150 Years of Italian CO_2 Emissions and Economic Growth

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Abstract

This paper examines the relationship between economic growth and carbon dioxide emissions in Italy considering the developments in a 150-year time span. Using several statistical techniques, allowing for structural changes and non-linearities, we find that GDP growth and carbon dioxide emissions are strongly interrelated, with a dramatic change of the elasticity of pollutant emissions with respect to output. Our findings highlight lack of a recent structural change in the reduction of the carbon dioxide, suggesting the difficulties for Italy to meet the emissions targets within the Europe 2020 strategy.

Keywords: Carbon Dioxide Emissions, Time Series Analysis, Italian Economy, Europe 2020.
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1 Introduction

Environmental awareness is becoming increasingly important in the policy debate, and in particular the transition towards a low-carbon economy represents one of the major economic challenges. In this context, the realization of the Europe 2020 strategy for sustainable economic growth relies on the use of sustainable energy sources, having three main headline targets, that is (i) drastic cut down of the carbon dioxide (CO_2) emissions, (ii) increasing the share of renewable energy sources in final energy consumption, and (iii) increasing energy efficiency. This set of EU-level goals have been translated into national objectives by each member state taking into account country-specific economic circumstances. Over the last decades Italy has made a unilateral commitment to reduce overall greenhouse gas emissions by 13% compared to 1990 levels, increase the share of renewable energy sources in final energy consumption to 17% and cut energy consumption by 27.90 mega tons of oil equivalent (mtoe), always with reference to 1990 levels.

In this paper we examine the relationship between economic growth and carbon dioxide emissions in Italy for a period which goes from the unification, 1861, up to 2011. To this end we adopt the most recent statistical reconstruction of the GDP series for the last 150 years.[1] After a period of prolonged low growth, the Italian economy has recently found itself at the center of a deep economic crisis. In this economic turmoil, at the moment only partially tamed, the environmental policy debate has been put aside and Italy has not yet clearly articulated its future energy strategies, even though greenhouse gases (GHG) emissions are largely derived from energy-related activities (OECD, 2012). The recent economic events represent an opportunity for Italy to restructure its economy looking at other alternative sources to satisfy its energy needs. In particular, Italy has limited domestic energy resources with high dependence on external energy supply, and thus it is characterized by an energy import dependency of 83.8% in 2010 against a EU-27 of 52.7%, see European Council (2012). Since 1990 final energy consumption has been increasing steadily, with transport, households and industry being the most energy-consuming sectors.[2] Italian per capita CO_2 emissions are well below the EU-27 average, however, the energy intensity in Italy is lower than the EU-27 average, but the carbon dioxide emissions are above the EU-27 mean level.[3] In particular, the Italian oil and gas shares in primary energy

[1]The GDP series has been part of a study published on occasion of the celebrations of the 150th anniversary of the unification of Italy; for more details, see Baffigi (2011); Vecchi (2011).

[2]In 2010, probably as a consequence of the strong slowdown of the economy, the households sector consumed more energy than the industry sector (25.2% against 24.9%, while the transport sector absorbed 33.6% of the total final energy consumption). See European Council (2012).

[3]In 2009 Italian carbon dioxide emissions per capita were equal to 7,200.8 kg /cap, while the EU-27 level was 8,105 kg/cap; the Italian CO_2 intensity was 2,549.9 kg CO_2/toe , while the average in the EU was equal to 2,381

supply are above the European average, while hydroelectricity and other renewable sources still play a minor role.[4] Given this scenario, it appears clear the need for Italy to start investing in a significant reduction of CO_2 emissions as a priority, before implementing new environmental policy interventions.[5]

With this analysis we contribute to the literature which studies the relationship between carbon dioxide emissions and economic activity by using different but complementary statistical approaches, allowing for structural changes and non-linearities and having as a focus the investigation of the economic trend and conditions of the Italian economy. Italy has been often analyzed within a panel of countries (Galeotti *et al.*, 2006; Richmond and Kaufmann, 2006; Martínez-Zarzoso and Bengochea-Morancho, 2004; Dijkgraaf and Vollebergh, 2005, *inter alia*). However, a more precise investigation of the relationship between economic growth and environment effects requires the study of the single country characteristics, underlying the importance of the specific historical experience (de Bruyn *et al.*, 1998; Stern, 1998a,b; Dijkgraaf and Vollebergh, 2005). Moreover, many studies which include Italy rely on linear cointegration techniques, while a nonlinear cointegration approach is recommended, see Hong and Wagner (2008).

To disentangle the effects of economic growth on carbon dioxide and emissions we adopt different approaches. Initially we study the time series properties testing for stationarity, and the existence of unit roots along with a Cointegrated VAR (CVAR or "restricted VAR") model by following the Juselius (2006) empirical approach. Subsequently, we consider a nonlinear representation of the same model by investigating whether and when nonlinear behaviors arise in our observed variables. More specifically, we study whether Italy has shown any transition between regimes, i.e. low emissions, high emissions, estimating a Smooth Transition Autoregressive (STAR) model for a univariate scenario (Chan and Tong, 1986; Teräsvirta, 1994; van Dijk *et al.*, 2002; Teräsvirta *et al.*, 2010). In this respect, our closest predecessor is the paper of Kim *et al.* (2010), who study the nonlinear dynamic relationship between CO_2 emissions and economic growth for Korea using a STAR model. In our paper we opt to use the multiple-regime STAR version introduced by Franses and van Dijk (1999) (MR-STAR) to test if more radical changes

kg CO_2/toe. Always in 2010 energy intensity in Italy was 123.6 toe/MEUR '05 (compared to 152.3 toe/MEUR '05 of the EU-27 average). For more details see European Commission (2012).

[4] In 2010 gross electricity generation in Italy is attributable to gases (52.1%), renewable sources (26.6%), solid fuels (13.2%), petroleum products (7.2%). In EU-27, in the same year, gross electricity generation is imputable to nuclear (27.4%), solid fuels (24.7%), gases (23.6%), renewable sources (20.9%), petroleum products (2.6%). In 2010 the energy gross inland consumption of Italy by product is so distributed: petroleum products (40.2%), gases (38.8%), renewables (10.3%), solid fuels (8.1%), waste (0.5%); in EU-27 energy gross inland consumption by product is due to petroleum products (35.1%), gases (25.1%), solid fuels (15.9%), nuclear (13.4%), renewables (9.8%), waste (0.6%). See European Commission (2012).

[5] For further details about energy policy in Italy, see International Energy Agency (2009).

affect the data, estimating the (multiple) regimes transitions. To the best of our knowledge we are the first to adopt this methodology for the study the relationship between emissions and GDP. The same problem is also considered in a multivariate context, where a simple Smooth Transition VAR model (Weise, 1999) is assumed. We then relax the observability hypothesis of the regime switching via simple Markov-Switching VAR (MS-VAR) in order to check how the quality of the results changes.[6] Finally, to complete the investigation, we test for the Environmental Kuznets Curve (EKC) hypothesis, according to which environmental degradation tends to increase as the economy develops, but begins to decline at higher levels of income (Grossman and Krueger, 1993, 1995; Stern, 1998a,b; Müller-Fürstenberger and Wagner, 2004; Selden and Song, 1994, *inter alia*).

Our results suggest that real GDP and carbon dioxide emissions are strongly interrelated, and the behavior of emission intensity, defined as CO_2 emissions to GDP ratio, is highly nonlinear. In particular, the CVAR analysis shows evidence of a common trend between real GDP and CO_2, which changes direction over time, globally increasing before 1975, and decreasing after that year. This could be probably due to the energy efficiency policies implemented in the aftermath of the oil crises of the 1970s. Consistently, the MR-STAR analysis suggests the presence of two structural shocks in 1881-1891 decade and in the second half of the Seventies. The MS-VAR analysis seems to be more sensitive to non-structural shocks, as shown by the change in regime after the post World Wars periods. In addition, according to this analysis the post-1975 reverse trend in CO_2 emissions seems less evident, resulting instead in a non-structural shock, while the state of high growth/high emissions appears to be permanent until the 2008 financial crisis. The results of the MS-VAR analysis would then suggest that no structural change in the reduction of the CO_2 emissions has occurred. Finally, our results on the EKC confirm that real GDP and carbon dioxide emissions are strongly interrelated and a sort of bell-shaped relationship seems to be present. However, the predicted turning point is at a very high level of per capita GDP. This may be due to the rigid structure of the standard quadratic EKC which shows to be outperformed by the flexible structure allowed by a cubic piecewise model.

The rest of the paper is organized as follows. In Section 2 we describe the dataset and discuss the historical evolution of carbon dioxide emissions and GDP in Italy. In Section 3 we study the properties of the time series by testing for unit roots and stationarity. The results on cointegration, structural change and non-linearities are presented in Section 4, and in Section 5 we

[6]This family of time series models have been introduced in econometrics by Hamilton (1989), in order to check if, and eventually when, the series under investigation can be described by two different unobserved regimes.

estimate a standard EKC model for carbon dioxide emissions. The main conclusions of the analysis are summarized in Section 6. Moreover, a separate online appendix describe the different statistical models and methods adopted in our analysis.

2 Data and Time Series Properties

To study the time evolution of the carbon dioxide emissions for Italy, we use annual data on total fossil fuel CO_2 emissions, real GDP and total population for the time period 1861-2011. Data on carbon dioxide emissions, stemming from fossil-fuel burning and the manufacture of cement, are from the database of the Carbon Dioxide Information and Analysis Center (CDIAC), provided by the Earth Sciences Division of the Oak Ridge National Laboratory which provides full information on the CO_2 emissions expressed in thousand metric tons of carbon.[7] The current dataset covers the period 1861-2009, while for the years 2010-2011 the CDIAC provides preliminary estimates obtained by extrapolation.[8]

For the 1861-2011 data on GDP we apply the most recent statistics based on the reconstruction of the national accounts, which is the result of a recent project coordinated by the Bank of Italy in cooperation with ISTAT, and University of Rome "Tor Vergata", see Baffigi (2011); Vecchi (2011) for full details. Notice that the GDP series is expressed in million of euros at 2005 constant prices, and from the same sources we extract data on population.

In Figures 1 - 4 we plot the historical patterns of GDP, and carbon dioxide emissions in Italy, for the period 1861-2011. More specifically, Figure 1 depicts the time series of per capita GDP for the whole period and for the two sub-samples 1861-1913 and 1950-2011. In line with the neutrality policy declared by Italy at the beginning of first global conflict (August 1914), the two sub-sample exclude the years 1914-1949 between the starting point of the World War I (WWI) and the years immediately after the World War II (WWII). During the 19th century the Italian economy was characterized by the presence of a large agricultural sector, which only at the end of the century gave way to an extensive industrialization. Indeed, although in Italy the industrial revolution began in the 1840s, only late in the 1890s modern infrastructures had begun to be built (Maddison, 2001; Malanima and Zamagni, 2010). Only at the end of the WWII, following the economics reconstruction, Italy experienced an unprecedented period of rapid economic growth which was known as "economic miracle". The growth of the industrial output in the years from

[7]The CDIAC maintains an extensive database on annual anthropogenic carbon dioxide emissions from each country, see Boden *et al.* (2012).

[8]See http://cdiac.ornl.gov/trends/emis/meth_reg.html for details.

1950 and 1974 drove a rise in per capita GDP to an average 5.3% per year, reaching a peak of 7.3% in 1961. In the early 70's due to the first oil crisis, the pace of growth slowed down causing a significant downturn of the Italian economy creating a wide economic disparities which caused in 1975 a drop in per-capita GDP of 2.7%. In the second half of the 1980s, the Italian economy was again prospering until the recession of the earlier 1990s. Over the last two decades Italy has been experiencing a prolonged period of slow growth with an average of 0.57% per annum. This poor performance, mainly due to a slowdown in the productivity, has been exacerbated by the recent crisis, see OECD (2012).

Figure 2 presents per capita carbon dioxide emissions for the whole sample and plots the series for the two sub-samples 1861-1913 and 1950-2011. At earlier stages of Italian economic development, we observe a slight increase in CO_2 emissions, and then two dramatic falls during the World Wars. From 1950 until the late 1970s, we notice a continuous, or even accelerating, growth of per capita CO_2 emissions. Immediately after the second oil shock in 1979, the growth of per capita CO_2 emissions with per capita gross domestic product levels out, as it emerges clearly from Figure 2c. This could be the result of the Italian economy's adjustment to the oil price shocks. Actually, the early 1980s saw some radical changes in the organization of Italian big industry with the introduction of automation and the dramatic reduction in the industrial work-force.[9] The recession in the early 1990s reduced the emissions slightly. From the second half of the 90's onwards there has been a constant, but slower, growth of carbon dioxide emissions amounting to around 2,228 kilos of carbon dioxide per capita in 2003. Since then we observe a decline up to 1,797 kilos of carbon dioxide per capita in 2011. Of course this sharp fall in emissions could be due to the recent crisis.

Figure 3 plots per capita carbon dioxide emissions against per-capita GDP, and as expected the period as a whole is characterized by a strong positive correlation between the two series.

Finally, Figure 4 reports the ratio between CO_2 emissions and GDP, expressed as CO_2 metric tons per million of euros. The CO_2/GDP ratio increases sharply from 1861, and then it falls during the World Wars. From 1950 until the earlier 1970s, we observe a prolonged increase in the ratio, up to a level of 135 metric tons per million of euros in 1973. Since then, the CO_2/GDP ratio has declined persistently up to a level of 76 CO_2 metric tons per million of euros in 2011. The reduction was mainly due to the increased efficiency in the use of energy sources, jointly

[9] In the period 1981-1983 Italy experienced economic stagnation. The large industry was facing the repercussions of a second oil shock and the consequences of low profit margins due to the wage-indexing mechanisms, which had been revised in the workers' favour after the first oil shock (see Zamagni (1993) for details).

with the new energy policies implemented in the aftermath of the oil crises of the 1970s, to which it followed a drop of the energy intensity in the manufacturing sector.

The observed historical pattern could reflect the existence of an inverted-U relationship between carbon dioxide emissions and GDP for Italy, along the lines suggested by the EKC literature. Moreover, inspection of the time series suggests the existence of five significant structural breaks in the data more likely explained by the World Wars, and the two oil shocks together with the recent crisis. In particular, an important caveat of our analysis is in the entity of the Second World War shock and the related statistical treatment. In this respect, in order to detect the presence of structural breaks and properly model the series, we use the Doornik (2009) Autometrics algorithm,[10] focussing our attention on the ratio CO_2/GDP (emissions intensity). As expected, when considering the length of the estimated residuals as criterion, we find two outliers in the error distribution in 1943 and 1946. This result suggests that a more deep investigation is needed. From Section 4 we account for this problem by simply using a transitory shift dummy acting for the years of the WWII.[11]

3 Univariate Properties of CO_2 Emissions and Real GDP in Italy

In the current section we test whether the time series of CO_2 emissions and GDP are driven by some trend, or whether the evolutions over time of these processes exhibit a unit root behavior, taking into account the possibility of structural break in the data. Both variables are expressed in per capita terms and in natural logarithms. Furthermore, to avoid any biases deriving from the quality of the data for the pre-war sample, we also present our results for the sub-sample 1950-2011.

Notably, conventional unit-root tests may produce wrong results when time series display structural breaks, especially when a time series exhibits systemic shifts we may fail to reject the null of unit root even in the absence of nonstationarity. In order to test the unit root hypothesis, taking into account the possibility of structural breaks in the data, we perform the Zivot and

[10]This is a computer-based approach for the selection of the best statistical model. The logic underlining this strategy is simple: first, it selects a general unrestricted model able to capture all the essential features of the data, that is an autoregressive model augmented by several lagged variables and dummies in order to capture the outlier observations. Then, it selects a value representing the significance level for several diagnostic tests (among them the Chow test for structural change). If the general unrestricted model does not pass these tests, it is reduced of one of the covariates. The procedure is iterated until the model does not pass all the tests.

[11]On the other hand, the Impulse Dummy Saturation method, applied on the same variable, conveys a large number of dummies, especially in the first half of the sample up to first years of the 1970s. Nevertheless, the lack of an impulse dummy for the 2001 or immediately after, is self explanatory of the impossibility to identify a structural change and a model for it. See Table 1, where the ratio CO_2/GDP is labeled EI (i.e. emissions intensity).

Andrews (1992) test (Zandrews test) and the tests proposed by Clemente, J. , Montañes, A. and Reyes, M. (1998). All results are reported in Tables 2 and 3. The Zandrews test allows us to examine for a single structural break in the intercept and in the trend of the time series. The optimal lag length was selected via a t-test. When taking into account the existence of different kinds of structural breaks, we fail to reject the null hypothesis of unit root for both time series in both samples. We notice that the shift in the intercept roughly corresponds to the season of the Italian *economic miracle* around the 1950's, while a structural change in trend is found in 1931 and 2001 for per capita GDP and in 1967 and 1988 for per capita CO_2 emissions.

According to Clemente-Montañés-Reyes unit root tests we proceed considering two alternative events within our time series: the "additive outlier" (CLEMAO) model that captures a sudden change in the series, and the "innovation outlier" (CLEMIO) model that allows a gradual shift in the mean of the series. For convenience, we test for unit root allowing for the existence of one or two structural breaks, in turn. According to the CLEMAO test results we fail to reject the null hypothesis of unit root for both samples and both variables, with the exception of per capita CO_2 in the period 1950-2011, allowing for an additive outlier in 1962. We can conclude that unit roots are present even when instantaneous structural breaks are accounted for. When instead we consider the possibility of innovation outliers, we reject the null for both variables. It is worth noting that when we conduct our analysis on the sub-sample 1950-2011, the CLEMIO test find breaks during the *economic miracle* and at the onset of the recent great recession that struck globally in 2008 and hit Italy harder than expected, after a prolonged period of low growth.

4 Cointegration, Structural Change and Non-Linearities

In this section we study the relationship between carbon dioxide emissions and gross domestic product using different, but complementary, statistical approaches. We start by assuming a "non-stratified" one sided scenario, where CO_2 emissions are a by-product of economic activity measured by GDP. Here we assume that both of them have an auto-regressive (AR) structure. This stylized representation permits a better investigation of the peculiarities of the two observed series, which should have same common dynamics. That is the two processes should be cointegrated.

We start by considering the following simple representation of the economy at time t:

$$CO_{2,t} = k_t + \phi GDP_t + \epsilon_t, \tag{1}$$

where both variables are expressed in per capita terms and in natural logarithm, k_t is a generic constant term, possibly including a trend, ϵ_t is an i.i.d error, indicating all the idiosyncratic elements in the specification of the relationship and ϕ represents the parameter capturing any effect that GDP may have on CO_2 emissions. This representation can be re-written in the following error correction form:

$$\epsilon_t = CO_{2,t} - (k_t + \phi GDP_t), \tag{2}$$

where $[1, -\phi]$ is the cointegrating vector and the linear combination of the two variable is assumed to be an $I(0)$-process. In particular, we expect to find some ϕ which is positive so to have a theory-consistent dynamics. The presence of one cointegrating relation may be deduced by the simple graph analysis conducted in Section 2. The graphical analysis clearly shows that the long-run relation between carbon dioxide emissions and gross domestic product in Italy has been changing over time, as result of continuous technology innovation and higher energy efficiency. Following the Juselius (2006) empirical approach, we study equation (2) estimating a Cointegrated VAR (CVAR or "restricted VAR") model. In particular, after having performed some linearity tests on both variables, we focus on testing if any transition between different regimes (i.e. low emissions, high emissions) are observed. If this latter is the case, then we apply a Multiple-Regime Smooth Transition Autoregressive (MR-STAR) model for a univariate scenario for our estimation. The same analysis is also considered in a multivariate context, but the switching regime is assumed to be unobserved, and this is done by estimating a simple Markov-Switching VAR (MS-VAR). Both variables under analysis are subject to logarithmic transformation in the CVAR analysis, while for the nonlinear scenarios growth rates are used.

4.1 Linear Scenario: CVAR

In this section we report the cointegration analysis results. We adopt the Juselius (2006) approach to macroeconomic modelling for its fully empirically-based nature. The structural break in 1975 observed in the graphical analysis (see Figures 1 - 4) has been modeled by a broken linear trend in the CVAR. The WWII has been treated by a transitory dummy variable, acting in the years 1943-46. This seems consistent with the graphical analysis and the statistical findings of Section 2, showing that such a period constitutes just a temporary episode, albeit dramatically heavier than other crisis events.

The main findings can be summarized as follows. First, the analysis of the roots of the companion matrix suggests the presence of one unit root in the bivariate process, as shown in Table 4. Second, the Johansen's Trace test is performed and this rejects the hypothesis of $r = 0$, that is no cointegration is observed (see Table 5). The distribution of the Rank Test is approximated by simulating 2500 random processes with length $T = 400$, and restricted linear trend with one break in 1975.

Given the above results we select $r = 1$ and introduce a linear trend in the cointegrating relation allowing for a break in 1975, to account for the self-evident change in the levels of emissions. The estimated cointegrating relation is shown in Figure 5. The fact that the resulting cointegrating vector is not stationary, as particularly evidenced by the differenced form in the bottom panel, seems to suggest that the deterministic change in trend imposed is the real driver of the results. This result, in turn, weakens the hypothesis of the presence of a stable long-run stochastic trend in the system.

In Table 6 we report the estimation results of the restricted VAR model with one cointegrating relation and normalized eigenvector β'. It shows the required signs in both components of the long-run matrix (this happens independently on the normalization which we settle for the emission variable). In particular, the loading sizes of the two variable of interest - [1.000 - 1.372]' - confirm the theoretical relation stated in Section 4. It is very interesting to notice the strength of the transitionary dummy (66% with respect to the normalized variable) and the role of the change in trend imposed after 1975, which can be quantified in 12.3 % with respect to the normalized variable (three times the effect of the global trend).

The resulting long-run matrix has also the expected signs, where a 1% increase in GDP is associated with a 2.5% marginal increase in the emissions growth rate. The transition dummy accounts for a 1.2 % marginal reduction of emissions if considering its effect on the whole sample. The break in the trend after 1975 is generally associated with a low impact on growth rates of the two variables of interest (0.2 % on DCO_2 and -0.1% on DGDP, respectively); the same story applies to the global trend.

The residual analysis confirms the non-normality of the residuals (although the statistics for CO_2 and GDP are close to the critical value of 6), the presence of an ARCH effect and some skewness and kurtosis problems.

Table 6 reports the results of four diagnostic tests discussed in (Juselius, 2006, CH 10 - 11). The first one is a test on the null hypothesis of redundancy of the variable from the original system of equations: if the model without the variable assumed to be redundant performs better, the investigator will be allowed to reduce the dimension of the VAR. In this case, when the VAR is augmented to include the global trend and the 1975-trend, both variables are not significant. The second test is a classical ADF test on each single equation of the system. We observe that for both variables the null of stationarity can be rejected consistently with the findings of the previous Section. This result suggests the need to estimate a restricted model able to capture a latent (stationary) trend. We test for omitted variables and for no correlation of the independent variables with the error term (weak endogeneity). The third third test shows that none of the two variables is significantly weakly exogenous. This finding is consistent with the result of the exclusion test according to which we need both equations in the system to capture the long-run dynamics. Finally, the fourth test suggests that the null of endogeneity is rejected for both of the variables. This means that none of the variables is permanently affected when a shock hits the system and it is not possible to detect which variable "drives" the other.

These diagnostic checks on the estimated CVAR model and in particular the results of the last two tests seem to suggest opposite conclusions. The bivariate nature of system and the by-product nature of the CO_2 with respect to GDP explain in some way the difficulty to distinguish between "pulling and pushing forces". This leads us to stop the investigation via CVAR model in favor of different approaches.

4.2 Nonlinear Scenario: Linearity Tests

Before moving to the nonlinear models, in this section we perform some tests for linearity on the variables under investigation. Table 7 provides the results of four different tests for linearity. In the Tsay (1989) test the null of linearity is rejected if a delay of 1 year is used for output (2 for emissions), while the Luukkonen *et al.* (1988) test requires at least $d = 2$ in order to have evidence of nonlinearity. In all cases one could not reject the null of linearity because all p-values are high. We apply the Tsay rule for detecting the right parameter d by searching the one for which the p-value is minimum. For $d = 2$ the p-value is relatively lower than in the other cases, thus we select SETAR(4; 2) and SETAR(1;2) models. The Hansen (1996) test for the no-threshold effect confirms the previous finding for the output series, while for the emissions it seems to be quite near to linearity (the p-value single LM-statistics is always higher than 5%). However, the SETAR estimates shown in Table 8 seem to leave no doubt that there is some change in the sample mean. In this analysis we include the emission intensity, defined as the natural log of the ratio CO_2/GDP, labeled EI.

4.3 Nonlinear Scenario: (MR-)STAR and STVAR

The nonlinear analysis via STAR model for GDP is shown in Table 8, which clearly displays evidence in favor of a nonlinear dynamics in all the series. According to the results of the SLT test and an application of the Teräsvirta rule (shown at the top panel of the same Table), the logistic STAR model seems to be the natural candidate. The resulting estimates (in the middle panel) of the slope parameter γ suggest that all the variables are smoothly changing. However, the low $p-$values of the three diagnostic tests (bottom panel) make us skeptical about the goodness of the selected STAR model; in particular, the null of no additive nonlinearity is strongly rejected for GDP (0.00), and just weekly accepted for CO_2 (0.14).
According to these findings, we then allow for the presence of more than two regimes and the corresponding transition. The results are reported in the same Table, in the last two columns of the first panel: the p-values for the hypothesis of linearity in the first transition (F_L^1) range between 0.14 and 0.16, while for the equivalent hypothesis for the second transition (F_L^2) are higher (between 0.23 and 0.39), so that adding a third regime could not be easily justified. Nevertheless, the previous discussion in Subsection 4.1, makes a deeper investigation necessary. To this aim, in the same Table we report the estimates for the equivalent MR-STAR. It is worth noting the difference between the two slopes measured for the same series, where the latter, measuring the transition corresponding to the oil shocks, is considerably higher than the former (73 and 573 for CO_2, 13 and 67 for EI). This time the diagnostics tests for the new parametrization are quite reassuring, so that we are reasonably sure of the presence of two smooth transitions in the sample.
According to these findings one could be interested in knowing which transition between the two detected is the effective driver of the nonlinearity in the series. In order to answer to this question, we assume that the vector $y_t = [CO_{2,t}, GDP_t]$ follows a Smooth Transition VAR (STVAR) (Weise, 1999; Camacho, 2004) model, and test it against the null of linear VAR(p) model. The p-value of the equivalent statistics is < 0.0001, so that linearity is rejected for both equations in the system. The estimated slope is high ($\gamma = 10.05$) and located in the last part of the Seventies, as clearly shown by Figure 7. This confirms the empirical finding of Subsection 4.1, in which the decline of carbon dioxide emissions after 1975 turns out to be the real driver

of the observed nonlinearity.[12]

Figure 6 shows the two estimated transition functions for the three variables. In all sub-plots, the first panel describes the G function versus the transition variable s_t. This enables us to visualize the path of the transition of the variable from state 0 (low emissions, low output) to state 1 (high emissions, high output), measured by the steep parameter γ. The second panel shows the same function versus time, allowing us to visualize the duration of each regime change expressed in number of years, and when such change has occurred. The transitions are clearly identified by the two structural shocks happened in 1881-1891 decade, and during the second half of the Seventies[13]. The first transition seems to be more persistent than the second one, in particular for the emissions, where the new regime is reached only at the end of WWI. It is worth noticing that the WWII is not considered as the beginning of a new regime.

4.4 Nonlinear Scenario: MS-VAR

In this subsection we move to a multivariate scenario by allowing for an unobserved change in the regime of the system from state 0 (high GDP growth, high CO_2 growth) to state 1 (low GDP growth, low CO_2 growth).

The coefficients of the selected VAR(1) reported in Table 9 show a preponderance of the state 0, especially for CO_2 emissions. In particular, there is an evident asymmetry of the duration with respect the two states (14 years vs. 3 years and a half on average). Figure 8 reports the estimated conditional means and standard deviations and the estimated state of the VAR process for each equation. With respect to the MR-STAR model, the MS-VAR model is more sensitive to the non-structural shocks, as shown by the change in regime after the years of the completion of Italian Reunification, the WWI, the Autarky period and WWII periods. It is important to notice that the oil crises seem to have just a transitory effect, as indicated by the return of the smoothed probability in the same state ante-1975 for both variables. In this way the reversed trend in CO_2 emissions is now more problematic to justify. State 0 appears to be more persistent since the mid '50s, with just a break occurring during the last years of the Seventies. As a matter of fact, it seems that the state of high growth/high emissions is permanent until the Great Recession in 2008.

[12] The estimated long run parameter matrices Φ_t and Θ_t do not convey substantial difference in the quality of the results and are not shown.

[13] On the contrasting interpretations of the economic events of 1880s, see Fenoaltea (2011) who remarks that two major external developments affected the Italian economy in that decade: (i) a strong increase in the supply of foreign capital, along with (ii) a sharp fall in the price of imported grain.

Finally, we notice the lack of any change in transition probabilities in the decade 1880s-1890s. This is not in contrast with the evidence of the smooth transition estimated for the same period and seems us to be an evidence of the Fenoaltea (2011) criticism of existence of a crisis in that period.

5 Testing the EKC for Italy

In this section we test for the existence of a systematic relationship between pollution and economic growth, commonly referred to as Environmental Kuznets Curve (EKC). According to the EKC hypothesis, environmental degradation tends to increase as the economy develops, but begins to decline at higher levels of income. The existence of a systematic a bell-shaped relationship between pollutant and income is still an open issue and the results of the empirical literature are controversial.[14]

Aware of the limits of this approach we test the EKC hypothesis adopting two strategies. First we estimate a standard polynomial relationship between per capita carbon dioxide emissions and per capita GDP for Italy. Then we replace the polynomial specification with a flexible non-linear model of per capita GDP. We model the polynomial relationship between carbon dioxide emissions and gross domestic product, as follows:

$$CO_{2,t} = \gamma_0 + \gamma_1 GDP_t + \gamma_2 GDP_t^2 + \varepsilon_t, \tag{3}$$

where ε_t denotes the error term and, as before, all variables are expressed in per capita terms and converted in natural logarithms. The turning point income, where pollutant emissions reach the peak is given by $\tau = e^{-\gamma_1/2\gamma_2}$. The parameters γ_1 and γ_2 are long-term elasticities of carbon dioxide per capita emissions with respect to per capita real GDP, and squared per capita real GDP, respectively. An inverted-U relationship between GDP and CO_2 requires that $\gamma_1 > 0$ and $\gamma_2 < 0$.

We estimate the EKC model (3) for the whole sample, 1861-2011, and for the subset 1950-2011, using GLS in order to consider possible serial correlation. In the presence of autocorrelated disturbances the standard errors estimated by OLS are likely to be too small. Results are reported in Table 10. The estimated coefficients of the linear term and of the quadratic term are

[14] For reviews of the EKC literature see e.g. Stern (1998a,b); Millimet *et al.* (2003); de Bruyn and Heintz (1999); Dinda (2004) *inter alia*.

highly significant, and exhibit the theoretically expected sign.

Test results show the presence of serially correlated residuals and of heteroskedasticity. According to the results of the Ramsey's RESET test, there is functional form misspecification. In general, we notice that the statistical quality of the estimation, in terms of measures of goodness of fit, is much better for the second sub-period 1950-2011 than for the whole sample. In the quadratic specification the turning points for CO_2 emissions are estimated to occur at a per capita real GDP value of $91,329$ and $74,078$ Euros, respectively. It should be noticed that in 2011 the per capita GDP of Italy was about $23,514$ Euros. With this regard, our estimates about the chances for Italy to curb carbon dioxide emissions are very pessimistic. Figure 9 plots the fitted values of the quadratic model over the period 1861-2011, while Figure 10 plots the residuals. We now turn to the results of the flexible non-linear model. In particular, we consider a restricted cubic spline model, where per capita carbon dioxide emissions are modeled as a restricted cubic spline function of per capita GDP for the whole sample. Restricted cubic splines are such that: (i) below the first and above the last knot the function should be linear; (ii) within each interval the function should be cubic; (iii) at each knot the function should be continuous and smooth with continuous first and second derivatives. Figure 11 plots the fitted values of the cubic spline model together with the observed data. To check the adequateness of the fitted spline we also compare the residuals of the spline model with those obtained with the quadratic model. See Figure 12. Clearly, the second specification appears to be more adequate in representing the behaviour of carbon dioxide emissions with respect to GDP, confirming that the more flexible structure allowed by the cubic piecewise model outperforms the rigid structure imposed by the standard quadratic model.

6 Conclusions

Environmental awareness has become a central issue in the policy debate. Given the heavy reliance of Italy on fossil fuels, the reduction of carbon dioxide emissions for the accomplishment of the Europe 2020 strategy remains a serious environmental and policy challenge.

In this paper we have analyzed the relationship between GDP and carbon dioxide emissions for Italy, in a historical perspective. Using several different statistical techniques, our results suggest that the CO_2 emission trajectory is closely related to the income time path, and that the behavior of emission intensity and of the main two series are highly nonlinear. There seems to be a common trend between real GDP and CO_2, which however changes direction in the middle of the Seventies, suggesting a possible slowdown in the emission intensity, probably due to the energy efficiency policies implemented in the aftermath of the oil crises of the 1970s. Consistently, according to the MR-STAR analysis, a structural shock may have occurred in the same period, marking a slowdown in the growth rate of carbon dioxide emissions. However, the MS-VAR suggests that the state of high growth/high emissions seems to be permanent until the recent recession. In addition, the EKC analysis shows the existence of a bell-shaped relationship between income and the pollutant, but according to the estimates, the predicted turning point turns out to be pessimistically high.

Overall, our results do not seem to unambiguously show a structural slowdown of carbon dioxide emissions in recent years, that is why we argue that meeting the climate change and energy sustainability goals of the Europe 2020 strategy represents a very challenging task calling for a radical policy shift.

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Tables and Graphs

Table 1: Outlier Detection in Emissions Intensity: Results from the Impulse Dummy Saturation Method

Parameter	Coefficient	Std. Error	t-value	t-prob	Part. R^2
EI_1	0.509074	0.02845	17.9	0.0000	0.8020
EI_2	0.433185	0.02750	15.8	0.0000	0.7585
Constant	0.247337	0.01993	12.4	0.0000	0.6609
I:1865	-0.250502	0.03032	-8.26	0.0000	0.4635
I:1866	-0.126186	0.03087	-4.09	0.0001	0.1746
I:1870	0.244488	0.02862	8.54	0.0000	0.4802
I:1871	-0.0887700	0.02923	-3.04	0.0032	0.1045
I:1872	0.108296	0.02875	3.77	0.0003	0.1523
I:1874	-0.101191	0.02825	-3.58	0.0006	0.1397
I:1875	-0.0672231	0.02807	-2.40	0.0190	0.0677
I:1876	0.226305	0.02805	8.07	0.0000	0.4517
I:1878	-0.154793	0.02802	-5.52	0.0000	0.2786
I:1880	0.113938	0.02786	4.09	0.0001	0.1748
I:1881	0.121361	0.02774	4.37	0.0000	0.1950
I:1884	0.0746910	0.02746	2.72	0.0080	0.0857
I:1885	0.105334	0.02749	3.83	0.0003	0.1567
I:1887	0.108512	0.02739	3.96	0.0002	0.1658
I:1888	0.108229	0.02758	3.92	0.0002	0.1632
I:1889	0.0585687	0.02728	2.15	0.0349	0.0551
I:1890	0.0706413	0.02725	2.59	0.0113	0.0784
I:1891	-0.114545	0.02725	-4.20	0.0001	0.1828
I:1892	-0.0999577	0.02751	-3.63	0.0005	0.1432
I:1893	-0.0936431	0.02726	-3.44	0.0009	0.1300
I:1894	0.145512	0.02734	5.32	0.0000	0.2639
I:1896	-0.140178	0.02740	-5.12	0.0000	0.2489
I:1899	0.0638100	0.02723	2.34	0.0216	0.0650
I:1901	-0.0765632	0.02722	-2.81	0.0062	0.0911
I:1905	0.0617278	0.02720	2.27	0.0260	0.0612
I:1906	0.139579	0.02722	5.13	0.0000	0.2497
I:1907	0.0981283	0.02736	3.59	0.0006	0.1400
I:1909	0.0728745	0.02717	2.68	0.0089	0.0835
I:1915	-0.115877	0.02719	-4.26	0.0001	0.1869
I:1916	-0.148503	0.02732	-5.44	0.0000	0.2722
I:1917	-0.434069	0.02734	-15.9	0.0000	0.7614
I:1919	0.101586	0.02766	3.67	0.0004	0.1458
I:1920	-0.0703908	0.02720	-2.59	0.0115	0.0781
I:1921	0.179566	0.02730	6.58	0.0000	0.3539
I:1922	0.170390	0.02785	6.12	0.0000	0.3214
I:1924	0.127088	0.02721	4.67	0.0000	0.2164
I:1926	0.0870894	0.02734	3.19	0.0021	0.1138
I:1927	0.200712	0.02742	7.32	0.0000	0.4042
I:1931	-0.155518	0.02723	-5.71	0.0000	0.2922
I:1932	-0.278732	0.02744	-10.2	0.0000	0.5663
I:1934	0.297901	0.02727	10.9	0.0000	0.6017
I:1935	0.203040	0.02812	7.22	0.0000	0.3975
I:1936	-0.267014	0.02728	-9.79	0.0000	0.5481
I:1937	0.107251	0.02856	3.75	0.0003	0.1514
I:1938	0.0832031	0.02802	2.97	0.0039	0.1004
I:1940	0.0660551	0.02719	2.43	0.0174	0.0695
I:1941	-0.0760859	0.02726	-2.79	0.0066	0.0898
I:1943	-1.59041	0.02718	-58.5	0.0000	0.9774
I:1944	-1.25768	0.05322	-23.6	0.0000	0.8761
I:1946	1.35936	0.02913	46.7	0.0000	0.9650
I:1947	0.883006	0.04522	19.5	0.0000	0.8284
I:1954	0.119338	0.02717	4.39	0.0000	0.1963
I:1955	0.103045	0.02732	3.77	0.0003	0.1526
I:1956	0.109873	0.02721	4.04	0.0001	0.1711
I:1960	0.0868046	0.02717	3.19	0.0020	0.1144
I:1961	0.0977302	0.02727	3.58	0.0006	0.1398
I:1962	0.133810	0.02724	4.91	0.0000	0.2340
I:1963	0.120334	0.02736	4.40	0.0000	0.1967
I:1964	0.0764604	0.02730	2.80	0.0064	0.0903
I:1965	0.0685609	0.02727	2.51	0.0140	0.0741
I:1966	0.0971623	0.02729	3.56	0.0006	0.1383
I:1967	0.0698652	0.02735	2.55	0.0126	0.0763
I:1970	0.0728221	0.02731	2.67	0.0093	0.0826
I:1971	0.0787019	0.02735	2.88	0.0051	0.0949
I:1972	0.0659988	0.02736	2.41	0.0182	0.0686

Table 4: The Roots of Companion Matrix

Roots	Real	Imaginary	Modulus	Argument
Root 1	1.000	0.000	1.000	0.000
Root 2	1.000	0.000	1.000	0.000
Root 3	0.988	0.000	0.988	0.000
Root 4	−0.087	0.764	0.769	1.684
Root 5	−0.087	0.764	0.769	1.684
Root 6	−0.474	−0.597	0.762	−2.242
Root 7	−0.474	−0.597	0.762	−2.242
Root 8	0.360	0.652	0.745	1.066
Root 9	0.360	0.652	0.745	1.066
Root 10	0.739	0.000	0.739	0.000

NOTE: software used: CATS for RATS

Table 2: Unit Root Tests with Structural Breaks for Per Capita GDP and CO_2 Emissions, 1861-2011

	Per Capita GDP		*Per Capita CO_2*	
	Test statistics	Year	Test statistics	Year
Zandrews (break in intercept)	−3.867(2)	1946	−3.015(3)	1954
Zandrews (break in trend)	−2.304(2)	1931	−2.557(3)	1988
CLEMAO1	−2.846	1964	−1.886	1941
CLEMAO2	−2.094	1954, 1975	−3.499	1889, 1957
CLEMIO1	−6.555**	1944	−5.640**	1943
CLEMIO2	−9.855**	1942, 1944	−9.977**	1941, 1944

NOTES: Variables in natural logs. Lags reported in parentheses. For the Zandrews statistics lags selected via *t* test. A single asterisk, *, indicates significance at 10% level, a double asterisk, **, at 5% level and a triple asterisk, ***, at 1%.

Table 3: Unit Root Tests with Structural Breaks for Per Capita GDP and CO_2 Emissions, 1950-2011

	Per Capita GDP		*Per Capita CO_2*	
	Test statistics	Year	Test statistics	Year
Zandrews (break in intercept)	0.997(0)	1959	−3.966(1)	1960
Zandrews (break in trend)	−1.087(0)	2001	−3.517(1)	1967
CLEMAO1	−2.675	1973	−3.603*	1962
CLEMAO2	−3.223	1969, 1989	−4.031	1963, 1973
CLEMIO1	−7.342**	1957	−8.764**	1958
CLEMIO2	−7.362**	1957, 2008	−8743**	1958, 2007

NOTES: Variables in natural logs. Lags reported in parentheses. For the Zandrews statistics lags selected via *t* test. A single asterisk, *, indicates significance at 10% level, a double asterisk, **, at 5% level and a triple asterisk, ***, at 1%.

Table 5: The Simulated Trace Test Distribution

Simulated Trace Test Distribution				
p-r	r	Eig.Value	Trace	P-Value
3	0	0.277	81.597	0.000
2	1	0.164	34.338	0.018
1	2	0.054	8.120	0.479

Quantiles of the Simulated Distribution									
p-r	r	Mean	S.E.	50%	75%	80%	85%	90%	95%
3	0	36.283	7.498	35.570	41.002	42.369	44.108	45.979	49.209
2	1	20.717	5.711	20.051	24.093	25.213	26.360	28.241	31.114
1	2	8.460	3.662	7.826	10.485	11.324	12.205	13.342	15.596

NOTE: software used: CATS for RATS

Table 6: The Estimated CVAR

Descriptive Statistics					
	Mean	Std. Dev.	Skewness	Kurtosis	
DCO_2	0.000	0.116	-0.080	5.956	
DGDP	-0.000	0.027	-0.607	4.526	
$D_{tr}43$	0.000	0.091	7.504	82.974	
	Maximum	Minimum	ARCH(5)	Normality	R^2
DCO_2	0.349	-0.507	21.286 [0.001]	38.560 [0.000]	0.757
DGDP	0.094	-0.089	31.514 [0.000]	11.538 [0.003]	0.693
$D_{tr}43$	0.950	-0.254	0.883 [0.971]	1551.238 [0.000]	0.398

Normalized β'					
	CO_2	GDP	$D_{tr}43$	T(1975)	TREND
	1.000 [NA]	−1.372 [−1.252]	0.660 [0.326]	−0.123 [−3.337]	−0.047 [2.542]

α	
DCO_2	−0.018 [−2.334]
DGDP	0.009 [4.673]
$D_{tr}43$	−0.010 [−0.079]

Π					
	CO_2	GDP	$D_{tr}43$	T(1975)	TREND
DCO_2	−0.018 [−2.334]	0.025 [2.334]	−0.012 [−2.334]	0.002 [2.334]	−0.001 [−2.334]
DGDP	0.009 [−0.726]	−0.012 [0.615]	0.006 [0.033]	−0.001 [−3.543]	0.000 [2.709]
$D_{tr}43$	−0.000 [−0.079]	0.001 [0.079]	−0.000 [−0.079]	−0.000 [−0.079]	0.000 [0.079]

Log-likelihood = 1223.432

Tests for autocorrelation[a]		Normality Test[b]	ARCH effects[a]	
Ljung-Box(36): $\chi^2(285)$	321.672 [0.067]	1659.561 [0.000]		
LM(1): $\chi^2(9)$	25.051 [0.003]		LM(1): $\chi^2(9)$	107.063 [0.000]
LM(2): $\chi^2(9)$	28.935 [0.001]		LM(2): $\chi^2(18)$	198.286 [0.000]

Diagnostic Tests[c]						
TEST	STATISTIC	CO_2	GDP	$D_{tr}43$	T(1975)	TREND
Exclusion	$LR(\nu_1)$	1.740 [0.187]	1.363 [0.243]	0.027 [0.871]	5.461 [0.019]	2.222 [0.136]
Stationarity	$LR(\nu_2)$	1.415 [0.493]	2.857 [0.240]	2.365 [0.306]		
Weak Exogeneity	$LR(\nu_1)$	4.504 [0.034]	16.505 [0.000]	0.003 [0.955]		
Unit vector in α	LR(ν_3)	18.334 [0.000]	4.577 [0.042]	20.420 [0.914]		

NOTES: Effective sample: 1866-2011 (146 obs.); No. observations - no. variables: 132; selected no. lags in VAR: 5; (a) Number of degree-of-freedom in parenthesis, p-values in squared brackets; (b) Distributed as $\chi^2(4)$, p-values in squared brackets; (c) All tests are distributed as $\chi^2(\nu_i)$, i=1, …, 3, $\nu_1 = rm$, m restrictions on each rank r, $\nu_2 = r - n_b$, with r rank restriction and n_b number of known cointegrating vectors, $\nu_3 = p - r$, with p rank restriction and n_b number of known cointegrating vectors; Software used: RATS

Table 7: Linearity Testing and TAR Model Estimates

SERIES	DGDP				DCO_2				DEI			
	d=1	d=2	d=3	d=4	d=1	d=2	d=3	d=4	d=1	d=2	d=3	d=4
Tsay test	$1.23e^{-6}$	0.058	0.456	$1.02e^{-05}$	0.989	**0.003**	0.218	0.630	0.644	0.045	0.068	0.056
LST test	0.191	**0.000**	0.000	0.000	0.172	**0.011**	0.024	0.003	0.1616	0.0072	0.0247	0.0398
	Hansen's test											
Statistic	SupLM	ExpLM	AveLM	No Thr.	SupLM	ExpLM	AveLM	No Thr.	SupLM	ExpLM	AveLM	No Thr.
bootstrap p-val	0.001	0.000	0.001	0.004	0.114	0.072	0.057	0.044	0.082	0.206	0.334	0.1674

TAR estimates

Threshold:	0.0025			0.0737			0.0815		
Regressor	Full Sample	<= Thrsh	>Thrsh	Full Sample	<= Thrsh	>Thrsh	Full Sample	<= Thrsh	>Thrsh
$const$	0.0080 [0.0036]	0.0080 [0.0115]	−0.0048 [0.0041]	0.0371 [0.0112]	0.0261 [0.0130]	0.1408 [0.0356]	0.0196 [0.0100]	0.0090 [0.0111]	0.1459 [0.0423]
ϕ_{t-1}	0.1714 [0.0837]	−0.1239 [0.2915]	0.3777 [0.0972]	−0.1044 [0.0774]	−0.0902 [0.1189]	−0.5605 [0.1760]	−0.1733 [0.0758]	−0.2242 [0.1142]	−0.6519 [0.1934]
ϕ_{t-2}	0.1067 [0.0830]	[0.2313]	0.1771 [0.0664]						
ϕ_{t-3}	0.2147 [0.0842]	−0.0028 [0.2705]	0.1902 [0.0655]						
ϕ_{t-4}	0.0119 [0.0851]	−0.4848 [0.2189]	0.2142 [0.0689]						

NOTES: No. of lags selected according to AIC and BIC for lags $\{1, \ldots, 12\}$; Software used: RATS

Table 8: Empirical application of the (MR-)STAR model to univariate data

Linearity tests (p-values)

Series	STAR[2]				Model[2]	s_t	MR-STAR[2]	
	F_L	F_3	F_2	F_1			F_L^1	F_L^2
CO_2	0.0000	0.3928	0.2146	0.0000	LSTAR1	y_{t-4}	0.1650	0.2356
GDP	0.0000	0.0034	0.0210	0.0001	LSTAR1	y_{t-1}	0.1486	0.3991
EI	0.0000	0.4602	0.6188	0.0000	LSTAR1	y_{t-3}	0.1600	0.3677

Estimates

Parameter	STAR						MR-STAR					
	CO_2		GDP		EI		CO_2		GDP		EI	
	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE
ϕ_0	1.4494	1.6872	0.2161	0.0349	-0.2423	0.3018	0.0332	0.0566	0.1870	0.0203	0.1853	0.1823
ϕ_1	0.4894	0.0001	4.0336	0.3887	0.7608	0.5972	0.1674	0.1564	2.8720	0.2415	-0.1478	0.4220
ϕ_2	-0.1283	0.5020	-3.7971	0.6789	-0.2219	0.2170	-0.0237	0.1052	-1.4991	0.2259	-0.0425	0.2523
ϕ_3	6.1375	4.9299	2.9564	0.7306	-0.7616	0.2274	-0.2195	0.1507	0.1947	0.2575	-0.6697	0.0977
ϕ_4	-4.0235	3.0268	-3.2546	0.8004	0.4390	0.3158	-0.0327	0.1653	-1.9133	0.2786	0.0747	0.4885
ϕ_5	0.7682	1.0984					0.0590	0.0950				
θ_{10}	-2.3569	0.0001	-0.2116	0.0354	0.6549	1.3485	0.7275	0.7037	5.6455	2.5364	5.2152	2.9780
θ_{11}	1.1042	0.0001	3.6016	0.3950	-2.1242	2.2454	-9.6598	11.3507	383.0179	10.4320	-2.7741	2.4671
θ_{12}	0.6432	0.7908	3.8492	0.6827	0.3687	0.4352	28.9409	28.5143	50.4489	38.4147	64.8190	34.1254
θ_{13}	-10.2453	8.3900	-2.8984	0.7337	0.4046	0.7754	19.5717	19.4648	46.2458	29.0967	5.5913	4.0329
θ_{14}	6.4354	4.1284	3.3245	0.8045	-0.6888	0.6938	-109.9989	103.6096	54.9049	33.2107	-0.0844	1.4516
θ_{15}	-1.0444	1.6561					-12.0544	12.1393				
θ_{20}							-0.8102	0.6975	-5.8316	2.5364	-5.3640	2.9785
θ_{21}							10.0559	11.3028	385.3599	10.4291	3.0676	2.2702
θ_{22}							-27.7827	28.5355	-48.9345	38.4406	-64.7948	34.1305
θ_{23}							-17.8466	19.4722	-46.3394	29.0857	-4.8679	4.0089
θ_{24}							111.2583	103.4933	-52.9247	33.1572	0.0160	1.0183
θ_{25}							11.3389	12.1489				
γ_1	0.4010	0.2869	12.3068	5.7500	1.0134	0.9868	73.4621	0.2568	20.0000	7.2502	13.6565	4.1615
γ_2							573.2448	16.3161	19.5001	7.4498	67.4174	89.7596
c_1	-0.2645	0.0101	-0.0584	0.0031	0.1157	0.4583	0.0464	2.5582	-0.5545	38.4147	-0.4673	34.1254
c_2							0.0687	0.0333	-0.5483	0.0586	-0.4350	0.0129

Diagnostics (p-values)

Diagnostic Test	STAR			MR-STAR		
No error autocorrelation						
q=1	0.0755	0.0076	0.9451	0.1254	0.0611	0.1672
q=2	0.2307	0.0073	0.8947	0.2195	0.1608	0.2181
q=4	0.3055	0.0327	0.0331	0.1916	0.3705	0.1886
q=10	0.0425	0.0234	0.0186	0.4250	0.7654	0.2386
No rem. nonlinearity	0.1447	0.0001	0.2440	1.0000	1.0000	1.0000
Parameter constancy						
H1	0.2726	0.0018	0.3636	1.0000	1.0000	1.0000
H2	0.2248	0.0001	0.7720	1.0000	1.0000	1.0000
H3	0.6289	0.0005	0.8700	0.9996	0.5114	0.3422

NOTE: (1)Values expressed in test-statistics, significance denoted by '*' (5%), '**' (1%); (2) Results acquired by JMulTi; (3)Results acquired by RATS

Table 9: MS-VAR: Estimates

Feature	Value
Final Log-likelihood:	464.5412
No. of estimated parameters:	50
No. of Effective Observations:	146
No. of VAR lags (according to BIC):	5
Expected Duration for Regime 0	14.23
Expected Duration for Regime 1	3.67

Parameter	State 0 Estimate	State 0 SE	State 1 Estimate	State 1 SE
k	0.05	0.01	−0.11	0.08
	0.01	0.01	−0.02	0.01
Π_1	0.39 1.11	0.05 0.33	−0.22 0.90	0.36 1.66
	0.10 0.52	0.01 0.10	0.00 0.16	0.05 0.23
Π_2	−0.31 0.00	0.06 2.50	−0.31 −1.71	0.48 3.51
	0.00 −0.38	0.01 0.09	0.00 0.27	5.05 0.31
Π_3	−0.60 −1.02	0.05 1.50	−0.15 2.54	0.44 1.50
	−0.13 0.16	0.09 0.08	0.00 0.11	0.09 0.44
Π_4	0.16 0.25	0.06 0.30	−0.48 0.92	0.37 0.82
	0.05 0.05	0.02 0.09	−0.04 −0.10	0.05 0.35
Π_5	0.01 0.06	0.06 0.20	−0.02 −0.32	3.50 1.95
	−0.04 0.25	0.01 0.07	−0.04 −0.10	0.05 0.35

Transition Matrix (p-value in brakets)

Final State Probability	$P_{i\|1}$		$P_{i\|2}$	
$P_{1\|j}$	0.93	<0.001	0.28	<0.001
$P_{2\|j}$	0.07	<0.001	0.73	<0.001

NOTE: Software used: MatLab 2009b

Table 10: Environmental Kuznets Curve for Italian CO_2 Emissions, 1861-1959

	1861 - 2011		1950 - 2011	
	Linear	Quadratic	Linear	Quadratic
constant	2.7744^{***} (0.3264)	2.2051^{***} (0.3768)	3.4041^{***} (0.4420)	2.1001^{***} (0.5285)
GDP_t	1.6088^{***} (0.1547)	2.6814^{***} (0.4883)	1.3699^{***} (0.1108)	2.7985^{***} (0.4669)
GDP_t^2		-0.2970^{**} (0.1312)		-0.3250^{***} (0.1012)
$D_{tr}43$	-1.3237^{***} (0.1027)	-1.2042^{**} (0.1169)		
ρ	0.9515	0.9152	0.9957	0.9875
turning point $\mathcal{T}$	NA	$91,329$	NA	$74,078$
obs.	151	151	62	62
F statistic	171.96^{***}	121.760^{***}	237.64^{***}	289.96^{***}
Adj. R^2	0.6951	0.7069	0.89	0.90
AIC	-178.452	-166.220	-236.5118	-243.6991
BIC	-169.401	-154.151	-232.2576	-237.3177
log-likelihood	92.2263	87.1110	120.2559	124.8496
RESET	7.57^{***}	5.45^{***}	5.25^{***}	5.17^{***}
BP	26.74^{***}	36.78^{***}	32.74^{***}	27.56^{***}
BG(1)	13.936^{**}	15.177^{***}	6.977^{**}	6.849^{***}
ARCH(1)	26.014^{**}	14.891^{**}	3.720^{**}	4.941^{**}
DW	2.51	2.55	2.35	2.35

NOTES: Variables in natural logs. The regressions are estimated by GLS based on the Prais-Winsten transformation. Standard errors are in parentheses. A single asterisk, *, indicates significance at 10% level, a double asterisk, **, at 5% level and a triple asterisk, ***, at 1%; ρ is the estimated autocorrelation parameter; obs. denotes the number of observations; NA: not applicable because the coefficients are not significant in the quadratic specification and the relationship appears to be increasing. AIC is the Akaike information criterion value; BIC is Schwarz's Bayesian information criterion; the RESET is the Ramsey specification test for omitted variables; BP is the Breusch-Pagan test for heteroskedasticity; BG is the Breusch-Godfrey LM test for the presence of first order autocorrelation; ARCH(1) is the Engle's LM test for autoregressive conditional heteroskedasticity of order 1; D-W is the Durbin-Watson d statistic to test for first-order serial correlation.

Figure 1: Per Capita GDP in Italy, 1861-2003 *(Thousands of 2005 Euros Per Capita)*

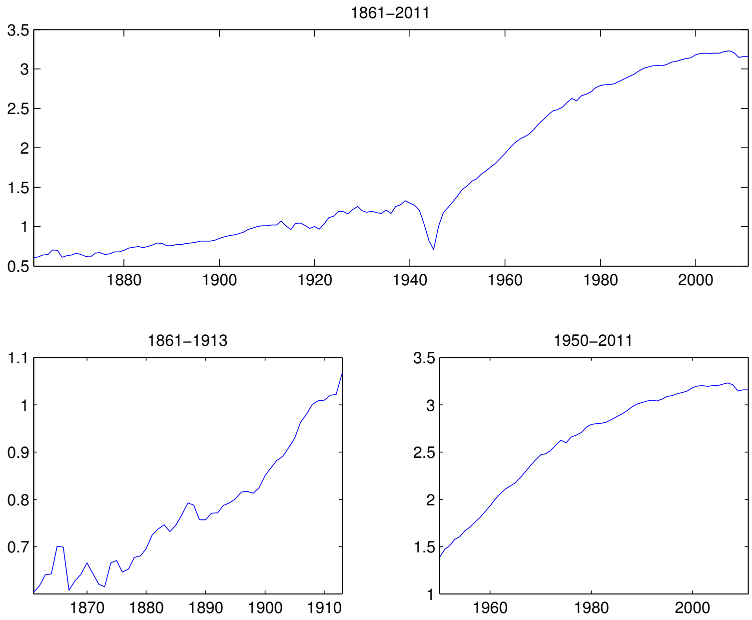


Figure 2: Per Capita CO_2 Emissions in Italy, 1861-2003 (Kilos Per Capita)

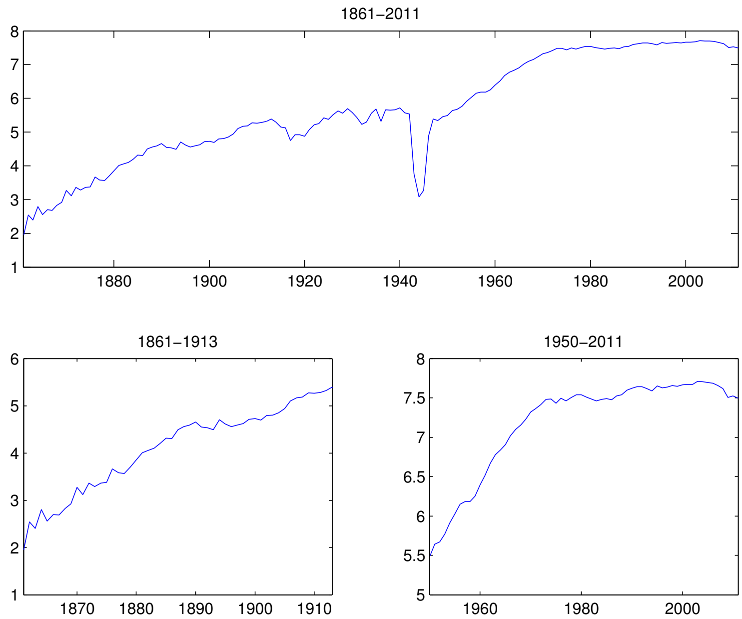


Figure 3: Per Capita CO_2 Emissions and Per Capita GDP in Italy, 1861-2003

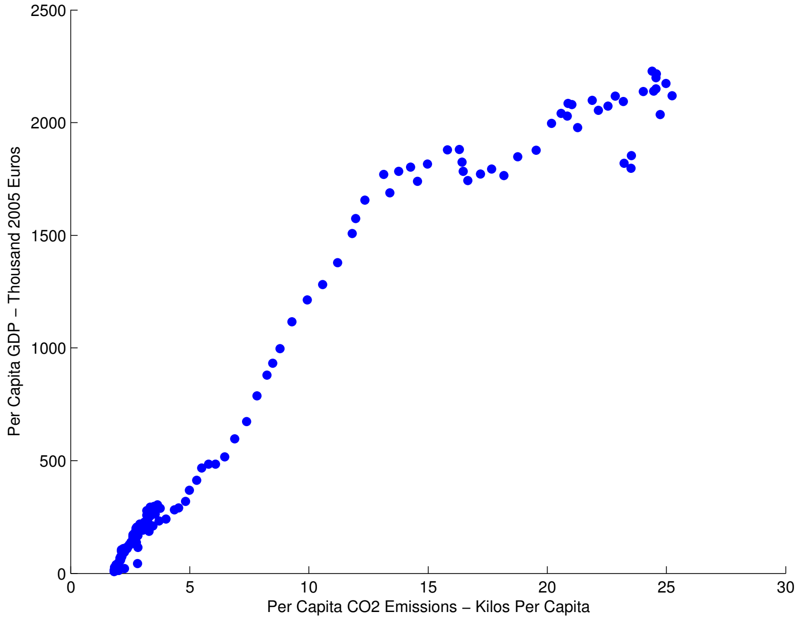


Figure 4: CO_2/GDP Ratio in Italy, 1861-2003 *(Metric Tons per Million of Euros)*

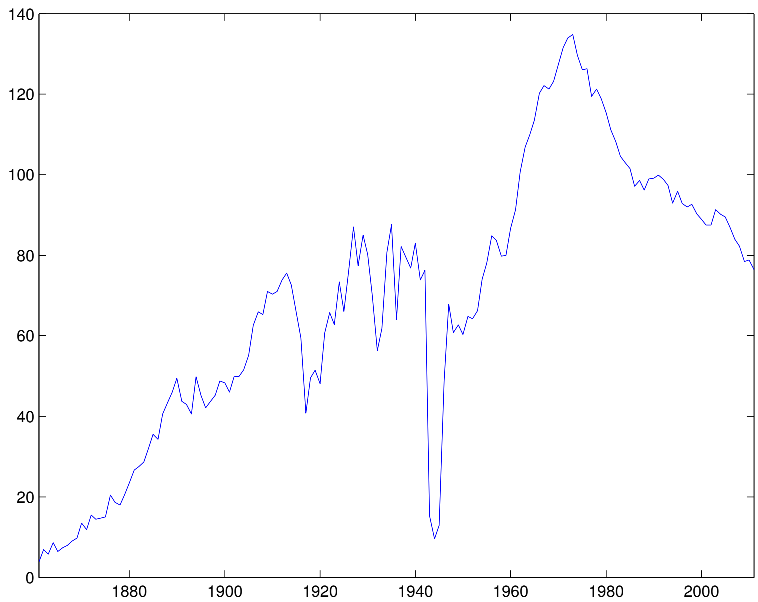


Figure 5: The Estimated Cointegrating Vector

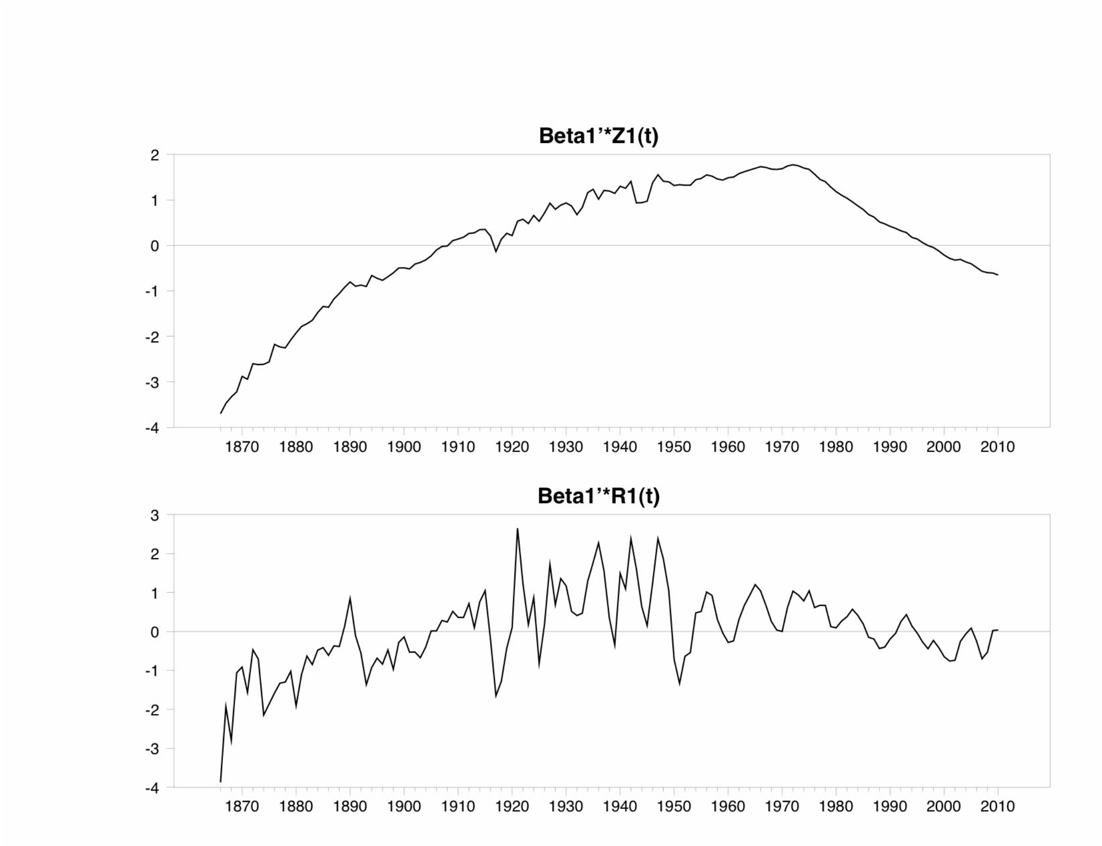
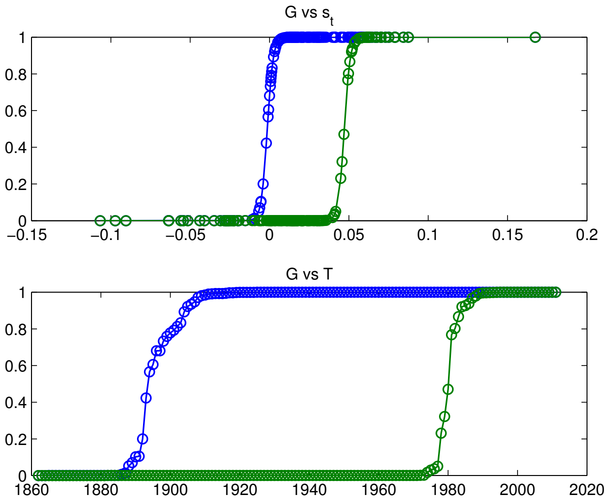
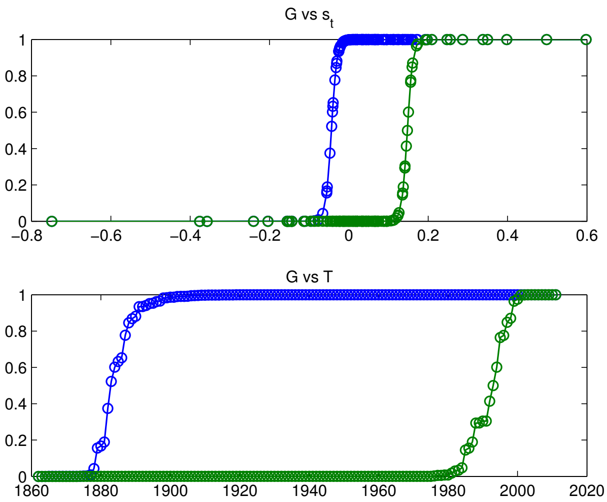


Figure 6: Estimated Transition Functions from MRSTAR model

(a) *GDP*



(b) CO_2



(c) *EI*

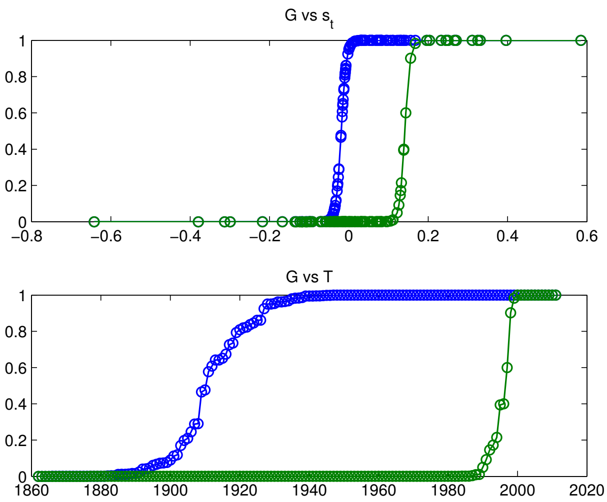


Figure 7: Estimated Transition Function from STVAR model

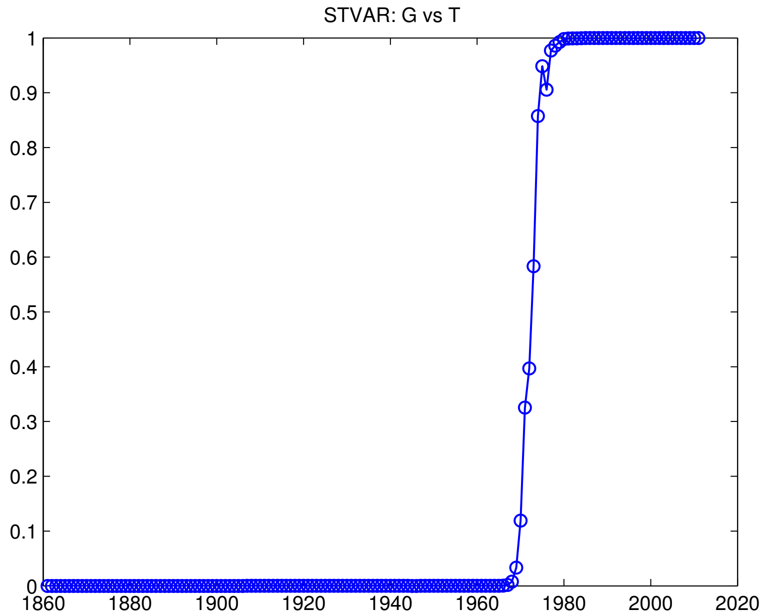


Figure 8: MS-VAR Results

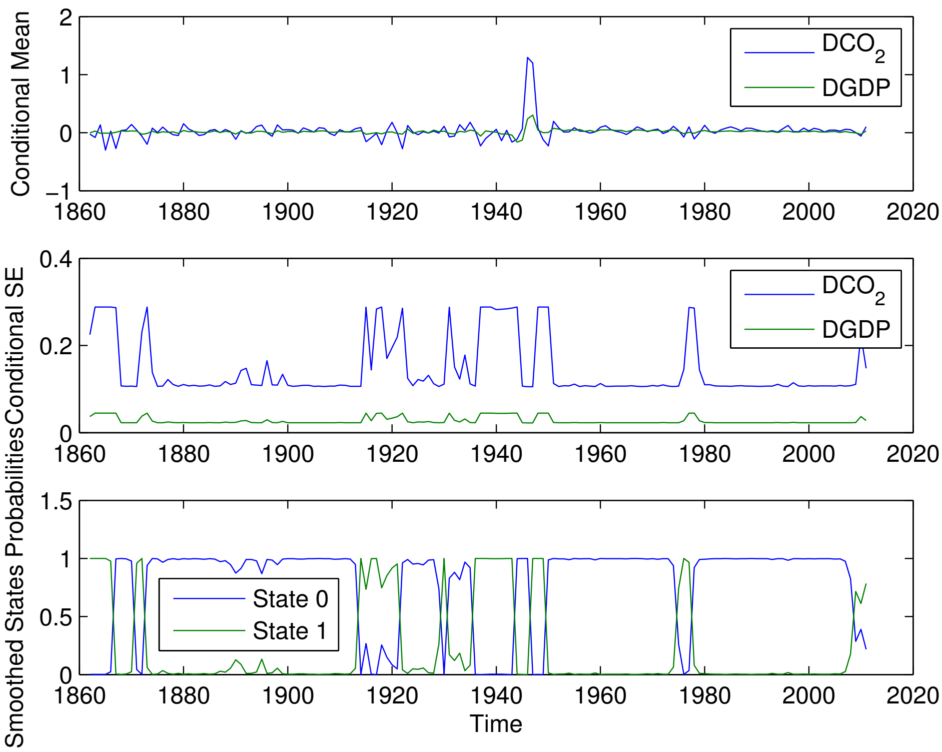


Figure 9: Fitted Quadratic Model

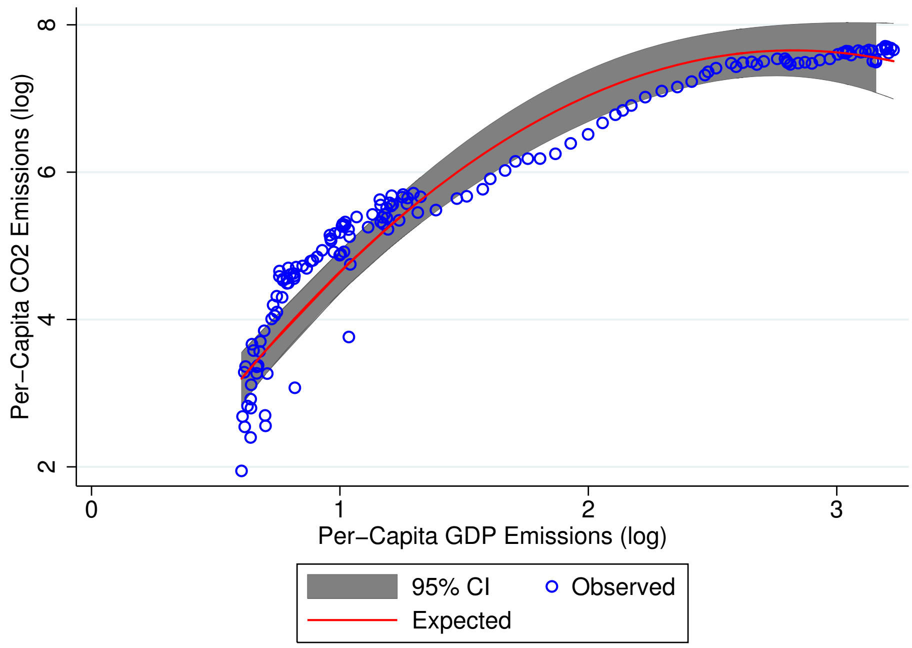


Figure 10: Quadratic Model Residuals

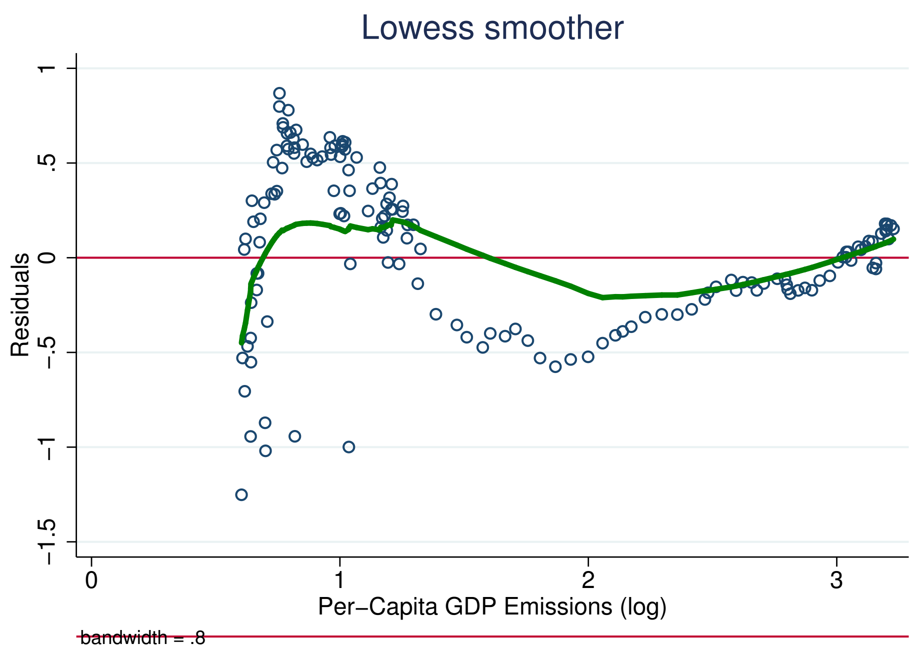


Figure 11: Fitted Cubic Spline

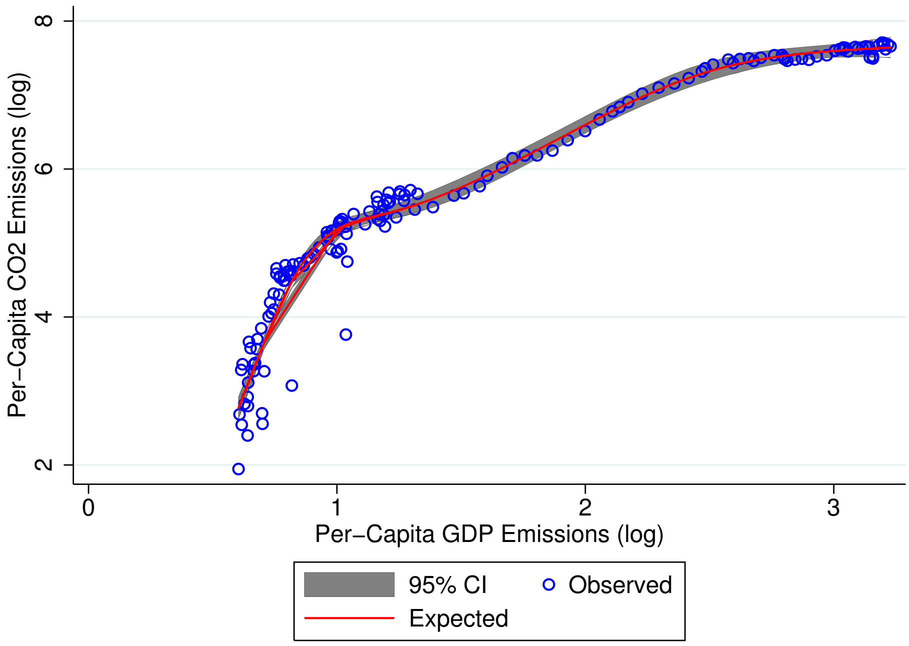
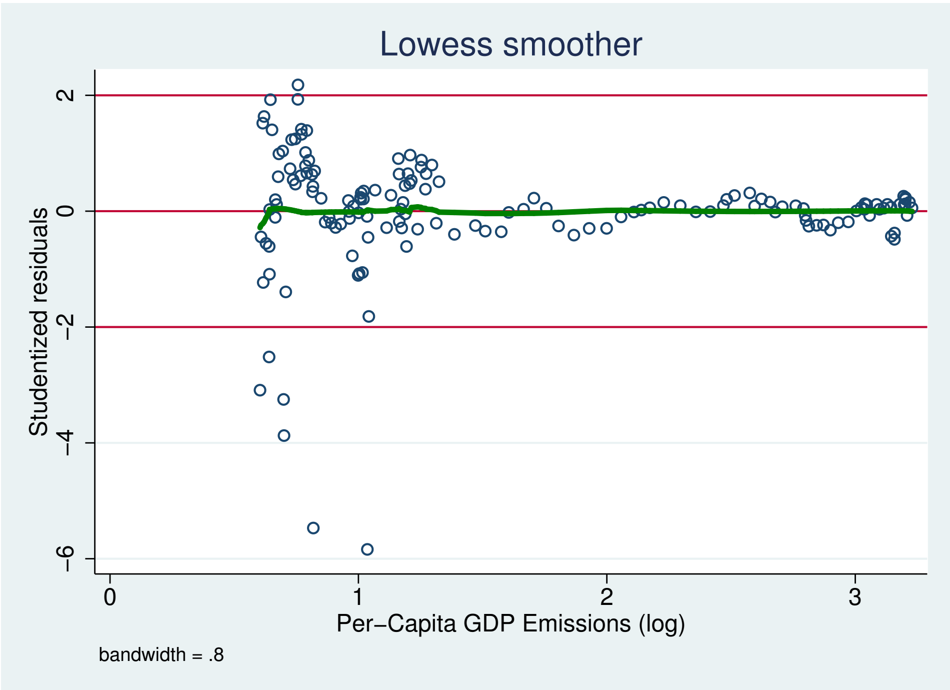


Figure 12: Cubic Spline Model Residuals



APPENDIX TO THE PAPER: “150 Years of Italian CO_2 Emissions and Economic Growth”

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A Statistical Models

A.1 CVAR

In order to model relations (1) or (2) in the main text, we first use a Cointegrated VAR model. This Subsection summarizes the analysis by (Juselius, 2006, CH 4, 10 and 11). Consider the bivariate vector $\boldsymbol{y_t} = [CO_{2,t}, GDP_t]'$ constituting the variables of interests. Then the VAR(p) model for this bivariate system is:

$$\boldsymbol{y_t} = \boldsymbol{\Pi_1 y_{t-1}} + \cdots + \boldsymbol{\Pi_p y_{t-p}} + \boldsymbol{\Phi D_t} + \boldsymbol{\epsilon_t} \quad t = 1, \ldots, T, \ \ \boldsymbol{\epsilon_t} \sim \mathcal{N}_p(\mathbf{0}, \boldsymbol{\Omega}) \tag{1}$$

The Error Correction form of model (1) is

$$\boldsymbol{\Delta y_t} = \boldsymbol{\Gamma_1^{(1)} \Delta y_{t-1}} + \boldsymbol{\Gamma_2^{(1)} \Delta y_{t-2}} + \cdots + \boldsymbol{\Gamma_{p-1}^{(1)} \Delta y_{t-p-1}} + \boldsymbol{\Pi y_{t-1}} + \boldsymbol{\Phi D_t} + \boldsymbol{\epsilon_t} \tag{2}$$

where: $\boldsymbol{\Gamma_1^{(1)}} = -(\boldsymbol{\Pi_2} + \boldsymbol{\Pi_3} + \ldots + \boldsymbol{\Pi_p})$, $\boldsymbol{\Gamma_2^{(1)}} = -(\boldsymbol{\Pi_3} + \cdots + \boldsymbol{\Pi_p})$ and $\boldsymbol{\Pi} = -(\boldsymbol{I} - \boldsymbol{\Pi_1} - \boldsymbol{\Pi_2} - \cdots - \boldsymbol{\Pi_p})$ are the short-run matrices and the long-run matrix, respectively where the integer (1) indicates the lag placement of ECM. Notice that $\boldsymbol{\Pi} = \boldsymbol{\alpha\beta'}$ is the reduced rank long-run matrix, with $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $p \times r$ matrices, $r \leq p$, $\boldsymbol{\Phi D_t} = \boldsymbol{\mu_0} + \boldsymbol{\mu_1} t$ are the unrestricted components (i.e. allowed to enter into the cointegrating relation) of deterministic trend. Equation (2) represents the CVAR model in Error Correction Form under I(1) hypothesis[1] and is re-written in the following reduced form:

$$\mathbf{Z}_{0t} = \boldsymbol{\alpha\beta'}\mathbf{Z}_{1t} + \boldsymbol{\Psi}\mathbf{Z}_{2t} + \boldsymbol{\epsilon_t}, \tag{3}$$

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[1] For further details, see Johansen (1991)

where: $\mathbf{Z}_{0t} = \Delta \mathbf{y}_t$, $\mathbf{Z}_{1t} = \mathbf{y}_{t-1}$, $\mathbf{Z}_{2t} = [\Delta \mathbf{y}'_{t-1}, \Delta \mathbf{y}'_{t-2}, \ldots, \Delta \mathbf{y}'_{t-p+1}, \Delta \mathbf{D}'_t]$. By Frisch-Waugh theorem, the coefficients estimators of the regressors in (3) are recovered by the following auxiliary regression:

$$\mathbf{Z}_{0t} = \hat{\mathbf{B}}_1' \mathbf{Z}_{2t} + \mathbf{R}_{0t} \tag{4}$$
$$\mathbf{Z}_{1t} = \hat{\mathbf{B}}_2' \mathbf{Z}_{2t} + \mathbf{R}_{1t} \tag{5}$$

where: $\hat{\mathbf{B}}_1' = \mathbf{M}_{02}' \mathbf{M}_{22}^{-1}$ and $\hat{\mathbf{B}}_2' = \mathbf{M}_{12}' \mathbf{M}_{22}^{-1}$ are OLS estimators and $\mathbf{M}_{ij} = \mathbf{\Sigma}_t (\mathbf{Z}_{it} \mathbf{Z}'_{jt})/T$. This makes us able to limit our attention on the concentrated (that is, cleaned of all short-run adjustments and deterministic kernel), more interpretable model:

$$\mathbf{R}_{0t} = \boldsymbol{\alpha\beta'} \mathbf{R}_{1t} + error \tag{6}$$

Equation (6) indicates that the process can be divided in two components, namely a stationary and a non-stationary, and these components can be analyzed via regression analysis. The peculiarity of this structure is the reduced-rank structure of $\mathbf{\Pi}$. One of the first problems to solve is the selection of the rank. This can be addressed by noting that the covariance matrix can be written as:

$$\mathbf{\Omega}(\beta) = \mathbf{S}_{00} - \mathbf{S}_{01} \beta (\beta' \mathbf{S}_{11} \beta)^{-1} \beta' \mathbf{S}_{10} \tag{7}$$

with $\mathbf{S}_{11} = T^{-1} \sum_{t=1}^{T} \mathbf{R_{1,t} R'_{1,t}}$, $\mathbf{S}_{01} = T^{-1} \sum_{t=1}^{T} \mathbf{R_{0,t} R'_{1,t}}$ and $\mathbf{S}_{00} = T^{-1} \sum_{t=1}^{T} \mathbf{R_{0,t} R'_{0,t}}$, so that the rank of $\mathbf{\Pi}$ and so the determinants of

$$\mathbf{S}_{00} \prod_{i=1}^{p} (1 - \lambda_i) \tag{8}$$

are the solution to the associated eigenvalue problem. The inference on the rank of $\mathbf{\Pi}$ corresponds to the hypothesis system:

$$H_0 : \text{rank} = p \;\text{ vs }\; H_1 : \text{rank} = p - r \tag{9}$$

and can be deduced by a simple Likelihood Ratio test with statistics

$$\tau_{p-r} = -T \ln(1 + \hat{\lambda}_{r+1}) \cdots (1 - \hat{\lambda}_p) \tag{10}$$

where $\hat{\lambda}_i$ is the i-esim estimated eigenvalue from $\boldsymbol{\Omega_t}$. The distribution of statistics (10) is non standard and has to be computed via simulation.
This framework allows to test for a number of hypotheses on the structure on α and β, which in turn are important for recover information about the common driving forces in the system. In particular, four hypotheses are important:

1. $H_{01} = \alpha = [\alpha_{\mathbf{1}} \;\; \mathbf{0}]'$;
2. $H_{02} = \alpha = [\mathbf{a}, \;\; \boldsymbol{\tau}]$;
3. $H_{03} = \mathbf{R_1}' \boldsymbol{\beta_1} = \mathbf{0}, \ldots, \mathbf{R_1}' \boldsymbol{\beta_r} = \mathbf{0}$;
4. $H_{04} = \boldsymbol{\beta^c} = [\text{b} \quad \phi]$;

In the first case, the $\alpha_{\mathbf{1}}$ represents the matrix of free parameters in α and the zero matrix indicates the null hypothesis that several cointegrating relation are spurious; thus, it corresponds to a test on weak exogeneity of some variables.
In the second case, $\boldsymbol{\tau}$ is a matrix of known numbers; since it is frequent that $r_k = 1$, it is commonly used as a test for unit vector in $\boldsymbol{\alpha}$ and thus it can be interpreted as a test on the null hypothesis that the long-run adjustment is purely adjusting.
In the third case, we are testing the hypothesis that any cointegrating relation identified by the restriction matrix $\mathbf{R_1}$ is null and, as a consequence, excludable from the system.
In the last case, we are testing the hypothesis that some of the $\boldsymbol{\beta}$ vector, defined by the n_k vector b are known (and so set at one) and $n - r_k$ unrestricted vectors ϕ. When this happens, the corresponding variable is stationary.

A.2 MR-STAR

To model for the change in the Italian economic structure during the 150 years of our sample we use the MR-STAR model. In particular, we consider the general additive non-linear model:

$$y_t = \boldsymbol{\phi}'\mathbf{z_t} + \boldsymbol{\theta}'\mathbf{z_t}\sum_{m=1}^{M} G(\boldsymbol{\gamma}, \mathbf{c}, s_t) + \epsilon_t \tag{11}$$

where y_t is the dependent variable, $\mathbf{z_t} = (1, y_1, \ldots, y_{t-p})'$, $\boldsymbol{\phi} = (\phi_0, \phi_1, \ldots, \phi_p)'$, $\boldsymbol{\theta} = (\theta_0, \theta_1, \ldots, \theta_p)'$ are parameter vectors, and $\epsilon_t \sim i.i.d.(0, \sigma^2)$. The transition function $G(\boldsymbol{\gamma}, \boldsymbol{c}, s_t)$ is a continuous function in the transition variable s_t, where the parameter vector $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_m, \ldots \gamma_M)$ controls the velocity of the M transitions with $\mathbf{c} = (c_1, \ldots, c_m, \ldots c_M)$ assumed as a vector of transition parameters. In what follows we suppose that the transition variable coincides with a lagged value of the endogenous variable y_t with lag denoted by delay $d > 0$.
One of the commonly used functions for $G(\cdot)$ is the (first order) logistic function:

$$G(\boldsymbol{\gamma}, \boldsymbol{c}, s_t) = \left(1 + exp\left\{-\gamma_M \prod_{k=1}^{K}(s_t - c_m)\right\}\right)^{-1}, \quad \gamma > 0, \tag{12}$$

where $\gamma_m > 0$ and $c_1 < \cdots < c_m < \cdots < c_M$ are identifying restrictions. Equations (12) and (11) define the first-order (Multiple-Regime) Logistic STAR (MR-LSTAR1) model. The most common choice is to set alternatively $K = 1$, whether the parameters $\boldsymbol{\phi} + \boldsymbol{\theta}G(\gamma, \boldsymbol{c}, s_t)$ change monotonically as a function of s_t from $\boldsymbol{\phi}$ to $\boldsymbol{\phi} + \boldsymbol{\theta}$, and $K = 2$, in case the parameters $\boldsymbol{\phi} + \boldsymbol{\theta}G(\gamma, \boldsymbol{c}, s_t)$ change symmetrically around the mid-point $(c_1 + c_2)/2$, where the logistic function attains its minimum, $min_G G(\cdot) \in [0, 1/2]$, that is:

$$min_G G(\cdot) = \begin{cases} 0 & if \ \gamma \to \infty \\ 1/2 & if \ c_1 = c_2 \ and \ \gamma < \infty \end{cases}$$

If $\gamma_m = 0$ the transition function will be $G(\gamma_m, \boldsymbol{c}, s_t) \equiv 1/2$, so that model (11) will nest a linear model. When $\gamma_m \to \infty$ the model (11) nests a SETAR model (Tong, 1983):

$$y_t = \sum_{j=1}^{r+1} (\boldsymbol{\phi}_j' \boldsymbol{y_t}) I(y_{t-d} \le c_j) + \sum_{j=1}^{r+1} (\boldsymbol{\phi}_j' \boldsymbol{y_t}) I(y_{t-d} > c_j) + \epsilon_{jt} \tag{13}$$

where $\boldsymbol{\phi}, \boldsymbol{y_t}$ are defined as before, s_t is a continuous switching random variable, $c_0, c_1, \ldots, c_{r+1}$ are threshold parameters, $c_0 = -\infty$, $c_{r+1} = +\infty$, $\epsilon_{jt} \sim i.i.d.(0, \sigma_j^2)$, $j = 1, \ldots, r$. The multiple regime hypothesis is investigated via LM test, and the most likely number of regimes can be obtained by iteration.

The linearity test is based on a third-order Taylor expansion of the transition function (12), $T_3(z) = g_1 z + g_3 z^3$ where $g_1 = \partial G/\partial z_{|z=0}$ and $g_3 = (1/6)\partial^3 G/\partial z^3_{|z=0}$, so that the approximation, $y_t = \boldsymbol{\phi'}\mathbf{z_t} + \boldsymbol{\theta'}\mathbf{z_t} T_3(\gamma(y_{t-d} - c)) + \boldsymbol{\epsilon}_t$, leads to the auxiliary regression:

$$\hat{\boldsymbol{\epsilon}}_t = \hat{\mathbf{z}}_{\mathbf{1t}}' \tilde{\boldsymbol{\beta}}_{\mathbf{1}} + \sum_{j=1}^{p} \beta_{2j} y_{t-j} y_{t-d} + \sum_{j=1}^{p} \beta_{3j} y_{t-j} y_{t-d}^2 + \sum_{j=1}^{p} \beta_{4j} y_{t-j} y_{t-d}^3 + v_t' \tag{14}$$

The null hypothesis for linearity against LSTAR is $H_0 : \beta_{2j} = \beta_{3j} = \beta_{4j} = 0$, $j = 1, \cdots, p$, which, under the condition that a linear autoregressive model holds and $E\epsilon_t < \infty$, is tested by statistics:

$$LM = (SSR_0 - SSR)/\hat{\sigma}^2 \sim \chi^2(3p) \tag{15}$$

where SSR are the sum of squared residuals from equation (14) or, alternatively, by setting the artificial model $y_t = g_1\gamma_0 + \gamma_1' \boldsymbol{z}_t + \gamma_2'(\mathbf{z}_t y_{t-d}) + \gamma_3'(\mathbf{z}_t y_{t-d}^2) + \gamma_4'(\mathbf{z}_t y_{t-d}^3) + v_t''$ where $v'' \sim nid(0, \sigma^2_{v''})$, $\gamma_j = (\gamma_{1j}, \cdots, \gamma_{jp})'$, and $j = 1, \cdots, 4$, and $H_0 : \gamma_2 = \gamma_3 = \gamma_4 = 0$. In terms of Taylor approximation we get:

$$\begin{aligned}
\gamma_2 &= g_1 \gamma \hat{\boldsymbol{\theta}} + 3 g_3 \gamma^3 c^2 \hat{\boldsymbol{\theta}} - 3 g_3 \gamma^3 c \theta_0 \boldsymbol{e}_d \\
\gamma_3 &= -3 g_3 \gamma^3 c \hat{\boldsymbol{\theta}} + g_3 \gamma^3 \theta_0 \boldsymbol{e}_d \\
\gamma_4 &= g_3 \gamma^3 \hat{\boldsymbol{\theta}}
\end{aligned} \tag{16}$$

where $\hat{\boldsymbol{\theta}}$ and $\mathbf{c}$ and d are previously defined. Similarly, if the model is an ESTAR(p) model, $\hat{\mathbf{z}}_{1t} = -\boldsymbol{z}_t$ and $\hat{\mathbf{z}}_{2t}(\boldsymbol{\pi}) = -(y_{t-d} - c)^2(\hat{\boldsymbol{\theta}}'\mathbf{z_t}) = -(\bar{\boldsymbol{\theta}}'\mathbf{z_t} y_{t-d}^2 + \theta_0 y_{t-d}^2 - 2c\bar{\boldsymbol{\theta}}'\mathbf{z}_t y_{t-d} + c^2\bar{\boldsymbol{\theta}}'\mathbf{z_t} - 2c\theta_0 y_{t-d} + c^2\theta_0)$. This yields the following auxiliary regression:

$$\hat{v}_t = \tilde{\boldsymbol{\beta}}_1' \hat{\mathbf{z}}_{1t} + \boldsymbol{\beta}_2' \mathbf{z}_t y_{t-d} + \boldsymbol{\beta}_3' \mathbf{z} y_{t-d}^2 + e_t' \tag{17}$$

where $\hat{v}_t$ is the analogue of $\hat{\boldsymbol{\epsilon}}_t$, e_t' is an error term and $\tilde{\boldsymbol{\beta}}_1 = (\beta_{10}, \boldsymbol{\beta_1'})'$ with $\beta_{10} = \phi_0 - c^2\theta_0$ and $\boldsymbol{\beta}_1 = \bar{\boldsymbol{\phi}} - c^2\bar{\theta} + 2c\theta_0\boldsymbol{e}_d$, $\boldsymbol{\beta_2} = 2c\bar{\theta} - \theta_0\boldsymbol{e}_d$. The null of linearity is $H_0' : \boldsymbol{\beta}_2 = \boldsymbol{\beta}_3 = 0$ which is tested by statistics

$$LM' = (SSR_0 - SSR)/\hat{\sigma}^2 \sim \chi^2(p) \tag{18}$$

where SSR is the sum of squared residuals from (17). In order to choose the correct transition function a nested hypothesis test (the so called "Teräsvirta rule") is adopted:

$$\begin{aligned}
&H_{04} : \gamma_4 = 0 \;\text{ against }\; H_{14} : \gamma_4 \neq 0 \;\text{ in }\; (16). \\
&H_{03} : \gamma_3 = 0 \mid \gamma_4 = 0 \;\text{ against }\; H_{13} : \gamma_3 \neq 0 \mid \gamma_4 = 0 \;\text{ in }\; (16). \\
&H_{02} : \gamma_2 = 0 \mid \gamma_3 = \gamma_4 = 0 \;\text{ against }\; H_{12} : \gamma_2 \neq 0 \mid \gamma_3 = \gamma_4 = 0 \;\text{ in }\; (16).
\end{aligned} \tag{19}$$

If the p-value of H_{03} is the smallest of the three, we select an ESTAR model, otherwise we select an LSTAR model.
When the time series is short and the number of lags high, the LM statistics has poor power properties. This problem can be addressed by the following procedure for carrying an equivalent F-test:

1. Estimate the GSTR model under the assumption of uncorrelated errors and compute the residual sum of squares $SSR_0 = \sum_{t=1}^{T} \hat{u}_t^2$.

2. Regress $\hat{u}_t$ on $\hat{v}_t$, $\mathbf{z_t}$, $\mathbf{z_t}\hat{G}(\mathbf{z_{t-d}})$, $\hat{G}_\gamma$, $\hat{G}_c$ and compute SSR;

3. Compute the test statistics

$$F_{LM} = \frac{SSR_0 - SSR}{3p} / \frac{SSR}{T-n-3p}, \tag{20}$$

where $n = \dim(\hat{\mathbf{z}}_\mathbf{t})$.

The test for serial independence assumes that the model's error term in (11) has the following structure:

$$\epsilon_t = a'v_t + u_t = \sum_{j=1}^{q} a_j L^j \epsilon_t + u_t, \quad u_t \sim N.I.D.(0, \sigma^2), \tag{21}$$

with L^j denoting the lag operator, $v_t = (u_{t-1}, \ldots, u_{t-q})'$, $a = (a_1, \ldots, a_q)'$, $a_q \neq 0$. Under the assumption of stationarity and ergodicity, the null hypothesis of serial independence is $H_0 : a = 0$. The resulting LM statistics is shown to be:

$$LM = \frac{1}{\hat{\sigma}} \left(\hat{\mathbf{u}}_\mathbf{t}' \hat{\mathbf{v}}_\mathbf{t} \right) \left\{ \hat{\mathbf{v}}_\mathbf{t}' \hat{\mathbf{v}}_\mathbf{t} - \hat{\mathbf{v}}_\mathbf{t}' \hat{\mathbf{z}}_\mathbf{t} \left(\hat{\mathbf{z}}_\mathbf{t} \hat{\mathbf{z}}_\mathbf{t}' \right)^{-1} \hat{\mathbf{z}}_\mathbf{t}' \hat{\mathbf{v}}_\mathbf{t} \right\}^{-1} \left(\hat{\mathbf{v}}' \hat{\mathbf{u}}_\mathbf{t} \right), \tag{22}$$

where $\hat{\mathbf{u}}_\mathbf{t} = (\hat{\mathbf{v}}_\mathbf{t-1}, \ldots, \hat{\mathbf{v}}_\mathbf{t-q})'$, $\hat{\mathbf{v}}_\mathbf{t-j} = y_{t-j} - \boldsymbol{\phi}'\mathbf{z}_\mathbf{t-j} - \boldsymbol{\theta}' G(\mathbf{z}_\mathbf{t-j}, \hat{\boldsymbol{\Xi}})$, $j = 1, \ldots, q$, $\hat{\boldsymbol{\Xi}}$ is the estimates of $\boldsymbol{\Xi}$ and $\hat{\mathbf{z}}_\mathbf{t} = \frac{\partial G(\mathbf{z_t}, \hat{\boldsymbol{\Xi}})}{\partial \hat{\boldsymbol{\Xi}}} = [\boldsymbol{\theta}'\mathbf{z_t} G_\gamma, \boldsymbol{\theta}'\mathbf{z_t} G_{\gamma c}]$ and $\hat{\sigma}^2 = \frac{1}{T}\sum_t u_t^2$. Under the null hypothesis, statistics (22) is asymptotically χ_q^2 distributed, or equivalently, the statistics (20) with q and $T-n-q$ degrees of freedom.
The test for no additive nonlinearity assumes the following model:

$$y_t = \boldsymbol{\phi}'\mathbf{z_t} + \boldsymbol{\theta}'\mathbf{z_t} G_1(\gamma^{\{1\}}, \boldsymbol{c}, s_t) + \boldsymbol{\pi}'\mathbf{z_t} G_2(\gamma^{\{2\}}, \boldsymbol{c}, s_t) + u_t, \tag{23}$$

with $u_t \sim iid\ (0, \sigma^2)$. The null of no neglected nonlinearity is:

$$H_0 : \gamma^{\{2\}} = 0 \quad \text{vs} \quad H_0 : \gamma^{\{2\}} > 0. \tag{24}$$

The test is implemented with the same procedure for serial correlation, the F-test has $(6p)$ and $(T-n-6p)$ degrees of freedom and the Teräsvirta rule can be applied to the Taylor-expanded version of (23) in order to select the form of the transition.
The test for parameter constancy assumes the model:

$$y_t = \boldsymbol{\phi(t)}'\bar{\mathbf{z}}_\mathbf{t} + \boldsymbol{\theta(t)}'\tilde{\mathbf{z}}_\mathbf{t} G(\gamma, \mathbf{c}, s_t) + u_t, \quad u_t \sim iid\ \left(0, \sigma^2\right), \tag{25}$$

with $\bar{\mathbf{z}}_{\mathbf{t}}$ denoting the $k \leq p+1$ element of $\mathbf{z_t}$ for which the corresponding element of ϕ is not assumed zero a priori, $\tilde{\mathbf{z}}_{\mathbf{t}}$ is the same $(l \times 1)'$ for the element of $\boldsymbol{\theta}$. Let $\tilde{\boldsymbol{\phi}}$ and $\tilde{\boldsymbol{\theta}}$ denote the equivalent $(k+1)$ and $(l+1)$ parameter vectors, $\boldsymbol{\phi(t)} = \tilde{\boldsymbol{\phi}} + \lambda_1 G_j(t; \boldsymbol{\gamma}, \mathbf{c}, s_t)^{(1)})$, and $\boldsymbol{\theta(t)} = \tilde{\boldsymbol{\theta}} + \lambda_2 G_j(t; \gamma, \mathbf{c}, s_t)$ with λ_1 and λ_2 being a $(k \times 1)$ and $(l \times 1)$ vectors, respectively. Then the null of parameter constancy in (25) is

$$H_0 : G_j(t; \gamma, \mathbf{c}, s_t) \equiv 0 \;\; (\text{or} \equiv \text{const}). \tag{26}$$

Three forms for G_j can be considered: the Logistic and Exponential smooth transition of the change in parameters (labeled as G_1 and G_2) and a cubic function (G_3) which allows for both monotonically and non-monotonically changing parameters and can be seen as a general case of G_1 and G_2 when building up a test. Under H_0, the statistics (20) has a χ^2 distribution with $3(k+l)$ degrees of freedom and the equivalent F-distribution has $3(k+l)$ and $T - 4(k+l) - 2$ degrees of freedom.

A.3 STVAR

Let consider the $T \times k$ vector $\boldsymbol{y_t}$ defined in A.1. Then the most immediate strategy to model the nonlinear behavior of the bivariate system is to assume a multivariate version of STAR model (11) (STVAR), which can be written as:

$$\boldsymbol{y_t} = \boldsymbol{\Phi}'\mathbf{z_t} + \boldsymbol{\Theta}'\mathbf{z_t} G(\gamma, \boldsymbol{c}, s_t) + \boldsymbol{\epsilon_t}, \quad \boldsymbol{\epsilon_t} \sim iid(\mathbf{0}, \Sigma_2^2) \tag{27}$$

where $G(s_t, \boldsymbol{c}, \boldsymbol{\gamma})$ is already defined, $\boldsymbol{\gamma}$ and $\boldsymbol{c}$ presents the same identifying restrictions. The modelling strategy is conceptually the same as the univariate model. The linearity test is the statistics (15), but with $3pk$ and $T - 3pk - n$ degrees of freedom (k and $T - pk - n$ if we refer to statistics (18)).

A.4 MS-VAR

The MR-STAR model assumes that transition between regimes is observed. This assumption can be removed by using a Markov Chain structure in the transition between the same (multiple) regimes. To this scope we use a Markov-Switching VAR model[2], having the p-th order autoregression for the K-dimensional time series vector $y_t = (y_{1t}, \ldots, y_{Kt})$, $t = 1, \ldots, T$,

$$\boldsymbol{y}_t = \boldsymbol{\mu}_0 + \boldsymbol{\Pi}_1(s_t) y_{t-1} + \cdots + \boldsymbol{\Pi}_p(s_t) y_{t-p} + \boldsymbol{u}_t, \tag{28}$$

where $\boldsymbol{\Pi}$ is defined as in the subsection A.1 with no interest for the $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ partition, $\boldsymbol{u}_t \sim IID(\mathbf{0}, \boldsymbol{\Sigma})$ and $y_0, \ldots, y_{1-p}$ are fixed, $s_t \in (1, \ldots, M)$ is the unobservable regime variable representing the probability of being in a different state of the world, which is governed by a discrete time, a discrete state, and a irreducible ergodic M state Markov process with the transition probabilities matrix defines as:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1M} \\ p_{21} & p_{22} & \cdots & p_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ p_{M1} & p_{M2} & \cdots & p_{MM} \end{bmatrix} \tag{29}$$

[2]See also Krozlig (1997).

where p_{ij} the probability of switching from state i to state j, that is

$$p_{ij} = Pr(s_{t+1} = j | s_t = i), \quad \sum_{j=1}^{M} p_{ij} = 1 \ \forall i, j \in \{1, \ldots, M\} \tag{30}$$

Denoting $A(L) = \mathbf{I}_K - \mathbf{\Pi}_1 L - \ldots, -\mathbf{\Pi}_p L^p$ as the $(K \times K)$ dimensional lag polynomial, we assume that there are no roots on or inside the unit circle $|\mathbf{\Pi}(z)| \neq 0$ for $|z| \leq 1$ where L is the lag operator, so that $y_{t-j} = L^j y_t$. If a normal distribution of the error is assumed, $u_t \sim NID(0, \Sigma(s_t))$, equation (28) is known as the intercept form of a stable Markov Switching Gaussian VAR(p) model.

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