Taylor Rules, Long-Run Growth and Real Uncertainty*

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Abstract

We study the effects of real uncertainty on long-run growth under different Taylor-type rules. We find a non-negligible relationship between real uncertainty and growth, which depends on the source of real uncertainty as well as the type of the Taylor rule considered. Importantly, when uncertainty is due to investment-specific shocks, it is highly detrimental for growth, unless the Central Bank follows a strong inflation targeting rule. Furthermore, we find that in the presence of real uncertainty, there is a positive correlation between average growth and average inflation under pure inflation targeting regimes and negative otherwise.

Keywords: Taylor Rules, Endogenous Growth, Real Uncertainty.

JEL codes: E32, E52, O42.

1 Introduction

Traditionally New Keynesian (NK) models abstract from growth to focus on business cycles. However, since the seminal paper by Ramey and Ramey (1995) many theoretical and empirical contributions have shown the importance of the relationship between short-run fluctuations and long-run growth. In an endogenous growth model, uncertainty affects growth-enhancing activities (i.e. savings, learning process, R&D) and modifies the growth trend. Despite these results, very few papers analyze the interaction between growth and business cycle uncertainty in the context of monetary models (e.g. Dotsey and Sarte 2000 and Varvarigos 2008). An even smaller subset consider nominal rigidities (e.g. Blackburn

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*The Appendix is available online on the authors’ homepages.
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and Pelloni 2004, 2005 and Annicchiarico et al. 2011a), but in the form of one-period nominal wage contracts. An exception are Annicchiarico et al. (2011b), who consider a NK model with staggered prices and wages to study the interplay between the two rigidities, nominal short-run volatilities and growth under different Taylor-type rules,\footnote{The authors find a negative relationship between nominal volatility and growth.} but neglecting the role played by monetary policy in shaping the relationship between real uncertainty and long-run growth.

To fill this gap, we consider a NK model embodying an AK growth mechanism à la Romer, and staggered prices à la Calvo, and we study the interaction between real uncertainty and growth under different Taylor-type interest rate rules.

We find a non-negligible relationship between real uncertainty and long-run growth, which depends on the type of the Taylor rule implemented and on the source of uncertainty. In particular, we show that the effect of uncertainty in technology on long-run growth is always negative, but in the case of strong inflation targeting. By contrast, uncertainty in public consumption has always a positive, though negligible, effect on long-run growth, but for a weak inflation targeting rule. Importantly, when uncertainty comes from investment-specific shocks, this is highly detrimental for growth, unless the Central Bank obeys to a strong inflation targeting rule. As will be clarified in the paper, this result is motivated by the fact that for all the Taylor rules considered, but for the strong inflation targeting rule, the price markup effect strongly dominates the precautionary savings effect, making real uncertainty detrimental for long-run growth. Consistently with these findings, our model predicts that growth uncertainty is positively correlated with average output growth only under strong inflation targeting. Similarly, the relationship between average inflation and the standard deviation of inflation is positive as long as the monetary authority reaction to inflation is sufficiently strong. A by-product of these results is that in the presence of real uncertainty, there is a positive correlation between average growth and average inflation only under pure inflation targeting regimes and negative otherwise.

The paper proceeds as follows. Section 2 describes the model. Section 3 investigates the relationship between long-run growth and real uncertainty.

## 2 The Model

We consider a NK-AK model with three sources of uncertainty: technology, the marginal efficiency of investment and government spending, assumed to be fully financed by lump-sum taxes. The monetary authority follows a Taylor-type rule.

2 The Model
2.1 Firms and Households

The final good $Y_t$ is produced by perfectly competitive firms, using the intermediate inputs produced by the intermediate sector: 

$$
Y_t = \left[ \int_0^1 Y_j(\theta_{p-1})/\theta_{p} \, dj \right]^{\theta_p/(\theta_p-1)}, \theta_p > 1. 
$$

A continuum of monopolistic competitive firms $j \in (0, 1)$ produce differentiated intermediate goods $Y_{j,t}$, with 

$$
Y_{j,t} = A_t K_{j,t}^{1-\alpha} (Z_t N_{j,t})^\alpha, \quad \alpha \in (0, 1), 
$$

where $K_{j,t}$ and $N_{j,t}$ denote capital and labor inputs, $Z_t$ represents an index of knowledge, taken as given by each firm, so that learning takes the form of a pure externality. We assume 

$$
Z_t = K_t = \int_0^1 K_{j,t} \, dj. 
$$

$A_t$ is an aggregate productivity shock, 

$$
\log A_t = (1 - \rho_A) \log A + \rho_A \log A_{t-1} + \varepsilon_{A,t}, \quad (2)
$$

with $0 < \rho_A < 1$ and $\varepsilon_{A,t} \sim i.i.d. N(0, \sigma_A^2)$. Prices are modeled à la Calvo, where a fraction $1 - \xi_p$ of firms optimally choose prices.

The representative household maximizes its lifetime utility subject to the budget constraint: 

$$
E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \mu_n \frac{N_t^{1+\phi}}{1+\phi} \right), \quad \phi, \mu_n > 0 \text{ and } 0 < \beta < 1, \quad (3)
$$

where $C_t$ is consumption, $N_t$ labor hours, $K_t$ physical capital, $I_t$ investments and $B_{t+1}$ represents riskless one-period bonds, paying one unit of the numéraire in $t+1$, while $B_t$ is the quantity of bonds carried over from $t-1$. $R_t$ is the gross nominal return on $B_t$, $R^K_t$ is the gross nominal return on capital, $T_t$ denotes lump-sum taxation and $D_t$ are firms’ dividends. There are investment adjustment costs and physical capital accumulates according to 

$$
K_{t+1} = (1 - \delta)K_t + \mu_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t, \quad (5)
$$

where $0 < \delta < 1$ and $\mu_t$ is an investment shock, 

$$
\log \mu_t = \rho_{\mu} \log \mu_{t-1} + \varepsilon_{\mu,t}, \quad (6)
$$

with $0 < \rho_{\mu} < 1$ and $\varepsilon_{\mu,t} \sim i.i.d. N(0, \sigma_{\mu}^2)$. The function $S(\cdot)$ measures the investment adjustment costs, such that $S(1) = S'(1) = 0$ and $S''(1) = \gamma_1 > 0$. 


2.2 Market Clearing, Monetary and Fiscal Policy

In equilibrium factor and good markets clear, hence:

\[ N_t = \int_0^1 N_{j,t} dj, \quad K_t = \int_0^1 K_{j,t} dj, \]

\[ Y_t = AK_t N_t^\alpha (D_{p,t})^{-1}, \quad (7) \]

and aggregate resource constraint,

\[ Y_t = C_t + I_t + G_t, \quad (8) \]

where \( G_t \) is public consumption, evolving as a constant fraction of output on the balanced growth path (BGP) and financed by lump-sum taxation \( T_t \):

\[ \log g_t = (1 - \rho_G) \log g + \rho_G \log g_{t-1} + \varepsilon_{G,t}, \quad (9) \]

where \( 0 < \rho_G < 1 \) and \( \varepsilon_{G,t} \sim i.i.d. \ N(0, \sigma_G^2) \).

The monetary authority implements a Taylor-type rule,

\[ \frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\phi_r} \left[ \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{y_t}{y} \right)^{\phi_y} \right]^{1-\phi_r}, \quad (10) \]

where \( \pi_t = P_t/P_{t-1}, \quad y_t = Y_t/K_t, \) and \( 0 \leq \phi_r < 1, \phi_\pi > 0, \phi_y \geq 0 \) are policy parameters.

To capture the role of monetary policy in transmitting the effects of external shocks and uncertainty on output growth, we will assign different values to these parameters.

2.3 Calibration

The model is calibrated consistently with the literature. Time is in quarters. We consider four different interest rate rules: (i) strong inflation targeting (SIT), with \( \phi_\pi = 5, \phi_r = \phi_y = 0 \); (ii) weak inflation targeting (WIT), where \( \phi_\pi = 1.2, \phi_r = \phi_y = 0 \); (iii) standard Taylor rule (TR), with \( \phi_\pi = 1.5, \phi_y = 0.125, \phi_r = 0 \); (iv) Taylor rule with smoothing (TRS), with \( \phi_\pi = 1, \phi_y = 0.125, \phi_r = 0.8 \).

The discount factor \( \beta \) is set at 0.99, the annual steady-state inflation rate is set at 4\%, \( \varphi \) is equal to 2, and \( \mu_\pi \) is calibrated to get steady-state labor hours equal to 1/3. The price elasticity \( \theta_p \) is set at 6, while \( \xi_p \) is set at 0.75. The labor return to scale is \( \alpha = 2/3 \). Capital depreciation rate \( \delta \) is 0.025. Function \( S(\cdot) \), is \( S \left( \frac{L}{L_{t-1}} \right) = \frac{\gamma_l}{2} \left( \frac{L}{L_{t-1}} - g_K \right)^2 \), where \( g_K \) is the growth rate of capital on BGP. The parameter governing the investment adjustment costs, i.e. \( \gamma_l \), is calibrated at 2.85 as in Justiniano et al. (2010).
We calibrate the remaining parameters to have $C/Y = 0.65$ in steady state and a 2% annual growth rate of output. As in Schmitt-Grohé and Uribe (2007) the persistence of technology shocks and government shocks are $\rho_a = 0.8556, \rho_g = 0.87$, while their standard deviations are $\sigma_a = 0.0064, \sigma_g = 0.016$. Following Justiniano et al. (2010), the investment specific shocks persistence and standard deviation are $\rho_\mu = 0.72$ and $\sigma_\mu = 0.0603$.

3 Growth and Real Uncertainty

We now study the role of monetary policy in transmitting the effect of real uncertainty on long-run growth. Table 1 reports the unconditional mean of the growth rate of output given the baseline standard deviations for the three real shocks, under the four monetary regimes. To clarify the transmission channel of uncertainty, the fifth column shows the results under flexible prices.

We start by discussing the results obtained under the assumption that all the three sources of uncertainty characterize the economy. We notice that only when the monetary policy obeys to a strong inflation targeting rule, the mean growth rate of output is higher than its deterministic counterpart (2% annual terms). In all other cases, in particular under a weak inflation targeting rule, the effect of overall real uncertainty on growth is negative. This result can be explained as follows. There are two different channels of transmission between uncertainty and growth: (i) the precautionary saving effect, (ii) the price markup effect. The precautionary saving channel is the source of a positive effect between uncertainty and growth, since agents react to uncertainty by reducing consumption and accumulating more capital, thus raising growth. Nominal rigidities and monopolistic competition are instead conducive to a negative relationship between uncertainty and growth, since higher uncertainty tends to boost average markup, so reducing the level of economic activity and the pace of growth. Why should higher uncertainty lead to higher markups? Intuitively, the Calvo’s pricing implies that producers resetting their prices choose a price that is a positive function of the weighted average of current and expected future marginal costs. Diminishing marginal productivity of labor implies that the marginal cost is a convex function of labor inputs. Therefore, a higher variability in labor inputs due to uncertainty, raises average nominal marginal costs and increases the price set by firms, implying that a higher markup will prevail in the economy. In this economy the precautionary saving effect is likely to prevail as long as the monetary authority reacts strongly to inflation, while the markup channel dominates when the reaction to inflation is weaker. Intuitively, when the monetary authority strongly reacts to inflation the effects of uncertainty on nominal marginal cost are dampened, since agents anticipate the vigorous response of the Central Bank to inflation. To switch off the markup channel, the fifth column of the table shows the results under flexible prices. In this case the markup is constant and does not depend on uncertainty. As expected, the precautionary saving channel induces positive effects of uncertainty on growth. Because of the presence
of flexible prices monetary policy is neutral and results do not depend on the policy rule.

In the baseline model, instead, the monetary policy affects the way in which uncertainty influences growth and, remarkably it is able to change the sign of this relationship. In respect to this sign there is a lack of consensus in the literature. From this point of view our analysis uncovers an additional channel, represented by the behavior of the monetary authorities.

When considering each source of fluctuations in isolation in the baseline model, we notice that the effect of uncertainty in technology on long-run growth is always negative, but in the case of strong inflation targeting. By contrast, uncertainty in public consumption has always a positive, though negligible, effect on long-run growth, provided that the interest rate rule is sufficiently responsive to inflation. When uncertainty is only due to investment-specific shocks, the effect on mean growth is positive if the monetary policy obeys to a strong inflation targeting rule and negative otherwise. This means that in all cases, but for the strong inflation targeting rule, the price markup effect strongly dominates the precautionary savings effect.

Figure 1 plots the annualized average growth rate of output with respect the annualized standard deviation of output growth, $\sigma_Y$, in the four monetary regimes. For each policy scenario, the plot was obtained by varying proportionally the standard deviations of each source of fluctuations, so as to keep the variance decomposition of output growth unchanged. Consistently with the findings of Table 1, our model predicts that more risky output growth is negatively correlated with average output growth only in the case of weak reaction of interest rate to current economic conditions.

Finally, our model has also predictions about the relationship between average inflation and the standard deviation of inflation. In Figure 2 we plot the annualized average inflation rate with respect to the annualized standard deviation of inflation, $\sigma_\pi$, for the four interest-rate rules. We find a negative relation between average inflation and its standard deviation under weak inflation targeting rule and positive otherwise.

A corollary of these results is that our model generates a positive correlation between average growth and average inflation under pure inflation targeting regimes and negative otherwise.\(^2\)

**References**


\(^2\) The model always displays a positive relationship between inflation and growth at business cycle frequency. See the Appendix.


Table 1: Mean Growth Rate of Output, Uncertainty and Monetary Policy
(Annualized Rates %)

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>SIT</th>
<th>WIT</th>
<th>TR</th>
<th>TRS</th>
<th>FLEX PRICES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi_x = 5$</td>
<td>$\phi_x = 1.2$</td>
<td>$\phi_x = 1.5$</td>
<td>$\phi_x = 1.5$</td>
<td>any rule</td>
</tr>
<tr>
<td></td>
<td>$\phi_y = 0$</td>
<td>$\phi_y = 0$</td>
<td>$\phi_y = 0.5/4$</td>
<td>$\phi_y = 0.5/4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\phi_z = 0$</td>
<td>$\phi_z = 0.8$</td>
<td>$\phi_z = 0$</td>
<td>$\phi_z = 0.8$</td>
<td></td>
</tr>
<tr>
<td>All Shocks</td>
<td>2.0976</td>
<td>0.9984</td>
<td>1.5788</td>
<td>1.8965</td>
<td>2.1089</td>
</tr>
<tr>
<td>Technology</td>
<td>2.0077</td>
<td>1.9931</td>
<td>1.9530</td>
<td>1.9798</td>
<td>2.0088</td>
</tr>
<tr>
<td>Public Consumption</td>
<td>2.0008</td>
<td>1.9993</td>
<td>2.0006</td>
<td>2.0010</td>
<td>2.0008</td>
</tr>
<tr>
<td>Investment</td>
<td>2.0890</td>
<td>1.0059</td>
<td>1.6250</td>
<td>1.9157</td>
<td>2.0992</td>
</tr>
</tbody>
</table>
Figure 1: Mean and Standard Deviation of Output Growth Rate (Annualized Rates %)
Figure 2: Mean and Standard Deviation of Inflation (Annualized Rates %)

- **ST**: 
  - Inflation vs. $\sigma_\pi$
  - Graph showing the relationship between inflation and $\sigma_\pi$.

- **WIT**: 
  - Inflation vs. $\sigma_\pi$
  - Graph showing the relationship between inflation and $\sigma_\pi$.

- **TR**: 
  - Inflation vs. $\sigma_\pi$
  - Graph showing the relationship between inflation and $\sigma_\pi$.

- **TRS**: 
  - Inflation vs. $\sigma_\pi$
  - Graph showing the relationship between inflation and $\sigma_\pi$. 