, growth rate dynamics and asset pricing, Other interesting contributions in this direction are Bianchi et al. (2014) and Cova et al. (2017). Croce et al. (2013) studies in a similar framework the effects of short- and long-run stabilization tax policies.

is driven by R&D, and we compare the results obtained with those arising when the expansion happens exogenously. We consider three different sources of uncertainty: the level of total factor productivity, of R&D productivity, and of real government purchases which are assumed to be fully financed by lump-sum taxes. As usual, we derive our results under the assumption that there is full commitment on the part of the monetary authority in determining the optimal allocation of resources, given the resource constraint of the economy and the additional constraints represented by the market general equilibrium conditions.

Our results are twofold, pertaining to optimal trend inflation and to the short-run response of inflation to shocks in the two growth settings.

Our first result is that significant deviations from zero trend inflation are optimal regardless of the assumptions on long-run growth. This finding is striking because the optimality of long-run zero inflation is a core result of NK models and one which is, in fact, robust across most modeling variations (see e.g. King and Wolman 1999, Schmitt-Grohé and Uribe 2008, Schmitt-Grohé and Uribe 2010 and Woodford 2011). Our second result is that an important quantitative determinant of the optimal long-run rate of inflation is the presence of innovation activities in the economy, along with the degree of nominal rigidities. A third result is that in the short run, optimal monetary policy requires moderate deviations from full inflation targeting in response to both technology and government spending shocks, no matter whether long-run growth is endogenous or exogenous. A final result is that the intensity of the reaction of the optimizing monetary authority to the considered supply and demand shocks turns out to be different in the two growth scenarios. The Ramsey planner would in fact find it optimal to allow for much larger deviations from price stability in response to technological shocks in the endogenous growth setting.

To understand our finding of positive trend inflation we first single out the market failures in the model, which the Ramsey planner can affect by setting inflation. The presence of the two non-competitive sectors implies a static inefficiency familiar from the standard analysis of monopoly. When technical progress is driven by R&D, a second, dynamic, market failure stems from the incomplete appropriability of the social surplus from a new invention. Higher markups in the final and intermediate goods sectors lower the level of economic activity and generate higher degrees of both these market failures.[3] Because of the presence of price adjustment costs the monetary authority has control to some degree over markups that can be eroded or enhanced by changes in inflation. The desirability of this control does not arise in a standard NK model where the time discount factor of the Ramsey planner is the same as that of the firms in the imperfectly competitive sectors. In our model the two are different because of growth and firms' entry in the intermediate sector, as we will show. In this context the Ramsey planner must find a compromise between the welfare gains of inflation, via the decrease in markups and increase in the level of economic activity, and the welfare losses via the adjustment costs in prices it entails. Consistently with our explanation, we find that when technical progress is driven by R&D, the optimal trend inflation is higher as the Ramsey planner factors in not only the static welfare benefit from a higher scale of production, but also the dynamic one accruing from higher growth.

In the short run, optimal monetary policy requires moderate deviations from full inflation targeting in response to both technology and government spending shocks, no matter whether long-

[3]This "downsizing" clearly emerges from the effects that market power itself has on the equilibrium level of production in the intermediate and final goods sectors.

run growth is endogenous or exogenous. However, the intensity of the reaction of the optimizing monetary authority to the considered supply and demand shocks turns out to be different in the two scenarios. The Ramsey planner would in fact find it optimal to allow for larger deviations from price stability in response to technological shocks when innovation is endogenous, because the distortions due to the lack of perfect competition reduce the market size for innovations.[4] On the contrary, when the economy is hit by government spending shocks the response of the Ramsey planner is more attenuated in an endogenous growth setting and tends to be accommodative at the earlier stages of the dynamic adjustment to the demand shock. This is because R&D spending is heavily crowded out by increases in government expenditures. This absorbs much of the effects of the shock and, therefore, stabilizes the response of the rest of the economy.[5] Overall, the small amount of short-run deviations of inflation from its long-run level observed in response to shocks suggests inflation targeting as a robust policy prescription.

This study mainly contributes to the business cycle NK literature on optimal monetary policy which is quite vast, but as its positive counterpart usually abstracts from the endogeneity of growth, e.g. Woodford (2002), Khan et al. (2003), Schmitt-Grohé and Uribe (2004), Benigno and Woodford (2005), Schmitt-Grohé and Uribe (2007), Faia (2008).[6] This study is also related to the literature on monetary policy, inflation and economic growth, where however, the focus is on the long-run relationship between inflation and growth, and between inflation and welfare, while the optimal transition dynamics in response to shocks is usually not investigated. Recent contributions include Arato (2009), Vaona (2012), Chu and Cozzi (2014), Chen (2015), Oikawa and Ueda (2015a), Arawatari et al. (2016), Chu and Ji (2016), Chu et al. (2017) and Zheng et al. (2017).[7]

The organization of the paper is as follows. In Section 2 we outline the main features of the NK endogenous growth model with innovation and those of the exogenous growth counterpart. Section 3 characterizes optimal monetary policy in the long- and in the short-run for both growth settings. Section 4 concludes.

2 The NK Model with R&D

There are three sectors in the economy, namely, a perfectly competitive R&D sector, a monopolistically competitive intermediate good sector and a monopolistically competitive final good sector. R&D activity leads to the creation of patents on new intermediate goods used along with existing ones in the production of final goods. An expansion in the variety of intermediate goods is the ultimate source of technological progress and, therefore, of sustained growth.

[4] This result is qualitatively consistent with the previous findings of Annicchiarico and Rossi (2013), but the response of inflation is much higher in this setting with endogenous R&D.

[5] In AK model the optimal monetary in response to a positive public consumption shock is such that the real rate is always positive along all the adjustment path. See Annicchiarico and Rossi (2013).

[6] The basic NK model has been extended along several dimensions. Some relevant examples include papers accounting for nominal wage rigidities (Erceg et al. 2000), various real frictions in the labor market (Faia 2009, Faia and Rossi 2013, Faia et al. 2014) and endogenous firm entry (Faia 2012, Bilbiie et al. 2014).

[7] For a positive analysis of the trade-off between the short-run positive and long-run negative effects of monetary easing, see Oikawa and Ueda (2015b) who conduct their study in a Schumpeterian model of creative destruction.

2.1 Final Good-Producing Firms

In the final good sector each firm has monopoly power over its particular good. As is common practice, we further assume the existence of an output aggregator who assembles the differentiated final goods $Y_{i,t}$, $i \in [0,1]$, into a single final product, Y_t, which we refer to as the final output index, by relying on a constant-return-to-scale technology of the type $Y_t = \left(\int_0^1 Y_{i,t}^{1-\frac{1}{\theta_Y}} di \right)^{\frac{\theta_Y}{\theta_Y - 1}}$, with $\theta_Y > 1$ denoting the elasticity of substitution between the various goods. If the price of each variety, $P_{i,t}$, is taken as given, by profit maximization we have the usual set of demand schedules $Y_{i,t} = (P_{i,t}/P_t)^{-\theta_Y} Y_t$ for all $i \in [0,1]$, where $P_t = \left(\int_0^1 P_{i,t}^{1-\theta_Y} di \right)^{\frac{1}{1-\theta_Y}}$ is the Dixit-Stiglitz aggregate price index. The aggregator will, in turn, sell units of the final output index at their unit cost P_t.

All final good firms have access to the same technology, represented by the following production function:

$$Y_{i,t} = A_t \left(K_{i,t}^{1-\alpha} N_{i,t}^{\alpha} \right)^v G_{i,t}^{1-v}, \tag{1}$$

where $\alpha \in (0,1)$, $v \in (0,1)$ and A_t measures aggregate productivity and is subject to shocks. The production of the generic final good $Y_{i,t}$, requires the use of capital $K_{i,t}$, labour inputs $N_{i,t}$ and of a CES composite of intermediate inputs $G_{i,t} = \left(\int_0^{Z_t} M_{i,j,t}^{1-\frac{1}{\theta_M}} dj \right)^{\frac{\theta_M}{\theta_M - 1}}$, where $M_{i,j,t}$ is intermediate good $j \in [0, Z_t]$, Z_t is a measure of product variety and $\theta_M > 1$ denotes the elasticity of substitution between the intermediate goods. Notice that we attach a time subscript to Z_t since product variety will be growing over time.

The optimal choice of factor inputs is the solution to a static cost minimization problem, where the nominal wage W_t, the rental cost of capital $P_t R_t^K$ and the price of each intermediate good $P_{j,t}^M$ are taken as given. In a symmetric equilibrium the first-order conditions are then found to be:

$$\frac{W_t}{P_t} = \alpha v MC_t \frac{Y_t}{N_t}, \tag{2}$$

$$R_t^K = (1-\alpha) v MC_t \frac{Y_t}{K_t}, \tag{3}$$

$$\frac{P_{j,t}^M}{P_t} = (1-v) MC_t Y_t \frac{M_{j,t}^{-\frac{1}{\theta_M}}}{G_t^{1-\frac{1}{\theta_M}}}, \text{ for } j \in [0, Z_t], \tag{4}$$

where MC_t denotes the real marginal cost.

Consider now the optimal price setting problem of the typical firm i. Formally, the firm sets the price $P_{i,t}$ by maximizing the present discounted value of expected profits, subject to the demand constraint $Y_{i,t} = (P_{i,t}/P_t)^{-\theta_Y} Y_t$, the available technology for production (1) and the adjustment cost of the Rotemberg type $\frac{\gamma_Y}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 Y_t$ with γ_Y measuring the degree of price rigidity. This price adjustment cost increases in magnitude with the size of the price change and with the size of economic activity. At the optimum, and after having imposed symmetry across firms, we have the following optimal pricing condition:

$$(1-\theta_Y) Y_t + \theta_Y MC_t Y_t - \gamma_Y (\Pi_{Y,t} - 1) \Pi_{Y,t} Y_t + \gamma_Y E_t \Lambda_{t,t+1}^R (\Pi_{Y,t+1} - 1) \Pi_{Y,t+1} Y_{t+1} = 0, \tag{5}$$

where E_t is the rational expectation operator, $\Pi_{Y,t} = P_t/P_{t-1}$ and $\Lambda^R_{t,t+1}$ is the real stochastic discount factor used at time t by shareholders to value date $t+1$ real profits and is related to the households' discount factor β and to their marginal utility of wealth λ_t (i.e. $\Lambda^R_{t,t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t}$). Equation (5) is often referred to as the New Keynesian Phillips curve and describes the equilibrium relationship between inflation and the marginal cost. In the limiting case of fully flexible prices (i.e. $\gamma_Y = 0$), condition (5) collapses to $MC_t = \frac{\theta_Y - 1}{\theta_Y}$. Note that this is equivalent to saying that in the absence of costs on price adjustment price markups are always at their desired level $\frac{\theta_Y}{\theta_Y - 1}$.

2.2 Intermediate Good-Producing Firms

The intermediate goods sector is populated by a continuum of firms belonging to an interval of length Z_t acting as monopolistic competitors, given the demand schedules set by the final good firms. Intermediate goods producers transform one unit of the CES composite of final goods into one unit of their respective intermediate good. In other words, the production is roundabout. As in the final good sector, in this sector firms are assumed to face nominal rigidity in the form of a quadratic adjustment cost function *à la* Rotemberg. At time t each intermediate firm j sets the price $P^M_{j,t}$ so as to maximize the present discounted value of expected profits, given the demand schedule (4) and the adjustment cost $\frac{\gamma_M}{2}\left(\frac{P^M_{j,t}}{P^M_{j,t-1}} - 1\right)^2 M_t$, with γ_M measuring the degree of price rigidity. Given the above assumptions the real profits for firm j in period t can be written as:

$$D_{j,t} = \frac{P^M_{j,t} - P_t}{P_t} M_t - \frac{\gamma_M}{2}\left(\frac{P^M_{j,t}}{P^M_{j,t-1}} - 1\right)^2 M_t. \tag{6}$$

In addition, we assume that firms operating in this sector face a positive probability of being hit by a negative shock forcing them to exit from the market. Let $\phi \in (0,1)$ denote the survival rate of of firms operating in this sector. At the optimum, anticipating that the equilibrium is symmetric, the following optimal pricing condition holds:

$$(\theta_M - 1) M_t p^M_t - \theta_M M_t + \gamma_M \left(\Pi_{M,t} - 1\right) \Pi_{M,t} M_t - \phi E_t \Lambda^R_{t,t+1} \gamma_M \left(\Pi_{M,t+1} - 1\right) \Pi_{M,t+1} M_{t+1} = 0, \tag{7}$$

where $\Pi_{M,t} = P^M_t / P^M_{t-1}$ and $p^M_t = P^M_t / P_t$ is the real relative price of the intermediate good, while $\phi E_t \Lambda^R_{t,t+1}$ is the discount factor adjusted for the survival rate.

Given the roundabout nature of the available technology, p^M_t measures the markup capturing the degree of market power prevailing in this sector. Under fully flexible prices (i.e. $\gamma_M = 0$), condition (5) collapses to $p^M_t = \frac{\theta_M}{\theta_M - 1}$.

After having imposed symmetry, (4) can be expressed as follows:

$$M_t = \left[\frac{1}{p^M_t} MC_t (1-v) A_t \left({K_t}^{1-\alpha} N^\alpha_t\right)^v Z_t^{\theta_M(1-v)/(\theta_M - 1) - 1}\right]^{\frac{1}{v}}. \tag{8}$$

From the above expression we notice that the equilibrium quantity of the intermediate good is negatively affected by the degree of market power in both the final goods sector and in the intermediate goods sector.

The value of owning exclusive rights to produce intermediate goods, denoted by V_t, is equal to the present discounted value of the current and future profits this allows:

$$V_t = D_t + \phi E_t \Lambda^R_{t,t+1} V_{t+1}. \tag{9}$$

where $D_t = (p^M_t - 1)M_t - \frac{\gamma_M}{2}\left(\Pi_{M,t} - 1\right)^2 M_t$. Clearly, in this context, the effect of imperfect competition in the intermediate goods on the value of patents is twofold. On the one hand, less competition has a direct positive effect on profits, through the effects on the markup. On the other hand, less competition has a negative effect on profits through the negative impact it has on M_t. We will see that the latter effect tends to dominate the former, i.e. profits are pro-cyclical. The pro-cyclicality of profits implies that the value of patents is also pro-cyclical. Since the value of patents is the payoff to innovation, as described below, this implies that the payoff to innovation is pro-cyclical as well.

2.3 R&D Sector

In the R&D sector researchers develop blueprints for new intermediate goods. The patents are then sold to firms that produce the new goods. For simplicity we assume that innovators finance their activity by borrowing from households. Assuming free entry in the R&D sector, the price of a new patent will be equal to its value for a new firm i.e. V_t. The R&D sector is characterized by a linear technology. Let S_t be the total amount of R&D expenditure in terms of the final good and ξ_t be its productivity level. Given the intermediate product survival rate ϕ, the law of motion for the measure of intermediate goods Z_t is then

$$Z_{t+1} = \xi_t S_t + \phi Z_t, \tag{10}$$

where, as in Comin and Gertler (2006), the technology coefficient ξ_t involves a congestion externality effect capturing decreasing returns to scale in the innovation sector (i.e. a "stepping on toes effect"):

$$\xi_t = \hat{\xi}\left(Z_t/S_t\right)^{1-\varepsilon}, \ \varepsilon \in (0,1), \tag{11}$$

with ε measuring the elasticity of new intermediate goods with respect to R&D and $\hat{\xi}$ being a scale parameter. Perfect competition in the R&D sector implies that the following break-even condition must hold:

$$E_t \Lambda^R_{t,t+1} V_{t+1}(Z_{t+1} - \phi Z_t) = S_t, \tag{12}$$

where V_{t+1} is the price of an innovation at time $t+1$. The above condition simply says that the expected sales revenues, $E_t \Lambda^R_{t,t+1} V_{t+1}(Z_{t+1} - \phi Z_t)$, must be equal to the cost S_t. This condition can be equivalently formulated using (10) as

$$1/\xi_t = E_t\left(\Lambda^R_{t,t+1} V_{t+1}\right), \tag{13}$$

which simply implies that the marginal cost $1/\xi_t$ equals the expected marginal revenue $E_t\left(\Lambda^R_{t,t+1} V_{t+1}\right)$.

2.4 Households

Consider now the infinitely lived representative household who faces the following time-separable expected utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \mu_n \frac{N_t^{1+\varphi}}{1+\varphi} \right), \tag{14}$$

where β is the subjective discount factor, μ_n is a positive scale parameter measuring the disutility of labor, $\varphi > 0$ measures the inverse of the Frisch elasticity of labour supply and C_t is consumption of the final good. Households make one-period loans to innovators, own monopoly rights on firms and also own the capital stock and let this capital to firms in a perfectly competitive rental market at the real rental rate R_t^K. The period budget constraint takes the form

$$P_t C_t + E_t \left(\Lambda_{t,t+1} B_{t+1} \right) = B_t + W_t N_t + P_t R_t^K K_t - P_t I_t + T_t, \tag{15}$$

for $t = 0, 1, 2...$, where K_t is physical capital carried over from period $t-1$, I_t denotes investments, T_t represents the lump-sum component of income, which includes dividends from the ownership of the firms and non-distortionary taxation. B_t is total loans the household makes at $t-1$ that are payable at t and $\Lambda_{t,t+1}$ is a vector of prices of state-contingent assets. Each element of $\Lambda_{t,t+1}$ is the price of an asset that will pay one unit of currency if a particular state of nature occurs in period $t+1$, while each element of the vector B_{t+1} represents the quantity of such contingent claim purchased at time t. Hence, the risk-free (gross) nominal interest rate is given by $R_t^{-1} = E_t \left(\Lambda_{t,t+1} \right)$.

Investment increases the household's stock of capital according to a standard law of motion:

$$K_{t+1} = (1-\delta) K_t + I_t, \tag{16}$$

where $\delta \in (0,1)$ is the depreciation rate of capital. The typical household will choose the sequences $\{C_t,\ B_{t+1},\ K_{t+1},\ I_t,\ N_t\}_{t=0}^{\infty}$ so as to maximize (14), subject to (15) and (16). The household maximization problem delivers the following optimality conditions:

$$C_t^{-1} = \lambda_t, \tag{17}$$

$$E_t \Lambda_{t,t+1} = \beta E_t \frac{\lambda_{t+1}/P_{t+1}}{\lambda_t / P_t} = \frac{1}{R_t}, \tag{18}$$

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(R_{t+1}^k + 1 - \delta \right), \tag{19}$$

$$\mu_n \frac{N_t{}^{\varphi}}{\lambda_t} = \frac{W_t}{P_t}, \tag{20}$$

where λ_t denotes the Lagrange multiplier associated to the flow budget constraint (15) and measures the marginal utility of consumption, condition (18) gives the price of the state-contingent asset and reflects the optimal choice between current and future consumption, (19) refers to the optimality condition with respect to capital, whereas (20) reflects the optimal choice for non-leisure activities. Clearly, $\Lambda_{t,t+1}$ can be interpreted as the nominal stochastic discount factor, so that its real counterpart is simply $\Lambda_{t,t+1}^R = \Lambda_{t,t+1} \frac{P_t}{P_{t+1}}$.

2.5 Market Clearing

Final output is used for consumption, investment in physical capital, factor input used in the production of intermediate goods, R&D, public expenditure and nominal adjustment costs on prices. In equilibrium factors and goods markets clear and, therefore, the following aggregate resource constraint must hold:

$$Y_t = C_t + I_t + Z_t M_t + S_t + c_t^G Y_t + \frac{\gamma_Y}{2}\left(\Pi_{Y,t} - 1\right)^2 Y_t + \frac{\gamma_M}{2}\left(\Pi_{M,t} - 1\right)^2 Z_t M_t, \tag{21}$$

where c_t^G denotes the public consumption to output ratio, therefore $c_t^G Y_t$ is public consumption, fully financed by lump-sum taxation. This assumption is made to capture the fact that government expenses grow with the economy. The ratio c_t^G is subject to shocks.

Using (8) into the production function (1) final output can be expressed as

$$Y_t = A_t^{\frac{1}{v}} \left[\frac{1}{p_t^M} MC_t(1-v)\right]^{\frac{1-v}{v}} \left({K_t}^{1-\alpha} N_t^{\alpha}\right) Z_t^{\frac{1-v}{v(\theta_M - 1)}}, \tag{22}$$

For the existence of a balanced growth path the aggregate production function must be homogeneous of degree one in the accumulating factors K_t and Z_t. Hence we need the following parameter restriction:

$$\frac{1-v}{v\left(\theta_M - 1\right)} = \alpha, \tag{23}$$

which also ensures stationarity of M_t.[8]

In this economy a number of variables, such as output, consumption etc. will not be stationary along the balanced-growth path. We therefore perform a change of variables, so as to obtain a set of equilibrium conditions that involve only stationary variables. We note that non-stationary variables at time t are cointegrated with Z_t, while the same variables at time $t+1$ are cointegrated with Z_{t+1}. We divide variables by the appropriate cointegrating factor and denote the corresponding stationary variables with lowercase letters. In particular, for any variable, X_t, we have $x_t = X_t/Z_t$. In addition we denote $w_t = \frac{W_t}{Z_t P_t}$ and $g_{Z,t+1} = Z_{t+1}/Z_t$. Variables that need not be transformed are: M_t, MC_t, N_t, p_t^M, R_t, R_t^K, V_t, $\Lambda_{t,t+1}^R$, ξ_t, $\Pi_{Y,t}$ and $\Pi_{M,t}$. The two sources of uncertainty A_t and c_t^G are assumed to evolve as $\log A_t = (1-\rho_A)\log A + \rho_A \log A_{t-1} + \varepsilon_t^A$, with $0 < \rho_A < 1$, $\varepsilon_t^A \sim i.i.d.N(0, \sigma_A^2)$, and $\log c_t^G = (1-\rho_G)\log c_t^G + \rho_G \log c_{t-1}^G + \varepsilon_t^G$, with $0 < \rho_G < 1$, $\varepsilon_t^G \sim i.i.d.N(0, \sigma_G^2)$. The equilibrium conditions of the model expressed in efficiency units are reported in Table 1.

2.6 The NK Model with Exogenous Growth

To isolate the role of innovation we also consider a version of the model incorporating an exogenous growth mechanism. The structure of the economy is the same, so to obtain easily comparable results, but we now assume that the intermediate good sector expands at an exogenously set growth rate:

$$Z_{t+1} = g_z Z_t, \tag{24}$$

[8] From (8) M_t is stationary provided that $(1-\alpha)v + \frac{\theta_M(1-v)}{\theta_M - 1} = 1$. It is straightforward to show that this restriction holds under (23) implying that (8) can be written as $M_t = \left[\frac{\theta_M - 1}{\theta_M} MC_t(1-v)A_t\right]^{\frac{1}{v}} \left(\frac{K_t}{Z_t}\right)^{1-\alpha} N_t^{\alpha}$.

where g_z denotes a deterministic growth factor, so that there is no more a role for R&D activity.[9] Therefore, the above equation replaces (10)-(13), while the resource constraint of the economy becomes

$$Y_t = C_t + I_t + Z_t M_t + c_t^G Y_t + \frac{\gamma_Y}{2} \left(\Pi_{Y,t} - 1\right)^2 Y_t + \frac{\gamma_M}{2} \left(\Pi_{M,t} - 1\right)^2 Z_t M_t, \tag{25}$$

which replaces (21). Using the same notation adopted in the previous section, the exogenous growth model in efficiency units is summarized in Table 2.

2.7 Market Failures

Before turning to the implications of the model for monetary policy, we describe in more detail the distortions which the monetary authorities try with their action to minimize.

A first source of inefficiency is due to price rigidities, here introduced according to the Rotemberg setting. This pricing assumption gives rise to a wedge between aggregate demand and aggregate output, since a part of output is used for adjusting prices (see equations 21 and 25). Clearly, this wedge is eliminated at zero inflation.

A second source of inefficiency stems from the existence of monopolistically competitive producers: the quantity produced of each intermediate and final good is too low as the price is higher than the marginal cost. This is the static inefficiency familiar from the standard analysis of monopoly.

A third source of inefficiency arises in the model with R&D where, as in Romer (1990), the invention of a new intermediate has a cost to be paid upfront. However only the present discounted value of profits, as opposed to all of social surplus originating from the invention, is taken into account by an entrepreneur when deciding whether to pay for it: the rate of innovation is therefore inefficiently low.

Finally, there is a disparity in the price setting behavior of firms in the two monopolistic sectors, no matter whether long-run growth is exogenous or endogenous. This disparity becomes evident when the equations are stationarized as in Tables 1 and 2, and arises because the firm size and the cost of price adjustment per firm do not grow asymptotically in the intermediate goods sector.

3 Optimal Ramsey Monetary Policy

We now consider the problem of a monetary authority (Ramsey planner) which maximizes the expected discounted utility of households, given the constraints represented by the general equilibrium conditions of the market economies outlined in Section 2.[10] As is common practice, we assume that the Ramsey planner is able to commit to the contingent policy rule it announces at time 0

[9]This way of introducing exogenous technical progress was chosen to focus sharply on the consequences for optimal monetary policy of the assumption of innovation-led growth. Comparing results obtained in the same model but with no growth or with A_t growing exogenously would have made the interpretation of the differences less transparent. However, the assumption of Z_t growing exogenously could also be taken to represent the limit case of a follower economy which copies some prototypes of goods invented abroad.

[10]The Ramsey approach allows to study the optimal policy around a distorted steady state, as the one in our model. See Khan et al. (2003), Benigno and Woodford (2005), Schmitt-Grohé and Uribe (2007), Faia (2009) for a discussion on welfare analysis with a distorted steady state.

(i.e. there is ex-ante commitment to a feedback policy so as to have the ability to dynamically adapt the policy to the changed economic conditions). We assume that the planner's discount rate is β. In the economy with endogenous growth the Ramsey planner then maximizes (14), which can be easily expressed in efficiency units, subject to the constraints listed in Table 1. In the economy with exogenous growth, in turn, the Ramsey planner maximizes (14), subject to the constraints listed in Table 2. See the Appendix for details.

In what follows we will make two uses of the first-order conditions from the Ramsey policy problem.[11] First, we use these conditions to characterize the optimal trend inflation by finding a fixed point of the deterministic version of these conditions (i.e assuming no shocks). Second, we analyze the dynamic response of the economy to exogenous shocks. In doing so we assume that the Ramsey planner will react to shocks when it has long been operating under the optimal policy.

3.1 Optimal Trend Inflation

The optimal trend inflation rate is computed by solving the first-order conditions of the Ramsey plan in steady state. The inflation rate so computed reflects the so-called *modified golden rule steady state*, that is the steady state to which it is optimal for the policy marker to converge.[12] Notably, in a simple cashless NK model with Rotemberg pricing the optimal trend inflation is zero.[13] In what follows we will see that this result does not necessarily hold in the two growth models of this paper, where growth is generated by the expansion of varieties in the intermediate goods sector, whether the rate of the expansion is determined by R&D expenditure, as in the model of Table 1, or by an exogenous process, as in the model of Table 2.

3.1.1 Quantitative Results

In order to determine the optimal trend inflation we are forced to resort to numerical methods and parametrize the model. For comparison purposes and to make the analysis more transparent our benchmark parametrization is based on the decentralized competitive equilibrium for both models. Therefore, starting from the stationary models of Tables 1 and 2 it is possible to compute the deterministic steady state of both models in the decentralized case and then proceed with the calibration consistently with the existing literature. The model frequency is quarterly. We start with the conventional parameters. The subjective discount factor β is set to 0.99. The labor share α is set equal to 2/3. The physical capital depreciation rate δ is 0.025. We opt to set the inverse of the Frisch elasticity of labor supply φ to 1 which represents an intermediate value for the range of macro and micro data estimates. The elasticity of substitution between differentiated final goods θ_Y is set at 6. The scale parameter μ_n is set to deliver a steady-state fraction of time spent working

[11] The first-order conditions stemming from these problems are optimal from a "timeless perspective", rather than from the perspective of the particular date at which the policy is actually adopted. This is to rule out the possibility that the Ramsey planner could renege on previous announcements. Technically speaking, given the above Ramsey problems, this "timeless perspective" implies that we can focus on the first-order conditions at time $t \geq 1$.

[12] Notably this inflation rate differs from the *golden rule steady state*, that is the inflation rate that maximizes total utility of households along a deterministic balanced growth path. The latter notion of optimality overlooks uncertainty and the transitional dynamics implied by the Ramsey solution. For details, see King and Wolman (1999).

[13] See e.g. Schmitt-Grohé and Uribe (2008) for an analysis of optimal monetary policy in a prototype NK model with Rotemberg pricing.

$N = 0.17$ in the decentralized equilibrium. Steady-state inflation in the decentralized equilibrium is set to zero, therefore $\Pi_P = \Pi_M = 1$. Finally, c_t^G i set at 0.1 in steady state. The parameters γ_Y and γ_M governing price stickiness are set to be consistent with a Calvo's pricing setting with a probability that price will stay unchanged of 0.75. Similarly to Schmitt-Grohé and Uribe (2007) the persistence of the technology shock is $\rho_a = 0.8556$, while that of the government spending shock is $\rho_g = 0.87$.

Now we turn our attention to the parameters related to R&D and innovation. Our calibration mainly follows Comin and Gertler (2006). We consider an annual trend growth rate of output of 2% in the decentralized equilibrium, i.e. $g_z = 1.02^{1/4}$, and an annual obsolescence rate for intermediate goods equal to 3%, yielding $\phi = (1 - 0.03)^{1/4}$. The productivity parameters $\hat{\xi}$ in the R&D technology is set consistently, $\hat{\xi} = 0.20$, while the technology parameter in the final good production function can be normalized to unity, $A = 1$. The gross markup in the intermediate goods sector is set at 1.6, i.e. $\theta_M = 2.67$. We set the elasticity of new intermediate goods with respect to R&D spending at $\varepsilon = 0.5$, so as to ensure real determinacy of the Ramsey equilibrium.

Given this baseline parametrization, we are able to numerically compute the steady state solution to the Ramsey problem and quantify the optimal long-run inflation rate for both models (i.e. the *modified golden rule steady state* inflation).[14] In the benchmark case we observe that the Ramsey optimal inflation rate is 0.663% per year in the endogenous growth model and 0.175% in the exogenous growth model. This result stands in sharp contrast to the common finding emerging in many cashless NK models, where the Ramsey planner cannot use inflation as a device to reduce market inefficiencies and, therefore, opts to neutralize the adjustment costs by setting trend inflation at zero.

However, when we remove the hypothesis of price rigidities in the intermediate good sector both growth models replicate the standard result of zero optimal trend inflation. We deduce that what drives the optimal trend inflation above zero is then the possibility of controlling the markup in the intermediate good sector. If $\gamma_M = 0$ the monetary authority will not be able to use inflation to affect the long-run markup in this sector. As in the standard NK model, the optimality of zero trend inflation in this case derives from the fact that the planner will find it optimal to fully neutralize the distortions induced by the cost on price adjustment in the final good sector which reduces the overall resources available and creates a wedge between aggregate demand and output.[15]

What is so special about the intermediate good sector then? In both growth settings, intermediate good producers when taking their pricing decisions attach a lower weight to the costs of future expected inflation. On a balanced growth path, in fact, this sector expands at the extensive margin, while production at the intensive margin is constant. For this reason the relevant discount factor for pricing decisions is lower in this sector than in the final good sector. This implies that even when considering the dynamic trade-offs implied by the Ramsey solution, the benefits of price

[14] In other words to compute the optimal long-run inflation we opt for a two-step procedure. First, we calibrate both versions of the model so as to characterize the decentralized long-run equilibrium (such that inflation is zero and annual growth is equal to 2%). Then, we use the first-order conditions of the Ramsey problem to compute an initial vector for the Lagrange multipliers. Using this equilibrium as an initial candidate we rely on a non-linear solver to numerically find the Ramsey steady state.

[15] In a previous version of this paper the analysis has been carried out under this hypothesis. See Annicchiarico and Pelloni (2016). Of course, this result also arises since the model does not embody any money demand distortions. See Khan et al. (2003).

changes smoothing do not vanish in steady state, leaving room for a positive trend inflation.[16]

Under the benchmark calibration, in the endogenous growth case the optimal trend inflation is higher than in the exogenous growth one. In fact the higher level of economic activity made possible by lower markups reduces not only the static externality due to monopolistic competition, but also the dynamic externality due the appropriability problem we described above. A higher scale of production favors R&D spending and a higher innovation rate. In the baseline calibration, in fact, the model generates a monotonically positive relationship between inflation and growth.[17]

Figure 1 plots optimal trend inflation for different parametrizations of the model. The higher the degree of nominal rigidities in the intermediate good sector the higher the level of optimal trend inflation in both growth models, while the opposite is true for high price adjustment cost in the final good sector. Intuitively, higher nominal adjustment costs in the intermediate good sector improve the ability of the Ramsey planner to use inflation as a device to reduce market inefficiencies. By contrast, higher nominal adjustment costs in the final good sector imply a decrease of the benefits of positive trend inflation. Remarkably, we notice that for low adjustment costs in the final good sector the optimal trend inflation emerging in the exogenous growth model is higher than that stemming out from an endogenous growth setting. From Figure 1 we also observe that a larger elasticity of new intermediate goods with respect to R&D spending implies a higher optimal trend inflation. Intuitively, a higher ε implies a higher marginal return of R&D spending, making more convenient for the Ramsey planner to decrease markups as a way to free up more resources to be channelled toward R&D activity. Similarly, a higher obsolescence rate will push the Ramsey planner to take more advantage of positive inflation as a means to engineer a reduction of markups. When the rate of substitution of the old ideas by the new ideas is high, the market size when they launch new products becomes crucial for innovators.

3.2 Dynamics under Optimal Monetary Policy

We are now ready to characterize numerically the dynamic properties of Ramsey allocations in response to a positive shock on technology and public consumption by showing the impulse response functions of the main economic variables. The model is calibrated according to the parameterization outlined in the previous section and then solved by using a 'pure' perturbation method based on a second-order approximation of the model around the non-stochastic Ramsey steady state.

Figure 2 shows the Ramsey optimal impulse response functions to a one percent jump in technology shock for output, consumption, investment, hours, inflation, nominal and real interest rates, markups, R&D spending and profits. All results are reported as percentage deviations from the steady state, except for inflation, nominal and real interest rates, which are expressed as percentage-point deviations. Continuous lines show the impulse response functions of the Ramsey plan in the endogenous growth model, while dotted lines refer to the Ramsey plan in the exogenous

[16] See the Appendix for details.

[17] As a contribution to the literature on the long-run relationship between inflation and growth, in the Appendix we show that, in the baseline calibration and in the absence of price adjustment costs in the intermediate-good sector, the model is able to generate a monotonically positive relationship between these two variables. In this case inflation by reducing markups and increasing the market size for the new products is found to foster growth. The relationship is instead found to be hump-shaped when price adjustment costs arise only in the intermediate good sector. In this case the ability of inflation of increasing demand is moderate and is soon counterbalanced by the negative effects of higher price adjustment costs and by the diminished incentives to innovation induced by lower markups.

growth model.

As expected, output, consumption, investment, hours and R&D spending positively react to the technology shock and then gradually reverse back to the steady-state level. However, inflation initially increases, while the nominal interest rate increases by more yielding a higher real rate. Later the economy experiences a decline of inflation and lower real interest rates. During all the adjustment path markups are below their steady state level. Clearly, the Ramsey planner will find it optimal to initially inflate the economy using inflation as an explicit tax on monopolistic profits, so as to engineer a temporary negative effect on the price markups of final and intermediate good producers.[18] Quantitatively, however, we observe a small amount of inflation variation during all the adjustment process.

Turning to the differences between the two growth settings, we notice that with endogenous innovation all these effects tend to be slightly stronger and/or more persistent. In a model with R&D, in fact, inflation initially increases by more, while the real interest rate stays above its steady state level for longer than in the exogenous growth model. Worked hours increase by more when growth is endogenous, while the expansion of consumption is lower. This is because in this case a fraction of the increased output goes to R&D to sustain higher growth rates of output. By contrast, we observe a sharp increase in R&D spending which sustains aggregate demand, so that the effects on output are slightly higher with endogenous innovation. These results can be easily explained by noting that a higher level of technology increases the marginal product of intermediate goods as well, so boosting the demand for them and driving up real profits received by intermediate goods producers from marketing the specialized intermediate good. In other words, the Ramsey planner finds it optimal to decrease markups in order to induce a positive market size effect on innovation incentives and make available the extra resources needed to sustain the higher R&D. Overall, despite these differences, the behavior of the economy is very similar under the two growth settings.

Figure 3 displays the response of the economy to a one percent positive government spending shock. We observe that in both settings it is not desirable for the Ramsey planner to stabilize consumption in the face of government purchase shocks. Moreover, the optimizing monetary authority tightens monetary policy to raise markups in both sectors when government demand is high, thus amplifying the volatility of consumption.[19] The negative effects of this policy reaction on aggregate demand is such to induce a slight decrease of output in both models. However, the inflation and the nominal interest rate responses are such that the real rate is always positive along the adjustment path in the model with exogenous innovation. In this context the optimal monetary policy calls for a higher real rate so as to moderate the temporary expansionary effects of aggregate demand on output. On the other hand, in the economy with endogenous innovation, we observe that, at

[18] Qualitatively similar results stem out in a simple AK model, however inflation is found to react less than in this setting. See Annicchiarico and Rossi (2013). These results are also consistent with those obtained by Faia (2008) in a NK model embodying capital accumulation and Rotemberg price adjustment, but differ substantially with those obtained by Khan et al. (2003) who develop their analysis in a simple NK model with labor as the only production input.

[19] Optimal monetary policy is then found to stabilize output but destabilize consumption in response to government purchase shocks. These results are consistent with those obtained in simple versions of NK with and without capital accumulation. See Goodfriend and King (2001), Khan et al. (2003), Faia (2008). However, in the presence of a subsidy that raises output to its efficient level the prediction of the standard NK model is that zero inflation is optimal irrespective of the nature of the shocks. See Woodford (2002).

least initially, the resulting real interest rate is slightly below its long-run level, suggesting that the Ramsey planner will find it optimal to undertake a slightly accommodative monetary policy. We also observe that the response of all variables is more attenuated. This can be easily explained by noting the sharp decrease of R&D spending which is itself able to absorb a part of the expansionary shock on aggregate demand. In addition, the lower level of output, and the smaller market size for innovation exacerbates this negative response of the R&D expenditure to the shock. In this sense, the existence of an R&D sector acts as a shock absorber. It turns out that with endogenous innovation the optimizing monetary authority will find it optimal to tighten monetary policy when government demand is high to a lesser extent than in a model with exogenous growth.[20] Overall, we observe that in both cases the Ramsey planner manages to stabilize the economy, being the deviations of the variables from their steady state quite modest.

We complete our analysis by exploring the optimal dynamic response to R&D productivity shocks in the endogenous growth model. In particular, we assume that the coefficient $\hat{\xi}$ in (11) is time varying and follows a process of the form $\log \hat{\xi}_t = (1 - \rho_{\hat{\xi}}) \log \hat{\xi} + \rho_{\hat{\xi}} \log \hat{\xi}_{t-1} + \varepsilon_t^{\hat{\xi}}$, with $0 < \rho_{\hat{\xi}} < 1$, $\varepsilon_t^{\hat{\xi}} \sim i.i.d.N(0, \sigma_{\hat{\xi}}^2)$. Figure 4 plots the dynamic responses to a one percent positive shock to R&D productivity under Ramsey monetary policy assuming a high autocorrelation of the shock, namely $\rho_{\hat{\xi}} = 0.9$. Also in this case the Ramsey planner tolerates temporary deviations of the price level from its optimal long-run trend. In the final goods sector markups and profits decline sharply, while in the intermediate good sector profits increase. By using monetary policy the Ramsey planner is able to sustain the positive effects on output and therefore to increase the market size for innovation and innovation incentives during the periods of higher R&D productivity.

Overall, the moderate short-run variation of prices around the non-zero trend inflation observed in response to all the three shocks would suggest inflation targeting as a robust policy recommendation.

4 Conclusion

In this paper we have studied optimal monetary policy in a standard NK model augmented with innovation-led growth. More specifically, in the model profit-motivated R&D activity leads to an expansion in the variety of intermediates. For easy comparability of results we have also considered how results change when the expansion takes place at an exogenous rate. We have shown that, no matter whether growth is endogenous or exogenous, optimal trend inflation can be significantly above zero because the Ramsey planner uses inflation as a device to affect the markups and reduce the deadweight losses arising in the imperfectly competitive sectors of the economy. This use of inflation is made possible by the entry of firms in steady state in the intermediate sector. An important determinant of the optimal long-run rate of inflation is the endogeneity of growth, along with the degree of nominal rigidities. In the economy with endogenous growth optimal trend inflation is always higher. In fact, the decrease of the markups and increase in the level of economic activity so engineered not only reduce the static distortions from monopoly but also the dynamic one from the spillover to innovation in the model.

[20] In AK Romer (1986) model the optimal monetary in response to a positive public consumption shock is such that the real rate is always positive along all the adjustment path similarly to what observed in the model with exogenous innovation. See Annicchiarico and Rossi (2013).

In the short run the Ramsey policy requires small deviations from full inflation targeting in response to both technology and government spending shocks. However, the intensity of the reaction to expansionary supply or demand shocks crucially depends on the underlying growth mechanism. In response to positive shocks on productivity, with endogenous growth, in fact, the Ramsey planner would tolerate larger deviations of the inflation rate above its optimal steady state in the attempt to induce a stronger reduction of the markups and sustain a higher expansion, so as to create the conditions for a stronger positive market size effect for the new products. On the other hand, in response to a positive government shock, where optimality calls for a decline in the price level, an increase in the real interest rate, a fall in consumption and higher markup in the final good sector, we observe that in the endogenous growth setting the optimizing monetary authority would tend to tighten monetary policy to a lesser extent than in the model with exogenous growth. This is due to the fact that the R&D sector is heavily displaced by increases in government spending, with the former absorbing much of the effects of the shock. Finally, when considering positive shocks to R&D productivity in the endogenous growth model, we observe that also in this case, the Ramsey monetary authority will use inflation as a way to lower the markups, so inducing an expansion of the market size for new products.

Overall, in this paper we find further reasons why optimal monetary policy might depart from price stability, by showing the non-trivial role played by the underlying growth mechanism in shaping the optimal policy. However, the moderate short-run variations of prices around a non-zero trend inflation in response to shocks would suggest inflation targeting as a robust policy prescription. We argue that macroeconomic stabilization policies must explicitly consider the additional transmission channel represented by the engine of growth which better describes the economy under study.

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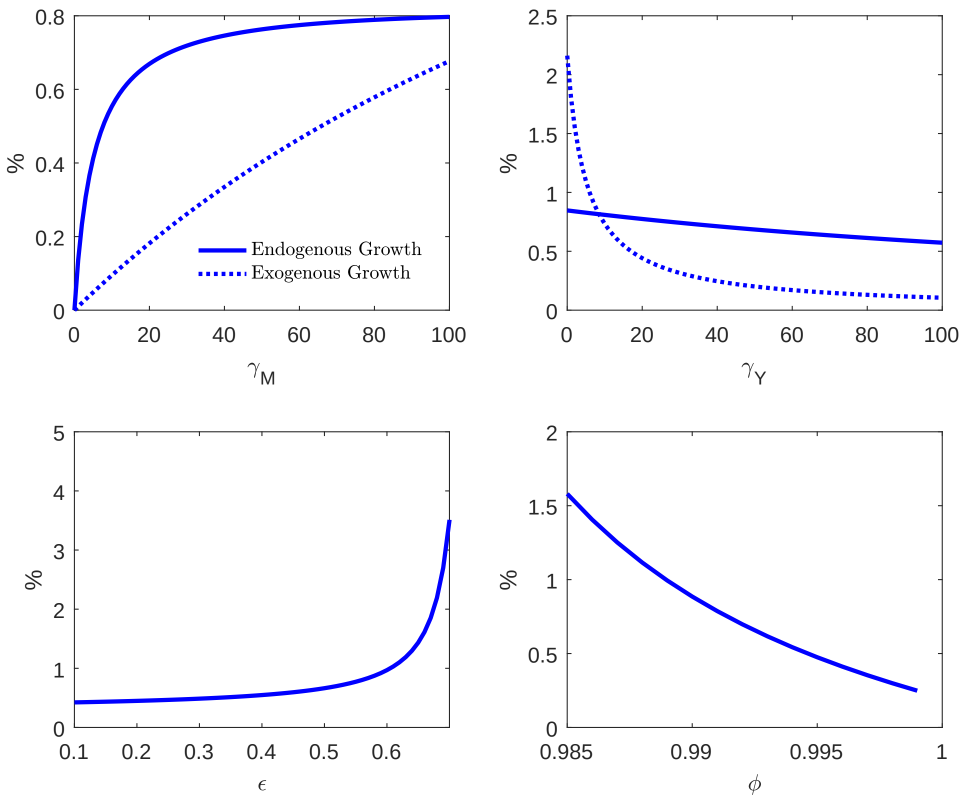
Table 1: Endogenous Growth Model in Efficiency Units

$y_t = c_t + i_t + M_t + s_t + c_t^G y_t + \frac{\gamma_Y}{2}\left(\Pi_{Y,t} - 1\right)^2 y_t + \frac{\gamma_M}{2}\left(\Pi_{M,t} - 1\right)^2 M_t$
$y_t = A_t^{\frac{1}{v}} \left[\frac{1}{p_t^M} MC_t(1-v)\right]^{\frac{1-v}{v}} k_t^{1-\alpha} N_t^{\alpha}$
$M_t = \left[\frac{1}{p_t^M} MC_t(1-v) A_t\right]^{\frac{1}{v}} k_t^{1-\alpha} N_t^{\alpha}$
$w_t = \alpha v MC_t \frac{y_t}{N_t}$
$R_t^K = (1-\alpha)\, v MC_t \frac{y_t}{k_t}$
$k_{t+1} g_{Z,t+1} = (1-\delta)\, k_t + i_t$
$E_t \frac{\Lambda_{t,t+1}^R}{\Pi_{Y,t+1}} = \frac{1}{R_t}$
$1 = \beta E_t \frac{c_t}{g_{Z,t+1} c_{t+1}} \left(R_{t+1}^k + 1 - \delta\right)$
$\mu_n \frac{N_t^{\varphi}}{\psi_t} = w_t$
$\theta_Y - 1 - \theta_Y MC_t + \gamma_Y(\Pi_{Y,t} - 1)\Pi_{Y,t} - \gamma_Y \beta E_t \frac{c_t}{c_{t+1}} \left(\Pi_{Y,t+1} - 1\right) \Pi_{Y,t+1} \frac{y_{t+1}}{y_t} = 0$
$g_{Z,t+1} = \xi_t s_t + \phi$
$\xi_t = \hat{\xi}\left(1/s_t\right)^{1-\varepsilon}$
$V_t = (p_t^M - 1) M_t - \frac{\gamma_M}{2}\left(\Pi_{M,t} - 1\right)^2 M_t + \phi E_t \Lambda_{t,t+1}^R V_{t+1}$
$p_t^M = p_{t-1}^M \frac{\Pi_{M,t}}{\Pi_{P,t}}$
$(\theta_M - 1)\, p_t^M - \theta_M + \gamma_M \left(\Pi_{M,t} - 1\right) \Pi_{M,t} - \phi E_t \Lambda_{t,t+1}^R \gamma_M \left(\Pi_{M,t+1} - 1\right) \Pi_{M,t+1} \frac{M_{t+1}}{M_t} = 0$
$1/\xi_t = E_t\left(\Lambda_{t,t+1}^R V_{t+1}\right)$
$\Lambda_{t,t+1}^R = \beta \frac{c_t}{g_{Z,t+1} c_{t+1}}$

Table 2: Exogenous Growth Model in Efficiency Units

$y_t = c_t + i_t + M_t + s_t + c_t^G y_t + \frac{\gamma_Y}{2}\left(\Pi_{Y,t} - 1\right)^2 y_t + \frac{\gamma_M}{2}\left(\Pi_{M,t} - 1\right)^2 M_t$
$y_t = A_t^{\frac{1}{v}} \left[\frac{1}{p_t^M} MC_t(1-v)\right]^{\frac{1-v}{v}} k_t^{1-\alpha} N_t^{\alpha}$
$M_t = \left[\frac{1}{p_t^M} MC_t(1-v) A_t\right]^{\frac{1}{v}} k_t^{1-\alpha} N_t^{\alpha}$
$w_t = \alpha v MC_t \frac{y_t}{N_t}$
$R_t^K = (1-\alpha)\, v MC_t \frac{y_t}{k_t}$
$k_{t+1} g_Z = (1-\delta)\, k_t + i_t$
$E_t \frac{\Lambda_{t,t+1}^R}{\Pi_{Y,t+1}} = \frac{1}{R_t}$
$1 = E_t \Lambda_{t,t+1}^R \left(R_{t+1}^k + 1 - \delta\right)$
$\mu_n N_t^{\varphi} c_t = w_t$
$\theta_Y - 1 - \theta_Y MC_t + \gamma_Y(\Pi_{Y,t} - 1)\Pi_{Y,t} - \beta E_t \frac{c_t}{c_{t+1}} \gamma_Y \left(\Pi_{Y,t+1} - 1\right) \Pi_{Y,t+1} \frac{y_{t+1}}{y_t} = 0$
$p_t^M = p_{t-1}^M \frac{\Pi_{M,t}}{\Pi_{P,t}}$
$(\theta_M - 1)\, p_t^M - \theta_M + \gamma_M \left(\Pi_{M,t} - 1\right) \Pi_{M,t} - \phi E_t \Lambda_{t,t+1}^R \gamma_M \left(\Pi_{M,t+1} - 1\right) \Pi_{M,t+1} \frac{M_{t+1}}{M_t} = 0$
$\Lambda_{t,t+1}^R = \beta \frac{c_t}{g_Z c_{t+1}}$

Figure 1: Annual Optimal Trend Inflation for Different Model Variations (%)



Note: The figure shows optimal trend inflation for different parametrizations, where γ_M is the degree of nominal rigidities in the intermediate good sector, γ_Y is the degree of nominal rigidities in the final good sector, ϵ measures the elasticity of new intermediate goods with respect to R&D and ϕ is the survival rate of intermediate good producers.

Figure 2: Impulse Responses to a 1% Technology Shock

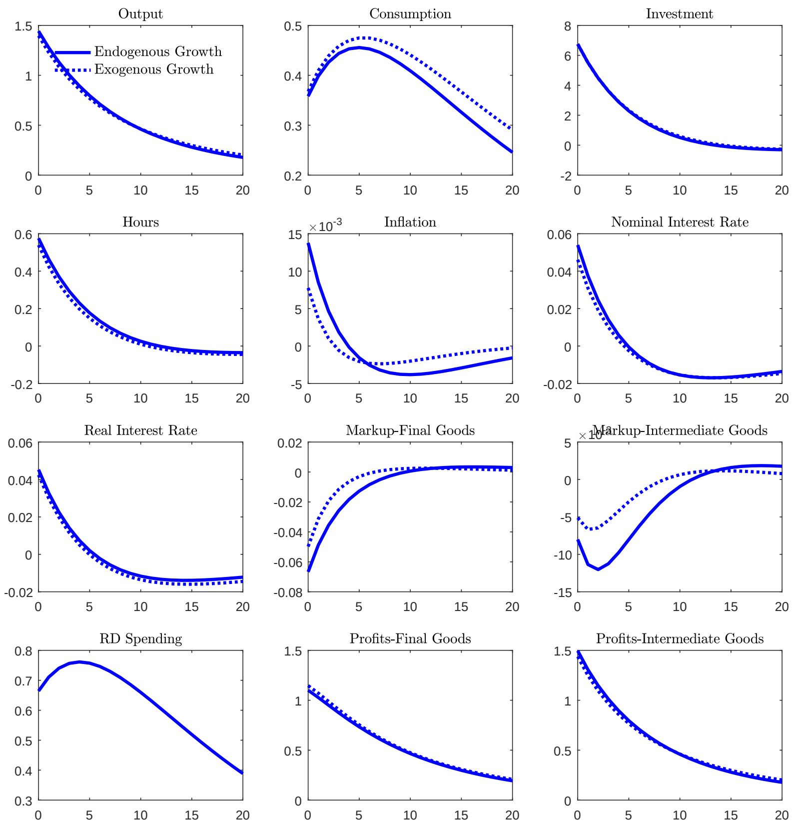


Figure 3: Impulse Responses to a 1% Public Spending Shock

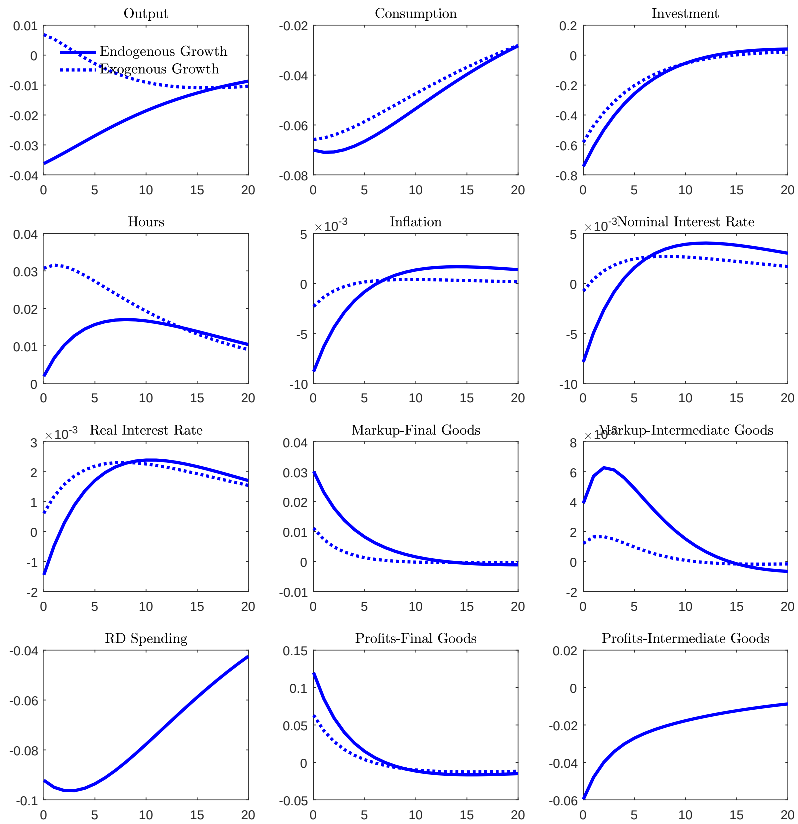
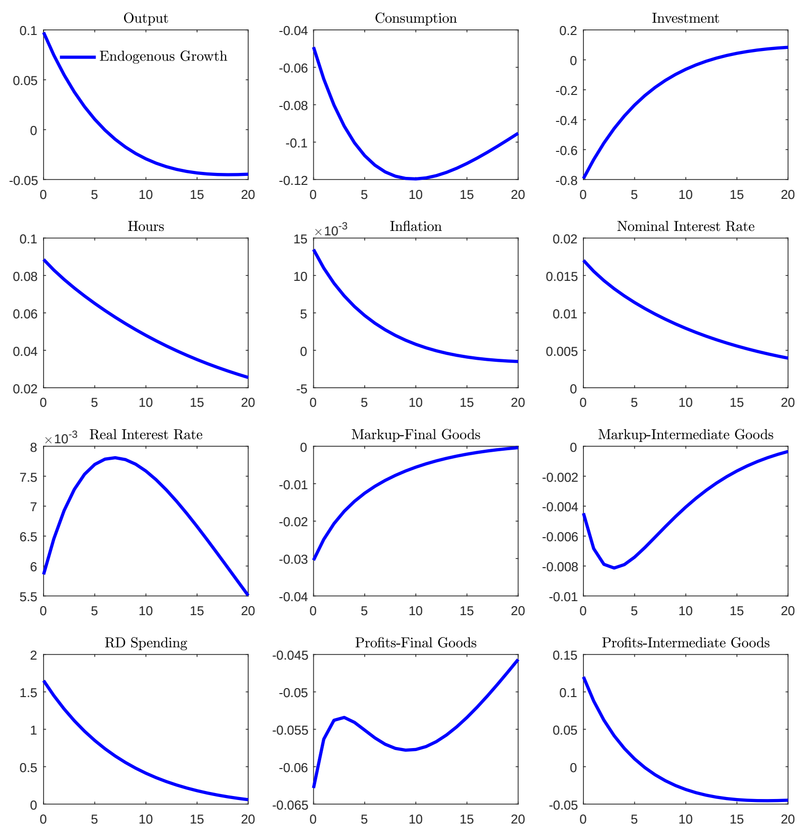


Figure 4: Impulse Responses to a 1% R&D Productivity Shock



Appendix

Welfare Measure in Stationary Variables

The lifetime utility function of the typical individual (21) can be written in recursive form as

$$V_t = \log C_t - \mu_n \frac{{N_t}^{1+\phi}}{1+\phi} + \beta E_t V_{t+1}. \tag{A-1}$$

By adding and subtracting $\frac{1}{1-\beta}\log Z_t$ and $\frac{\beta}{1-\beta}\log Z_{t+1}$ we get

$$\begin{aligned} V_t = {} & \log C_t - \mu_n \frac{{N_t}^{1+\phi}}{1+\phi} + \\ & - \log Z_t + \frac{1}{1-\beta}\log Z_t - \frac{\beta}{1-\beta}\log Z_t + \\ & + \frac{\beta}{1-\beta}\log Z_{t+1} - \frac{\beta}{1-\beta}\log Z_{t+1} + \beta E_t V_{t+1}. \end{aligned} \tag{A-2}$$

where we have used the fact that $\frac{1}{1-\beta}\log Z_t = \log Z_t + \frac{\beta}{1-\beta}\log Z_t$. Collecting terms and defining $\upsilon_t = V_t - \frac{1}{1-\beta}\ln Z_t$ yield:

$$\upsilon_t = E_t \sum_{j=0}^{\infty} \beta^j \left(\log c_{t+j} - \mu_n \frac{N_{t+j}^{1+\phi}}{1+\phi} + \frac{\beta}{1-\beta}\log g_{z,t+1+j} \right). \tag{A-3}$$

Ramsey Monetary Policy in the Endogenous Growth Model

We start by considering the Ramsey problem in the endogenous growth model. For the sake of simplicity we solve the Ramsey problem starting from the constraints already expressed in efficiency units. Having reduced the number of constraints of Table 1, the Lagrangian representation of the Ramsey problem is found to be:

$$
\underset{\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}}{Min}\ \underset{\{\mathbf{d}_t\}_{t=0}^{\infty}}{Max}\ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t E_t \left[\left(\log c_t - \mu_n \frac{N_t^{1+\phi}}{1+\phi} + \frac{\beta}{1-\beta} \log g_{Z,t+1} \right) + \right. \right. \tag{A-4}
$$

$$
\begin{aligned}
&+ \lambda_{1,t} \left(y_t - c_t - k_{t+1} g_{Z,t+1} + (1-\delta) k_t - s_t - M_t - c_t^G y_t - \frac{\gamma_M}{2} (\Pi_{M,t} - 1)^2 M_t - \frac{\gamma_Y}{2} (\Pi_{Y,t} - 1)^2 y_t \right) + \\
&+ \lambda_{2,t} \left(A_t^{\frac{1}{v}} \left(\frac{1}{p_t^M} MC_t (1-v) \right)^{\frac{1-v}{v}} {k_t}^{1-\alpha} N_t^{\alpha} - y_t \right) + \\
&+ \lambda_{3,t} \left(\beta \left((1-\alpha) v \frac{\mu_n {N_{t+1}}^{\varphi+1}}{\alpha v k_{t+1}} + \frac{1-\delta}{c_{t+1}} \right) - \frac{g_{Z,t+1}}{c_t} \right) + \\
&+ \lambda_{4,t} \left[\left((\theta_Y - 1) \frac{y_t}{c_t} - \theta_Y MC_t \frac{y_t}{c_t} + \gamma_Y (\Pi_{Y,t} - 1) \Pi_{Y,t} \frac{y_t}{c_t} - \beta \gamma_Y E_t (\Pi_{Y,t+1} - 1) \Pi_{Y,t+1} \frac{y_{t+1}}{c_{t+1}} \right) \right] + \\
&+ \lambda_{5,t} \left(\hat{\xi} s_t^{\varepsilon} + \phi - g_{Z,t+1} \right) + \\
&+ \lambda_{6,t} \left(-V_t \frac{g_{Z,t+1}}{c_t} + (p_t^M - 1) M_t \frac{g_{Z,t+1}}{c_t} - \frac{\gamma_M}{2} (\Pi_{M,t} - 1)^2 M_t \frac{g_{Z,t+1}}{c_t} + \phi \beta E_t \frac{V_{t+1}}{c_{t+1}} \right) + \\
&+ \lambda_{7,t} \left(-\frac{1}{\hat{\xi}} s_t^{1-\varepsilon} \frac{g_{Z,t+1}}{c_t} + \beta E_t \frac{V_{t+1}}{c_{t+1}} \right) + \\
&+ \lambda_{8,t} \left(\left(\frac{1}{p_t^M} MC_t (1-v) A_t \right)^{\frac{1}{v}} {k_t}^{1-\alpha} N_t^{\alpha} - M_t \right) + \\
&+ \lambda_{9,t} \left(\frac{c_t \mu_n {N_t}^{\varphi+1}}{\alpha v y_t} - MC_t \right) + \\
&+ \lambda_{10,t} \left(p_t^M \frac{\Pi_{P,t}}{\Pi_{M,t}} - p_{t-1}^M \right) + \\
&\left. \left. + \lambda_{11,t} \left((\theta_M - 1) M_t \frac{g_{Z,t+1}}{c_t} p_t^M - \theta_M M_t \frac{g_{Z,t+1}}{c_t} + \gamma_M \frac{g_{Z,t+1}}{c_t} (\Pi_{M,t} - 1) M_t \Pi_{M,t} - \beta \phi E_t \frac{1}{c_{t+1}} \gamma_M (\Pi_{M,t+1} - 1) M_{t+1} \Pi_{M,t+1} \right) \right] \right\},
\end{aligned}
$$

where $\{\mathbf{\Lambda}_t\}_{t=0}^{\infty} = \{\lambda_{1,t},\ \lambda_{2,t},\ \lambda_{3,t},\ \lambda_{4,t},\ \lambda_{5,t},\ \lambda_{6,t},\ \lambda_{7,t},\ \lambda_{8,t},\ \lambda_{9,t},\ \lambda_{10,t},\ \lambda_{11,t}\}_{t=0}^{\infty}$ denote the Lagrange multipliers attached to the constraints and $\{\mathbf{d}_t\}_{t=0}^{\infty} = \{c_t,\ g_{Z,t+1},\ k_{t+1},\ M_t,\ MC_t,\ N_t,\ p_t^M,\ s_t,\ V_t,\ y_t,\ \Pi_{M,t},\ \Pi_{Y,t}\}_{t=0}^{\infty}$. Notice that in the absence of monetary frictions, the nominal interest rate only enters the consumption Euler equation, $\beta E_t \frac{c_t}{\Pi_{Y,t+1} g_{Z,t+1} c_{t+1}} = \frac{1}{R_t}$, that is why this last condition can be omitted from the set of constraints. Basically, it is the intertemporal Euler equation that determines the nominal rate of interest R_t. In what follows we focus on the first order conditions with respect to inflation.

We start by considering the special case in which $\gamma_M = 0$. In the case of flexible prices in the intermediate good sector p_t^M is constant and equal to $\frac{\theta_M}{\theta_M - 1}$, while $\Pi_{M,t}$ is always equal to $\Pi_{Y,t}$. At the optimum, the following first-order conditions with respect to $\Pi_{Y,t}$ must hold:

$$
-\lambda_{1,t} \gamma_Y (\Pi_{Y,t} - 1) y_t + (\lambda_{4,t} - \lambda_{4,t-1}) \gamma_Y \frac{y_t}{c_t} (2\Pi_{Y,t} - 1) = 0, \tag{A-5}
$$

where the first term reflects the marginal effects of inflation on welfare deriving from the negative effects of nominal adjustment costs on the resource constraint, while the second term measures the marginal benefits of smoothing out price changes. Clearly, in steady state the above condition boils down to $\lambda_1 \gamma_Y (\Pi_Y - 1) y = 0$. Since $\lambda_1 > 0$, the optimal steady state inflation rate is then found to be equal to zero, i.e. $\Pi_Y = 1$. The effects of intertemporal cost smoothing vanishes in steady state, thus the Ramsey planner cannot use inflation as a device to reduce the markup. As discussed in the main text, in this case the endogenous growth model replicates the standard prediction of the baseline NK model, namely that the optimal long-run inflation rate is zero.

In the general case, instead, the relevant first-order conditions describing the optimal time paths of inflation for both sectors are found to be:

$$-\lambda_{1,t}\gamma_Y\left(\Pi_{Y,t}-1\right)y_t+\lambda_{4,t}\gamma_Y\frac{y_t}{c_t}(2\Pi_{Y,t}-1)-\lambda_{4,t-1}\gamma_Y\frac{y_t}{c_t}\left(2\Pi_{Y,t}-1\right)+\lambda_{10,t}\frac{p_t^M}{\Pi_{M,t}}=0, \tag{A-6}$$

$$\begin{aligned}&-\lambda_{1,t}\gamma_M\left(\Pi_{M,t}-1\right)M_t-\lambda_{6,t}\gamma_M\left(\Pi_{M,t}-1\right)M_t\frac{g_{Z,t+1}}{c_t}-\lambda_{10,t}p_t^M\frac{\Pi_{P,t}}{\Pi_{M,t}^2}+\\&+\lambda_{11,t}\gamma_M\frac{g_{Z,t+1}}{c_t}\left(2\Pi_{M,t}-1\right)M_t-\lambda_{11,t-1}\frac{\phi}{c_t}\gamma_M\left(2\Pi_{M,t}-1\right)M_t=0.\end{aligned} \tag{A-7}$$

Imposing the steady state and combining the two above conditions we obtain

$$\begin{aligned}&-\lambda_1\left(\gamma_M M+\gamma_Y Y\right)\left(\Pi_Y-1\right)-\lambda_6\gamma_M\left(\Pi_Y-1\right)M\frac{g_Z}{c}+\\&+\lambda_{11}\gamma_M\frac{g_Z}{c}\left(1-\frac{\phi}{g_Z}\right)\left(2\Pi_Y-1\right)M=0,\end{aligned} \tag{A-8}$$

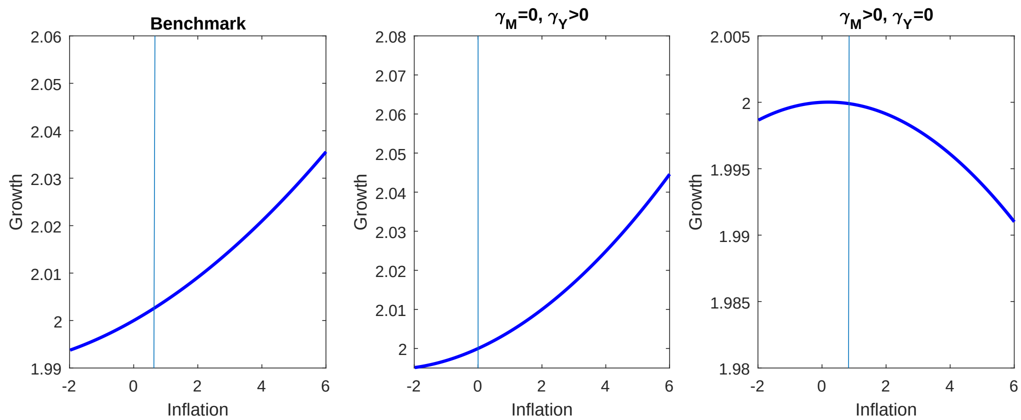
where we have used the fact $\Pi_{Y,t}=\Pi_{M,t}$. The first term measures the marginal negative effect of inflation on welfare through the resource constraint, the second term reflects the negative effects of inflation deriving from a lower firm's value, while the last term refers to marginal benefits of smoothing out price changes between two consecutive periods. Clearly, this last term does not vanish at zero inflation.

Ramsey Monetary Policy in the Exogenous Growth Model

Starting from the exogenous growth model expressed in efficiency units of Table 2 and reducing the number of constraints by substitution, the Ramsey problem can be written as:

$$\begin{aligned}&\underset{\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}}{Min}\ \underset{\{\mathbf{d}_t\}_{t=0}^{\infty}}{Max}\ E_0\left\{\sum_{t=0}^{\infty}\beta^t E_t\left[\left(\log c_t-\mu_n\frac{N_t^{1+\phi}}{1+\phi}+\frac{\beta}{1-\beta}\log g_Z\right)+\right.\right. \quad \text{(A-9)}\\
&+\lambda_{1,t}\left[y_t-c_t-k_{t+1}g_Z+(1-\delta)k_t-M_t-\frac{\gamma_Y}{2}\left(\Pi_{Y,t}-1\right)^2y_t-c_t^Gy_t-\frac{\gamma_M}{2}\left(\Pi_{M,t}-1\right)^2M_t\right]+\\
&+\lambda_{2,t}\left(A_t^{\frac{1}{v}}\left[\frac{1}{p_t^M}MC_t(1-v)\right]^{\frac{1-v}{v}}k_t^{1-\alpha}N_t^{\alpha}-y_t\right)+\\
&+\lambda_{3,t}\left[\beta\left((1-\alpha)v\frac{\mu_nN_{t+1}{}^{\varphi+1}}{\alpha vk_{t+1}}+\frac{1-\delta}{c_{t+1}}\right)-\frac{g_Z}{c_t}\right]+\\
&+\lambda_{4,t}\left[(\theta_Y-1)\frac{y_t}{c_t}-\theta_YMC_t\frac{y_t}{c_t}+\gamma_Y(\Pi_{Y,t}-1)\Pi_{Y,t}\frac{y_t}{c_t}-\beta\gamma_YE_t\left(\Pi_{Y,t+1}-1\right)\Pi_{Y,t+1}\frac{y_{t+1}}{c_{t+1}}\right]+\\
&+\lambda_{5,t}\left(\left[\frac{1}{p_t^M}MC_t(1-v)A_t\right]^{\frac{1}{v}}k_t^{1-\alpha}N_t^{\alpha}-M_t\right)+\\
&+\lambda_{6,t}\left(\frac{c_t\mu_nN_t{}^{\varphi+1}}{\alpha vy_t}-MC_t\right)+\\
&+\lambda_{7,t}\left(p_t^M\frac{\Pi_{P,t}}{\Pi_{M,t}}-p_{t-1}^M\right)+\\
&+\lambda_{8,t}\left((\theta_M-1)M_t\frac{p_t^M}{c_t}g_Z-\theta_MM_t\frac{g_Z}{c_t}+\gamma_M\frac{g_Z}{c_t}\left(\Pi_{M,t}-1\right)M_t\Pi_{M,t}-\beta E_t\frac{1}{c_{t+1}}\gamma_M\left(\Pi_{M,t+1}-1\right)M_{t+1}\Pi_{M,t+1}\right),\end{aligned}$$

Figure A-1: Growth and Inflation in Steady State - Decentralized Equilibrium (annual rates %)



Note: The figure shows the relationship between long-run growth and trend inflation in the benchmark case (γ_M, $\gamma_Y > 0$), in the case of nominal rigidities only in the final good sector ($\gamma_M = 0$, $\gamma_Y > 0$) and in the case of nominal rigidities only in the intermediate good sector ($\gamma_M > 0$, $\gamma_Y = 0$).

where $\{\mathbf{\Lambda}_t\}_{t=0}^{\infty} = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}, \lambda_{7,t}, \lambda_{8,t}\}_{t=0}^{\infty}$ denote the Lagrange multipliers attached to the constraints and $\{\mathbf{d}_t\}_{t=0}^{\infty} = \{c_t,\ k_{t+1}, M_t, MC_t,\ N_t, p_t^M,\ y_t,\ \Pi_{M,t}, \Pi_{Y,t}\}_{t=0}^{\infty}$.

Proceeding as done in the previous section, once can easily show that in the special case of flexible prices in the intermediate good sector in steady state the first order condition with respect to Π_Y becomes $\lambda_1 \frac{\gamma_Y}{2} (\Pi_Y - 1)\, y = 0$, implying the optimality of zero inflation. In the generale case, the first order conditions with respect to $\Pi_{Y,t}$ and $\Pi_{M,t}$, computed in steady state, can be combined to obtain:

$$-\lambda_1 (\gamma_M M + \gamma_Y Y)(\Pi_Y - 1) - \lambda_6 \gamma_M (\Pi_Y - 1) M \frac{g_Z}{c} + \lambda_{11} \gamma_M \frac{g_Z}{c} \left(1 - \frac{1}{g_Z}\right)(2\Pi_{_Y} - 1) M = 0, \tag{A-10}$$

where the last term representing the marginal benefits of intertemporal smoothing price changes does not vanish at zero inflation, opening up to the possibility of exploiting inflation as device to reduce markups.

Inflation and Growth

In this appendix we show the relationship between inflation and growth of the model of Table 1. Using the baseline calibration illustrated in Section 3, we compute the steady state of the model under different inflation rates and for three different parametrizations of the nominal adjustment cost on price. Figure A-1 presents the results. The vertical continuous lines refer to the optimal trend inflation stemming from the Ramsey policy.